Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

5-Inverse-trig-functions/5.3-Inverse-tangent/151-5.3.5-u-a+b-arctan-c+d-x-^p

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December 9, 2023 Compiled on December 9, 2023 at 6:46am

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [70]. This is test number [151].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (70)	0.00 (0)
Maple	98.57 (69)	1.43 (1)
Mathematica	95.71 (67)	4.29 (3)
Maxima	52.86 (37)	47.14 (33)
Mupad	42.86 (30)	57.14 (40)
Fricas	40.00 (28)	60.00 (42)
Sympy	32.86 (23)	67.14 (47)
Giac	12.86 (9)	87.14 (61)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

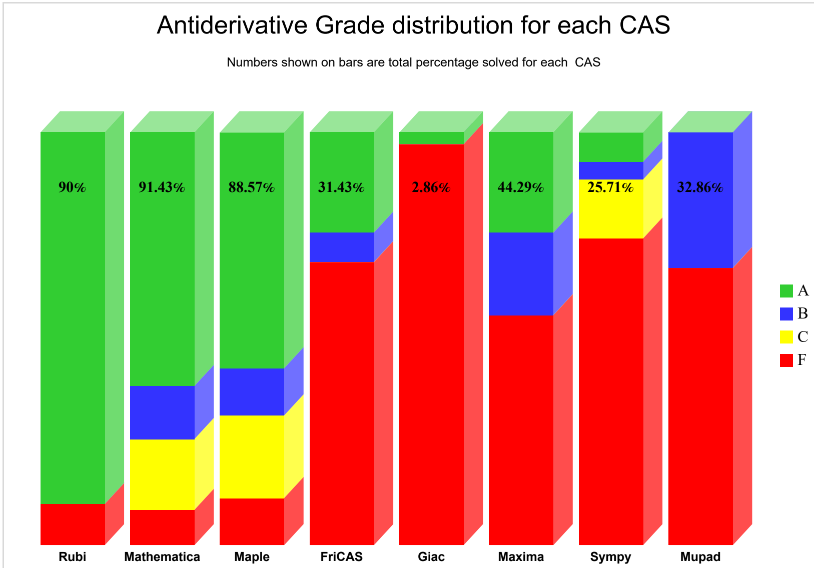
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

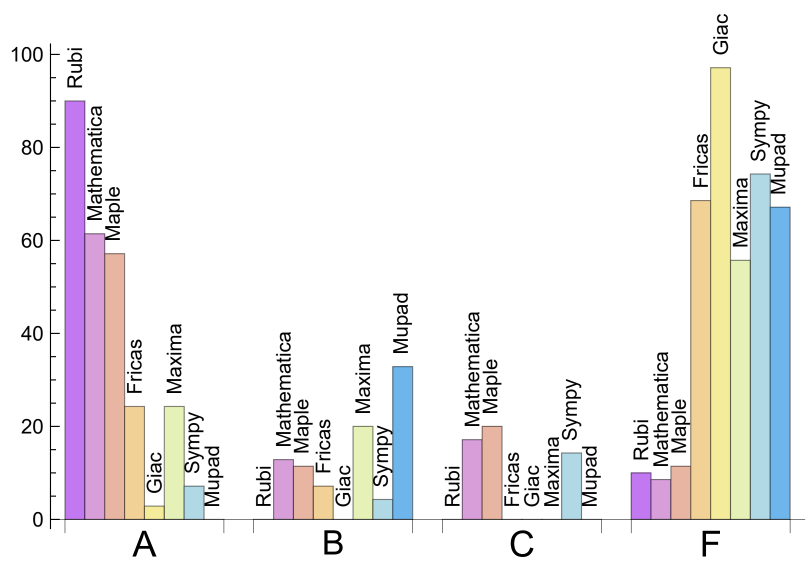
System	% A grade	% B grade	% C grade	% F grade
Rubi	90.000	0.000	0.000	10.000
Mathematica	61.429	12.857	17.143	8.571
Maple	57.143	11.429	20.000	11.429
Fricas	24.286	7.143	0.000	68.571
Maxima	24.286	20.000	0.000	55.714
Sympy	7.143	4.286	14.286	74.286
Giac	2.857	0.000	0.000	97.143
Mupad	0.000	32.857	0.000	67.143

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	3	100.00	0.00	0.00
Rubi	0	0.00	0.00	0.00
Maple	1	100.00	0.00	0.00
Maxima	33	93.94	0.00	6.06
Mupad	40	0.00	100.00	0.00
Fricas	42	97.62	0.00	2.38
Sympy	47	40.43	59.57	0.00
Giac	61	75.41	21.31	3.28

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.31
Rubi	0.60
Maxima	1.02
Mathematica	1.03
Mupad	1.47
Maple	1.50
Sympy	15.01
Giac	90.38

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Giac	9.78	0.30	3.00	0.15
Fricas	140.75	1.45	81.00	1.25
Mupad	178.37	1.75	102.50	1.35
Rubi	228.77	0.98	153.00	0.99
Sympy	236.30	2.65	168.00	1.98
Mathematica	261.99	1.42	163.00	1.08
Maple	668.71	2.40	186.00	1.05
Maxima	1036.14	4.56	165.00	1.42

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

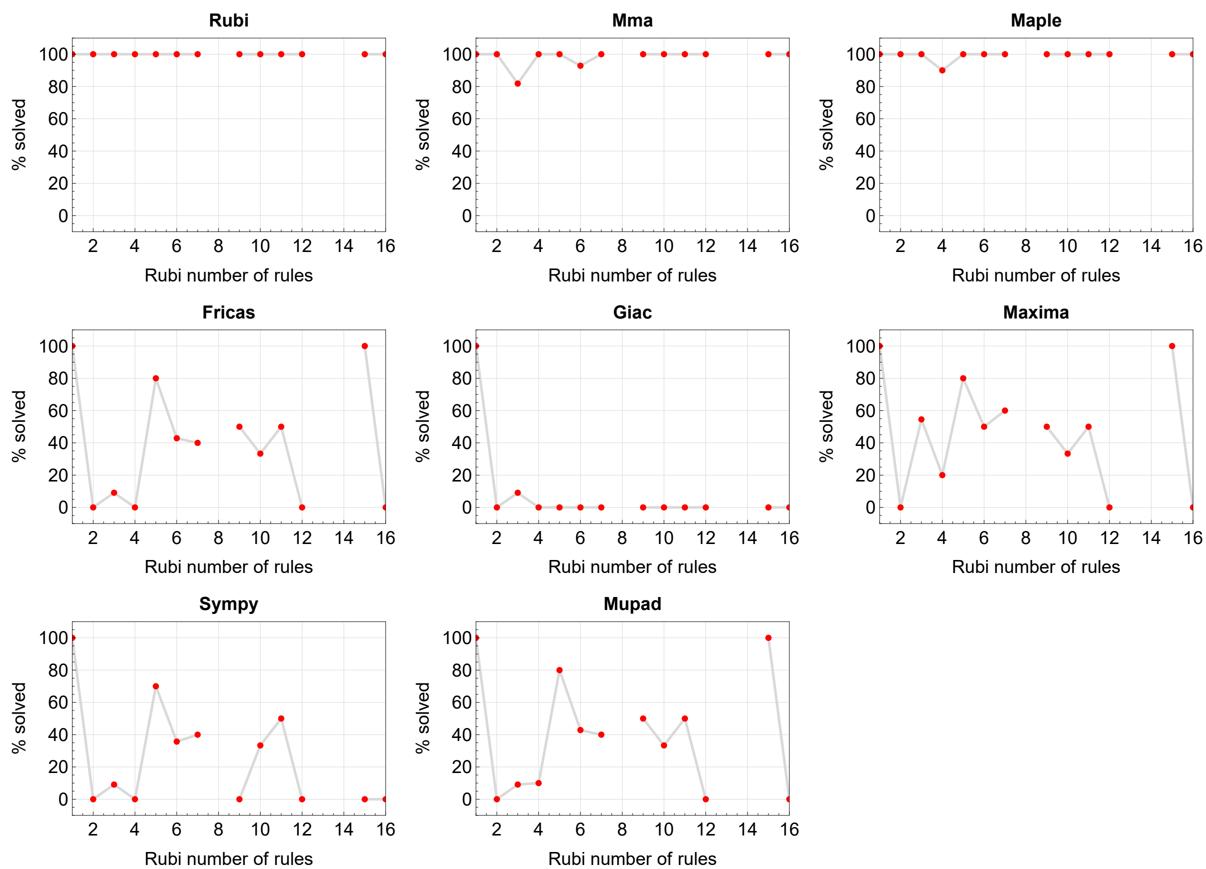


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

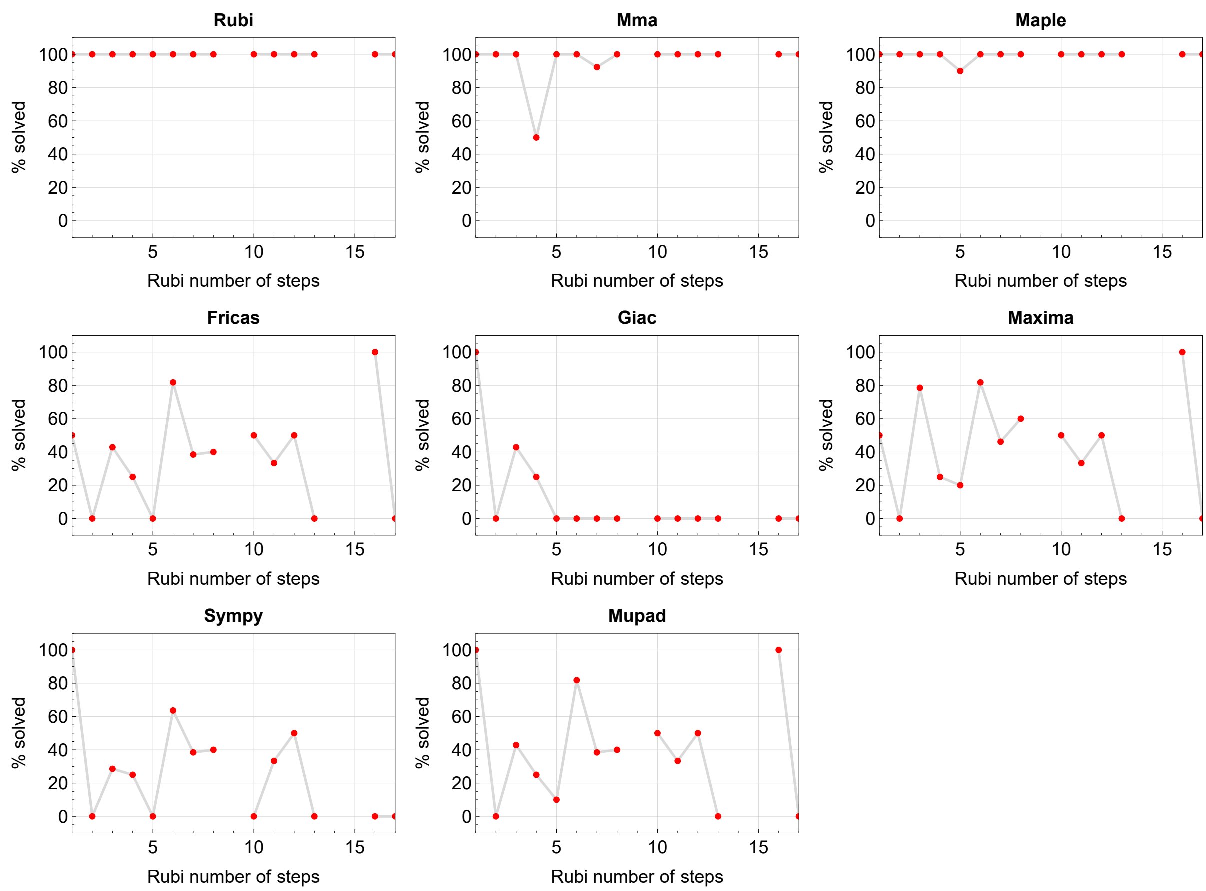


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

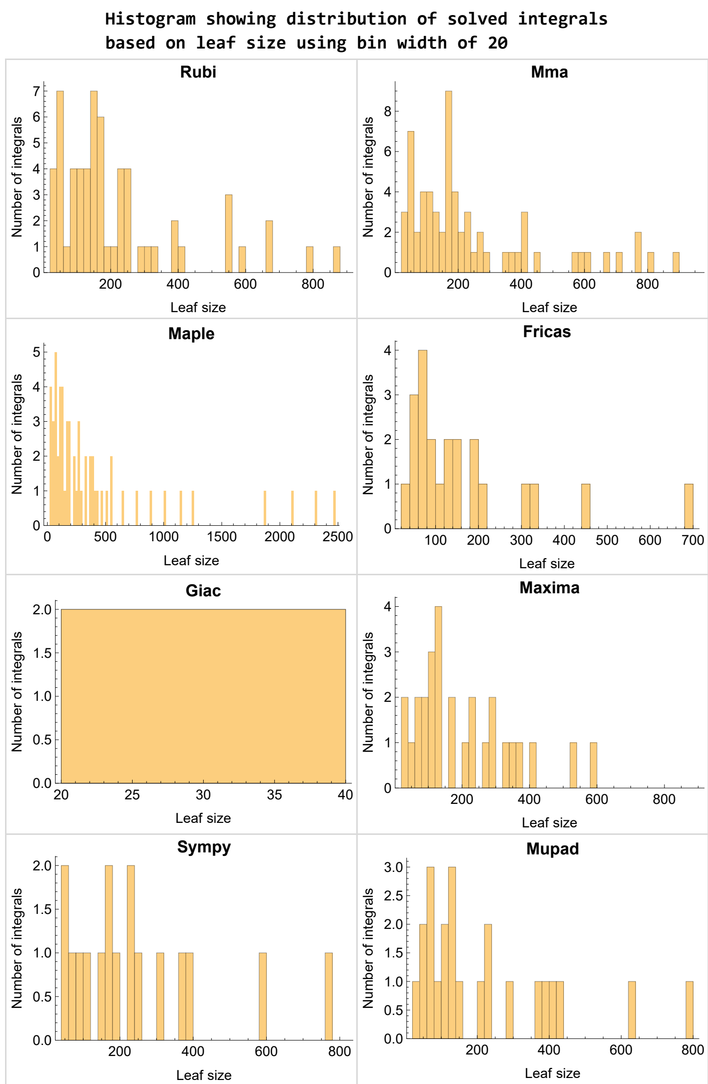


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

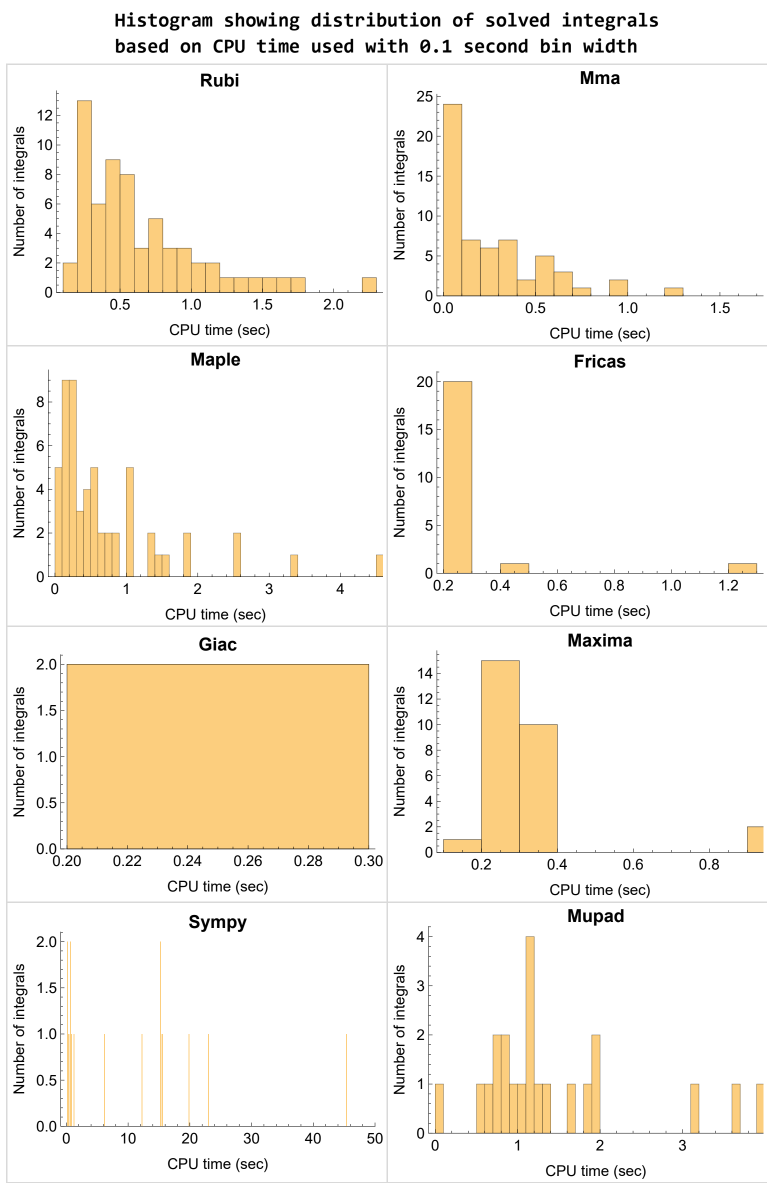


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

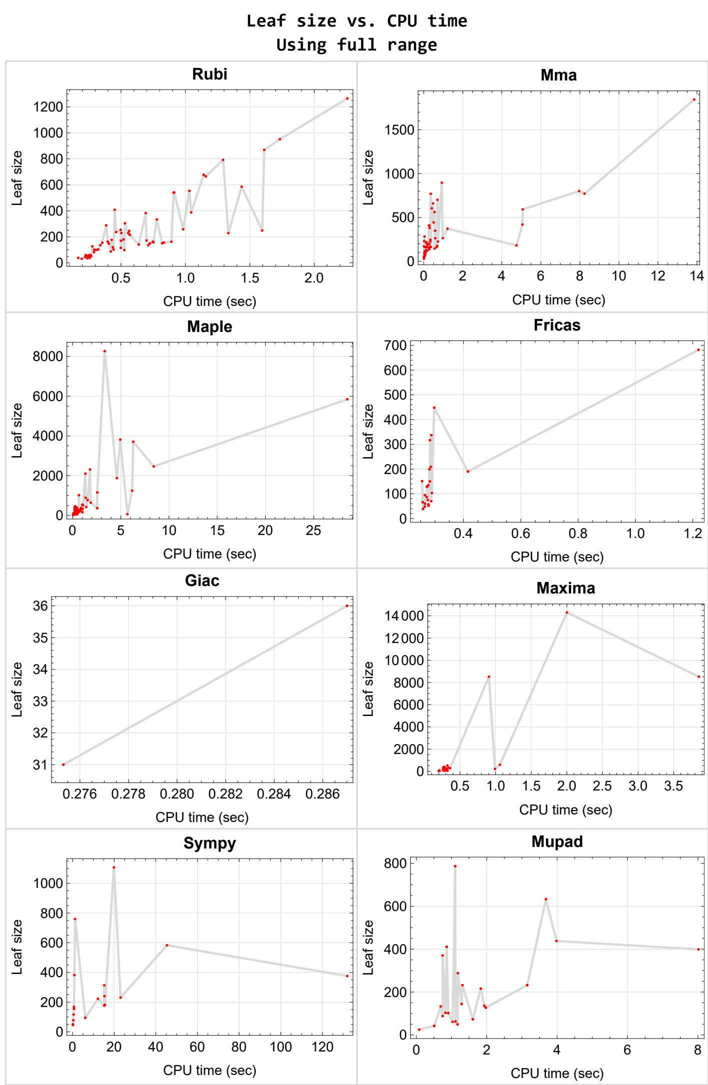


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{23, 42, 43, 65, 66, 69, 70\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{65, 66, 69, 70\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{2, 7\}$

Mathematica $\{36\}$

Maple $\{10, 15, 17, 18, 20, 34, 36, 37, 39, 40, 52, 57, 59\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

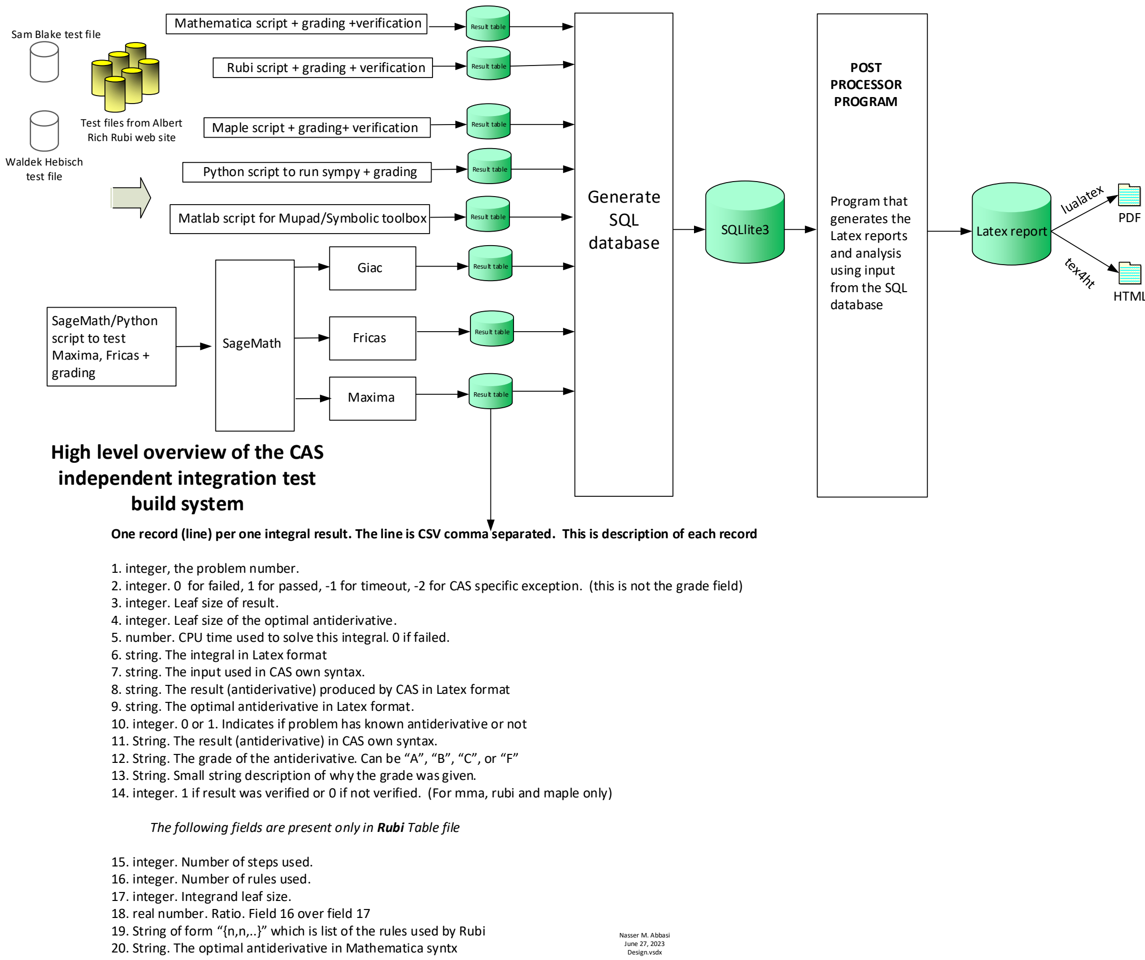
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 27, 28, 32, 33, 35, 37, 38, 41, 47, 48, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68 }

B grade { 10, 17, 31, 36, 55, 65, 66, 69, 70 }

C grade { 6, 24, 25, 26, 29, 30, 44, 45, 46, 49, 50, 51 }

F normal fail { 34, 39, 40 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 14, 21, 22, 25, 26, 27, 28, 29, 30, 32, 33, 35, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 60, 61, 63, 64, 67, 68 }

B grade { 11, 13, 16, 19, 24, 31, 38, 62 }

C grade { 10, 15, 17, 18, 20, 34, 36, 37, 39, 40, 52, 57, 58, 59 }

F normal fail { 41 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 3, 5, 6, 9, 12, 24, 25, 26, 27, 29, 44, 45, 46, 47, 49, 50, 51 }

B grade { 1, 2, 7, 14, 30 }

C grade { }

F normal fail { 4, 8, 10, 11, 13, 15, 16, 17, 18, 19, 20, 21, 22, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68 }

F(-1) timedout fail { }

F(-2) exception fail { 23 }

2.1.5 Maxima

A grade { 5, 24, 25, 26, 27, 29, 30, 44, 45, 46, 47, 48, 49, 50, 51, 55, 60 }

B grade { 1, 2, 3, 6, 7, 9, 12, 14, 21, 22, 53, 54, 56, 61 }

C grade { }

F normal fail { 4, 8, 10, 11, 13, 15, 16, 17, 18, 19, 20, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 52, 57, 58, 59, 63, 64, 67, 68 }

F(-1) timedout fail { }

F(-2) exception fail { 23, 62 }

2.1.6 Giac

A grade { 27, 47 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 15, 16, 21, 22, 24, 25, 26, 28, 29, 30, 31, 32, 33, 36, 37, 38, 41, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68 }

F(-1) timedout fail { 10, 11, 12, 13, 14, 17, 18, 19, 20, 34, 35, 39, 40 }

F(-2) exception fail { 58, 59 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 5, 6, 7, 9, 12, 14, 21, 24, 25, 26, 27, 29, 30, 44, 45, 46, 47, 49, 50, 51 }

C grade { }

F normal fail { }

F(-1) timedout fail { 4, 8, 10, 11, 13, 15, 16, 17, 18, 19, 20, 22, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 27, 44, 45, 46, 47 }

B grade { 1, 3, 6 }

C grade { 2, 5, 7, 9, 12, 25, 26, 49, 50, 51 }

F normal fail { 4, 8, 10, 11, 13, 15, 16, 17, 18, 19, 20, 21, 22, 32, 33, 37, 38, 60, 63 }

F(-1) timedout fail { 14, 24, 28, 29, 30, 31, 34, 35, 36, 39, 40, 41, 42, 43, 48, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 64, 67, 68 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	59	56	64	370	151	231	0	371
N.S.	1	0.82	0.78	0.89	5.14	2.10	3.21	0.00	5.15
time (sec)	N/A	0.246	0.015	5.695	0.285	0.255	23.076	0.000	0.748

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	51	54	61	238	129	182	0	144
N.S.	1	0.76	0.81	0.91	3.55	1.93	2.72	0.00	2.15
time (sec)	N/A	0.249	0.018	0.230	0.312	0.272	15.567	0.000	1.285

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	45	40	51	120	60	95	0	73
N.S.	1	0.94	0.83	1.06	2.50	1.25	1.98	0.00	1.52
time (sec)	N/A	0.220	0.012	0.107	0.311	0.265	6.121	0.000	1.604

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	52	52	65	0	0	0	0	0
N.S.	1	0.83	0.83	1.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.265	0.016	0.348	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	53	52	58	92	75	223	0	88
N.S.	1	0.87	0.85	0.95	1.51	1.23	3.66	0.00	1.44
time (sec)	N/A	0.237	0.020	0.279	0.207	0.276	12.222	0.000	0.747

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	52	51	57	120	70	314	0	103
N.S.	1	0.83	0.81	0.90	1.90	1.11	4.98	0.00	1.63
time (sec)	N/A	0.239	0.016	0.469	0.265	0.287	15.213	0.000	0.828

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	136	216	152	597	337	583	0	633
N.S.	1	0.87	1.38	0.97	3.80	2.15	3.71	0.00	4.03
time (sec)	N/A	0.725	0.146	0.489	1.060	0.288	45.360	0.000	3.682

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	150	163	276	0	0	0	0	0
N.S.	1	0.82	0.89	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.697	0.362	0.845	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	87	107	113	218	150	240	0	216
N.S.	1	0.92	1.13	1.19	2.29	1.58	2.53	0.00	2.27
time (sec)	N/A	0.416	0.097	0.195	0.991	0.282	15.311	0.000	1.831

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	183	172	381	1154	0	0	0	0	0
N.S.	1	0.94	2.08	6.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.686	0.301	2.563	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	115	135	325	0	0	0	0	0
N.S.	1	0.97	1.13	2.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.485	0.277	1.053	0.000	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	98	194	129	268	209	1107	0	232
N.S.	1	0.84	1.66	1.10	2.29	1.79	9.46	0.00	1.98
time (sec)	N/A	0.511	0.202	0.537	0.320	0.286	19.884	0.000	3.150

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	163	163	368	0	0	0	0	0
N.S.	1	0.84	0.84	1.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.724	0.663	2.558	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	156	245	164	534	448	0	0	438
N.S.	1	0.92	1.44	0.96	3.14	2.64	0.00	0.00	2.58
time (sec)	N/A	0.797	0.339	1.024	0.322	0.298	0.000	0.000	3.981

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	271	229	349	1249	0	0	0	0	0
N.S.	1	0.85	1.29	4.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.312	0.583	6.181	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	151	196	321	0	0	0	0	0
N.S.	1	0.92	1.20	1.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.809	0.285	0.553	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	279	258	562	2313	0	0	0	0	0
N.S.	1	0.92	2.01	8.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.985	0.548	1.814	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	163	158	263	2104	0	0	0	0	0
N.S.	1	0.97	1.61	12.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.728	0.586	1.334	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	163	225	423	0	0	0	0	0
N.S.	1	0.91	1.25	2.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.863	0.334	1.419	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	287	249	371	2465	0	0	0	0	0
N.S.	1	0.87	1.29	8.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.524	1.217	8.420	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	35	31	26	44	0	0	0	25
N.S.	1	1.13	1.00	0.84	1.42	0.00	0.00	0.00	0.81
time (sec)	N/A	0.232	0.007	0.061	0.323	0.000	0.000	0.000	0.082

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	39	34	38	123	0	0	0	0
N.S.	1	0.95	0.83	0.93	3.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.250	0.008	0.184	0.309	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	0	0	17	3	18
N.S.	1	1.00	1.11	0.89	0.00	0.00	0.94	0.17	1.00
time (sec)	N/A	0.173	3.739	0.133	0.000	0.000	3.179	127.074	1.016

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	230	157	464	346	317	0	0	787
N.S.	1	0.99	0.67	1.99	1.48	1.36	0.00	0.00	3.38
time (sec)	N/A	0.483	0.226	0.250	0.263	0.283	0.000	0.000	1.106

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	163	118	265	220	199	376	0	411
N.S.	1	1.05	0.76	1.71	1.42	1.28	2.43	0.00	2.65
time (sec)	N/A	0.390	0.128	0.201	0.291	0.281	131.969	0.000	0.863

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	103	163	113	116	103	177	0	136
N.S.	1	1.06	1.68	1.16	1.20	1.06	1.82	0.00	1.40
time (sec)	N/A	0.316	0.053	0.121	0.285	0.289	15.212	0.000	1.929

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	49	35	36	48	51	36	49
N.S.	1	1.00	1.29	0.92	0.95	1.26	1.34	0.95	1.29
time (sec)	N/A	0.166	0.011	0.061	0.200	0.265	0.168	0.287	1.169

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	180	160	198	0	0	0	0	0
N.S.	1	1.11	0.99	1.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.515	0.093	0.275	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	148	121	160	177	190	0	0	127
N.S.	1	0.98	0.80	1.06	1.17	1.26	0.00	0.00	0.84
time (sec)	N/A	0.398	0.167	0.246	0.261	0.415	0.000	0.000	1.973

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	245	175	245	409	682	0	0	399
N.S.	1	1.08	0.77	1.08	1.80	3.00	0.00	0.00	1.76
time (sec)	N/A	0.559	0.684	0.598	0.271	1.220	0.000	0.000	8.015

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	383	801	1018	0	0	0	0	0
N.S.	1	1.00	2.10	2.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.702	7.963	0.666	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	228	264	413	0	0	0	0	0
N.S.	1	1.03	1.19	1.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.542	0.972	0.417	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	109	139	0	0	0	0	0
N.S.	1	1.00	1.07	1.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.440	0.092	0.286	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	261	288	0	1877	0	0	0	0	0
N.S.	1	1.10	0.00	7.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.383	0.000	4.594	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	568	586	419	776	0	0	0	0	0
N.S.	1	1.03	0.74	1.37	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.417	5.053	1.564	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	564	554	1844	5843	0	0	0	0	0
N.S.	1	0.98	3.27	10.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.024	13.855	28.488	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	337	334	592	8267	0	0	0	0	0
N.S.	1	0.99	1.76	24.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.772	5.071	3.333	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	141	212	279	0	0	0	0	0
N.S.	1	0.99	1.48	1.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.629	0.116	0.779	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	372	408	0	3817	0	0	0	0	0
N.S.	1	1.10	0.00	10.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.450	0.000	4.948	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	1233	1265	0	3708	0	0	0	0	0
N.S.	1	1.03	0.00	3.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.238	0.000	6.291	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	177	237	162	0	0	0	0	0	0
N.S.	1	1.34	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.453	0.327	0.000	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	504	36	0	3	22
N.S.	1	1.00	1.10	1.00	25.20	1.80	0.00	0.15	1.10
time (sec)	N/A	0.271	4.368	0.187	8.769	0.267	0.000	111.038	0.542

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	659	52	0	3	22
N.S.	1	1.00	1.10	1.00	32.95	2.60	0.00	0.15	1.10
time (sec)	N/A	0.272	0.534	0.204	11.470	0.281	0.000	112.198	0.555

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	99	95	131	104	87	155	0	133
N.S.	1	0.93	0.90	1.24	0.98	0.82	1.46	0.00	1.25
time (sec)	N/A	0.300	0.058	0.126	0.269	0.271	0.688	0.000	0.693

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	82	114	102	85	66	117	0	102
N.S.	1	1.04	1.44	1.29	1.08	0.84	1.48	0.00	1.29
time (sec)	N/A	0.280	0.047	0.094	0.269	0.257	0.514	0.000	0.906

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	59	90	63	68	52	78	0	61
N.S.	1	0.98	1.50	1.05	1.13	0.87	1.30	0.00	1.02
time (sec)	N/A	0.255	0.026	0.084	0.291	0.278	0.370	0.000	1.030

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	31	39	30	31	39	46	31	42
N.S.	1	0.94	1.18	0.91	0.94	1.18	1.39	0.94	1.27
time (sec)	N/A	0.193	0.017	0.064	0.210	0.258	0.166	0.275	0.506

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	171	94	134	0	0	0	0
N.S.	1	1.00	1.42	0.78	1.12	0.00	0.00	0.00	0.00
time (sec)	N/A	0.432	0.011	0.170	0.318	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	60	67	61	77	57	168	0	63
N.S.	1	0.97	1.08	0.98	1.24	0.92	2.71	0.00	1.02
time (sec)	N/A	0.223	0.051	0.103	0.277	0.277	0.644	0.000	1.113

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	102	92	84	112	95	382	0	232
N.S.	1	1.06	0.96	0.88	1.17	0.99	3.98	0.00	2.42
time (sec)	N/A	0.285	0.082	0.138	0.293	0.265	0.872	0.000	1.309

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	136	128	115	165	135	760	0	288
N.S.	1	1.05	0.99	0.89	1.28	1.05	5.89	0.00	2.23
time (sec)	N/A	0.323	0.119	0.167	0.272	0.277	1.231	0.000	1.180

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	863	869	701	380	0	0	0	0	0
N.S.	1	1.01	0.81	0.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.568	0.698	0.885	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	541	409	542	8520	0	0	0	0
N.S.	1	1.00	0.75	1.00	15.69	0.00	0.00	0.00	0.00
time (sec)	N/A	0.895	0.284	1.041	3.854	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	170	231	186	284	0	0	0	0
N.S.	1	1.12	1.52	1.22	1.87	0.00	0.00	0.00	0.00
time (sec)	N/A	0.491	0.020	0.379	0.358	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	254	771	295	284	0	0	0	0
N.S.	1	1.04	3.16	1.21	1.16	0.00	0.00	0.00	0.00
time (sec)	N/A	0.491	8.240	0.345	0.359	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	668	678	660	647	8518	0	0	0	0
N.S.	1	1.01	0.99	0.97	12.75	0.00	0.00	0.00	0.00
time (sec)	N/A	1.115	0.475	1.885	0.905	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	933	951	896	511	0	0	0	0	0
N.S.	1	1.02	0.96	0.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.722	0.925	1.092	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	673	665	604	364	0	0	0	0	0
N.S.	1	0.99	0.90	0.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.138	0.407	0.238	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	770	792	770	388	0	0	0	0	0
N.S.	1	1.03	1.00	0.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.273	0.358	0.210	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	304	283	226	328	0	0	0	0
N.S.	1	1.11	1.03	0.82	1.20	0.00	0.00	0.00	0.00
time (sec)	N/A	0.522	0.041	0.779	0.340	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	541	409	542	14300	0	0	0	0
N.S.	1	1.00	0.75	1.00	26.34	0.00	0.00	0.00	0.00
time (sec)	N/A	0.894	0.267	1.011	2.001	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	388	443	890	0	0	0	0	0
N.S.	1	1.06	1.21	2.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.035	0.507	1.363	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	127	67	135	0	0	0	0	0
N.S.	1	0.96	0.51	1.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	0.072	0.427	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	155	95	176	0	0	0	0	0
N.S.	1	0.72	0.44	0.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.354	0.068	0.513	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	163	26	28	28	29	3	28
N.S.	1	1.00	5.82	0.93	1.00	1.00	1.04	0.11	1.00
time (sec)	N/A	0.238	0.311	0.246	0.259	0.262	0.882	56.779	0.574

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	33	33	165	31	33	33	31	3	33
N.S.	1	1.00	5.00	0.94	1.00	1.00	0.94	0.09	1.00
time (sec)	N/A	0.259	0.152	0.203	0.285	0.279	6.574	59.742	0.561

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	177	145	180	0	0	0	0	0
N.S.	1	0.95	0.78	0.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.414	0.559	0.677	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	215	189	222	0	0	0	0	0
N.S.	1	0.77	0.67	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.553	0.190	0.524	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	181	33	35	44	36	3	35
N.S.	1	1.00	5.17	0.94	1.00	1.26	1.03	0.09	1.00
time (sec)	N/A	0.313	4.753	0.230	0.357	0.289	3.725	172.488	0.594

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	225	38	40	49	37	3	40
N.S.	1	1.00	5.62	0.95	1.00	1.22	0.92	0.08	1.00
time (sec)	N/A	0.371	0.728	0.211	0.398	0.280	26.309	173.556	0.588

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [46] had the largest ratio of [.7500000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	0.82	21	0.238
2	A	7	6	0.76	21	0.286
3	A	6	5	0.94	19	0.263
4	A	5	4	0.83	21	0.190
5	A	8	7	0.87	21	0.333
6	A	6	5	0.83	21	0.238
7	A	12	11	0.87	23	0.478
8	A	12	11	0.82	23	0.478
9	A	7	6	0.92	21	0.286
10	A	7	6	0.94	23	0.261
11	A	7	6	0.97	23	0.261
12	A	11	10	0.84	23	0.435
13	A	11	10	0.84	23	0.435
14	A	16	15	0.92	23	0.652
15	A	13	12	0.85	23	0.522
16	A	11	10	0.92	21	0.476
17	A	8	7	0.92	23	0.304
18	A	8	7	0.97	23	0.304
19	A	10	9	0.91	23	0.391
20	A	17	16	0.87	23	0.696
21	A	5	4	1.13	12	0.333
22	A	5	4	0.95	19	0.211

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	N/A	1	0	1.00	18	0.000
24	A	6	5	0.99	18	0.278
25	A	6	5	1.05	18	0.278
26	A	6	5	1.06	16	0.312
27	A	1	1	1.00	10	0.100
28	A	7	6	1.11	18	0.333
29	A	10	9	0.98	18	0.500
30	A	6	6	1.08	18	0.333
31	A	5	4	1.00	20	0.200
32	A	5	4	1.03	18	0.222
33	A	7	6	1.00	12	0.500
34	A	4	3	1.10	20	0.150
35	A	7	6	1.03	20	0.300
36	A	5	4	0.98	20	0.200
37	A	5	4	0.99	18	0.222
38	A	7	6	0.99	12	0.500
39	A	4	3	1.10	20	0.150
40	A	7	6	1.03	20	0.300
41	A	5	4	1.34	18	0.222
42	N/A	3	0	1.00	20	0.000
43	N/A	3	0	1.00	20	0.000
44	A	7	6	0.93	10	0.600
45	A	6	5	1.04	10	0.500
46	A	7	6	0.98	8	0.750
47	A	4	3	0.94	6	0.500
48	A	8	7	1.00	10	0.700
49	A	8	7	0.97	10	0.700
50	A	6	5	1.06	10	0.500
51	A	7	6	1.05	10	0.600
52	A	3	3	1.01	16	0.188
53	A	3	3	1.00	16	0.188
54	A	7	6	1.12	14	0.429
55	A	3	3	1.04	16	0.188

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	3	3	1.01	16	0.188
57	A	3	3	1.02	16	0.188
58	A	5	4	0.99	18	0.222
59	A	6	5	1.03	18	0.278
60	A	3	3	1.11	14	0.214
61	A	3	3	1.00	16	0.188
62	A	2	2	1.06	19	0.105
63	A	3	2	0.96	28	0.071
64	A	4	3	0.72	33	0.091
65	N/A	3	0	1.00	28	0.000
66	N/A	3	0	1.00	33	0.000
67	A	5	4	0.95	35	0.114
68	A	6	5	0.77	40	0.125
69	N/A	3	0	1.00	35	0.000
70	N/A	3	0	1.00	40	0.000

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (ce+dex)^3(a+b\arctan(c+dx))\,dx$

3.1.1 Optimal result

Integrand size = 21, antiderivative size = 72

$$\int (ce+dex)^3(a+b\arctan(c+dx))\,dx = \frac{1}{4}be^3x-\frac{be^3(c+dx)^3}{12d}-\frac{be^3\arctan(c+dx)}{4d}+\frac{e^3(c+dx)^4(a+b\arctan(c+dx))}{4d}$$

output
```
1/4*b*e^3*x-1/12*b*e^3*(d*x+c)^3/d-1/4*b*e^3*arctan(d*x+c)/d+1/4*e^3*(d*x+
c)^4*(a+b*arctan(d*x+c))/d
```

3.1.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int (ce+dex)^3(a+b\arctan(c+dx))\,dx = \frac{e^3\left(-\frac{1}{4}b\left(-dx+\frac{1}{3}(c+dx)^3+\arctan(c+dx)\right)+\frac{1}{4}(c+dx)^4(a+b\arctan(c+dx))\right)}{d}$$

input
```
Integrate[(c*e + d*e*x)^3*(a + b*ArcTan[c + d*x]),x]
```

output
```
(e^3*(-1/4*(b*(-(d*x) + (c + d*x)^3/3 + ArcTan[c + d*x])) + ((c + d*x)^4*(
a + b*ArcTan[c + d*x]))/4))/d
```

3.1.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5566, 27, 5361, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^3(a+b\arctan(c+dx))\,dx$$

$$\Big\downarrow \ 5566$$

$$\frac{\int e^3(c+dx)^3(a+b\arctan(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{e^3\int (c+dx)^3(a+b\arctan(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \ 5361$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\arctan(c+dx))-\frac{1}{4}b\int \frac{(c+dx)^4}{(c+dx)^2+1}d(c+dx)\right)}{d}$$

$$\Big\downarrow \ 254$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\arctan(c+dx))-\frac{1}{4}b\int \left((c+dx)^2+\frac{1}{(c+dx)^2+1}-1\right)d(c+dx)\right)}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\arctan(c+dx))-\frac{1}{4}b\left(\arctan(c+dx)+\frac{1}{3}(c+dx)^3-c-dx\right)\right)}{d}$$

input
```
Int[(c*e + d*e*x)^3*(a + b*ArcTan[c + d*x]),x]
```

output
```
(e^3*(-1/4*(b*(-c - d*x + (c + d*x)^3/3 + ArcTan[c + d*x])) + ((c + d*x)^4
*(a + b*ArcTan[c + d*x]))/4))/d
```

3.1.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5566
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.1.4 Maple [A] (verified)

Time = 5.70 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{e^3 a (dx+c)^4}{4}+e^3 b \left(\frac{(dx+c)^4 \arctan(dx+c)}{4}-\frac{(dx+c)^3}{12}+\frac{dx}{4}+\frac{c}{4}-\frac{\arctan(dx+c)}{4}\right)}{d}$
default	$\frac{\frac{e^3 a (dx+c)^4}{4}+e^3 b \left(\frac{(dx+c)^4 \arctan(dx+c)}{4}-\frac{(dx+c)^3}{12}+\frac{dx}{4}+\frac{c}{4}-\frac{\arctan(dx+c)}{4}\right)}{d}$
parts	$\frac{e^3 a (dx+c)^4}{4d}+\frac{e^3 b \left(\frac{(dx+c)^4 \arctan(dx+c)}{4}-\frac{(dx+c)^3}{12}+\frac{dx}{4}+\frac{c}{4}-\frac{\arctan(dx+c)}{4}\right)}{d}$
parallelrisch	$3d^5 e^3 b \arctan(dx+c) x^4+3x^4 a d^5 e^3+12bc d^4 e^3 \arctan(dx+c) x^3+12x^3 ac d^4 e^3+18x^2 \arctan(dx+c) b c^2 d^3 e^3-x^3 b d^4 e^3+1$
risch	$\frac{i e^3 d^2 bc x^3 \ln(1-i(dx+c))}{2}+\frac{i e^3 d^3 b x^4 \ln(1-i(dx+c))}{8}+\frac{i e^3 b c^3 x \ln(1-i(dx+c))}{2}+\frac{3i e^3 db c^2 x^2 \ln(1-i(dx+c))}{4}+$

input
```
int((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*e^3*a*(d*x+c)^4+e^3*b*(1/4*(d*x+c)^4*arctan(d*x+c)-1/12*(d*x+c)^3
+1/4*d*x+1/4*c-1/4*arctan(d*x+c)))
```

3.1.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(64) = 128.

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.10

$$\int (ce+dex)^3(a+b\arctan(c+dx))\,dx = \frac{3\,ad^4e^3x^4+(12\,ac-b)d^3e^3x^3+3\,(6\,ac^2-bc)d^2e^3x^2+3\,(4\,ac^3-bc^2+b)de^3x+3\,(bd^4e^3x^4+4\,bcd^3e^3x^3}{12\,d}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(3*a*d^4*e^3*x^4 + (12*a*c - b)*d^3*e^3*x^3 + 3*(6*a*c^2 - b*c)*d^2*e
^3*x^2 + 3*(4*a*c^3 - b*c^2 + b)*d*e^3*x + 3*(b*d^4*e^3*x^4 + 4*b*c*d^3*e^
3*x^3 + 6*b*c^2*d^2*e^3*x^2 + 4*b*c^3*d*e^3*x + (b*c^4 - b)*e^3)*arctan(d*
x + c))/d
```

3.1.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(61) = 122.

Time = 23.08 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.21

$$\int (ce+dex)^3(a+b\arctan(c+dx))\,dx = \begin{cases} ac^3e^3x+\frac{3ac^2de^3x^2}{2}+acd^2e^3x^3+\frac{ad^3e^3x^4}{4}+\frac{bc^4e^3\operatorname{atan}(c+dx)}{4d}+bc^3e^3x\operatorname{atan}(c+dx)+\frac{3bc^2de^3x^2\operatorname{atan}(c+dx)}{2}-bc \\ c^3e^3x(a+b\operatorname{atan}(c)) \end{cases}$$

input
```
integrate((d*e*x+c*e)**3*(a+b*atan(d*x+c)),x)
```

output
```
Piecewise((a*c**3*e**3*x + 3*a*c**2*d*e**3*x**2/2 + a*c*d**2*e**3*x**3 + a
*d**3*e**3*x**4/4 + b*c**4*e**3*atan(c + d*x)/(4*d) + b*c**3*e**3*x*atan(c
 + d*x) + 3*b*c**2*d*e**3*x**2*atan(c + d*x)/2 - b*c**2*e**3*x/4 + b*c*d**
2*e**3*x**3*atan(c + d*x) - b*c*d*e**3*x**2/4 + b*d**3*e**3*x**4*atan(c +
d*x)/4 - b*d**2*e**3*x**3/12 + b*e**3*x/4 - b*e**3*atan(c + d*x)/(4*d), Ne
(d, 0)), (c**3*e**3*x*(a + b*atan(c)), True))
```

3.1.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(64) = 128$.

Time = 0.29 (sec) , antiderivative size = 370, normalized size of antiderivative = 5.14

$$\int (ce+dex)^3(a+b\arctan(c+dx))\,dx = \frac{1}{4}ad^3e^3x^4 + acd^2e^3x^3 + \frac{3}{2}ac^2de^3x^2$$
$$+\frac{3}{2}\left(x^2\arctan(dx+c) - d\left(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log(d^2x^2+2\,cdx+c^2+1)}{d^3}\right)\right)bc^2de^3$$
$$+\frac{1}{2}\left(2\,x^3\arctan(dx+c) - d\left(\frac{dx^2-4\,cx}{d^3} - \frac{2\,(c^3-3\,c)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^4} + \frac{(3\,c^2-1)\log(d^2x^2+2\,cdx\,-}{d^4}\right.\right.$$
$$+\frac{1}{12}\left(3\,x^4\arctan(dx+c) - d\left(\frac{d^2x^3-3\,cdx^2+3\,(3\,c^2-1)x}{d^4} + \frac{3\,(c^4-6\,c^2+1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^5} - \frac{6\,(c}{}\right.\right.$$
$$+ac^3e^3x + \frac{\left(2\,(dx+c)\arctan(dx+c) - \log\left((dx+c)^2+1\right)\right)bc^3e^3}{2\,d}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*a*d^3*e^3*x^4 + a*c*d^2*e^3*x^3 + 3/2*a*c^2*d*e^3*x^2 + 3/2*(x^2*arcta
n(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*
x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*c^2*d*e^3 + 1/2*(2*x^3*arctan(d*x + c) -
d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^
2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*b*c*d^2*e^3 + 1/12*(3*x^4*ar
ctan(d*x + c) - d*((d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 - 1)*x)/d^4 + 3*(c^4 -
6*c^2 + 1)*arctan((d^2*x + c*d)/d)/d^5 - 6*(c^3 - c)*log(d^2*x^2 + 2*c*d*x
 + c^2 + 1)/d^5))*b*d^3*e^3 + a*c^3*e^3*x + 1/2*(2*(d*x + c)*arctan(d*x +
c) - log((d*x + c)^2 + 1))*b*c^3*e^3/d
```

3.1.8 Giac [F]

$$\int (ce+dex)^3(a+b\arctan(c+dx))\,dx=\int (dex+ce)^3(b\arctan(dx+c)+a)\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 371, normalized size of antiderivative = 5.15

$$\int (ce+dex)^3(a+b\arctan(c+dx))\,dx$$
$$=\mathrm{atan}(c+d\,x)\,\left(b\,c^3\,e^3\,x+\frac{3\,b\,c^2\,d\,e^3\,x^2}{2}+b\,c\,d^2\,e^3\,x^3+\frac{b\,d^3\,e^3\,x^4}{4}\right)-x^3\,\left(\frac{d^2\,e^3\,(b-20\,a\,c)}{12}+\frac{2\,a\,c\,d^2\,e^3}{3}\right)+x^2\,\left(\frac{c\,\left(\frac{d^2\,e^3\,(b-20\,a\,c)}{4}+2\,a\,c\,d^2\,e^3\right)}{d}+\frac{d\,e^3\,(10\,a\,c^2-b\,c+a)}{2}-\frac{a\,d\,e^3\,(4\,c^2+4)}{8}\right)+x\,\left(\frac{c\,e^3\,(20\,a\,c^2-3\,b\,c+6\,a)}{2}+\frac{(4\,c^2+4)\,\left(\frac{d^2\,e^3\,(b-20\,a\,c)}{4}+2\,a\,c\,d^2\,e^3\right)}{4\,d^2}-\frac{2\,c\,\left(\frac{2\,c\,\left(\frac{d^2\,e^3\,(b-20\,a\,c)}{4}+2\,a\,c\,d^2\,e^3\right)}{d}+d\,e^3\,(10\,a\,c^2-b\,c+a)-\frac{a\,d\,e^3\,(4\,c^2+4)}{4}\right)}{d}\right)+\frac{a\,d^3\,e^3\,x^4}{4}-\frac{b\,e^3\,\mathrm{atan}\left(\frac{\frac{b\,c\,e^3\,\left(c^2+1\right)\,(c-1)\,(c+1)}{4}+\frac{b\,d\,e^3\,x\,\left(c^2+1\right)\,(c-1)\,(c+1)}{4}}{\frac{b\,e^3}{4}-\frac{b\,c^4\,e^3}{4}}\right)\,(c^2+1)\,(c-1)\,(c+1)}{4\,d}$$

input
```
int((c*e + d*e*x)^3*(a + b*atan(c + d*x)),x)
```

output
```
atan(c + d*x)*((b*d^3*e^3*x^4)/4 + b*c^3*e^3*x + (3*b*c^2*d*e^3*x^2)/2 + b
*c*d^2*e^3*x^3) - x^3*((d^2*e^3*(b - 20*a*c))/12 + (2*a*c*d^2*e^3)/3) + x^
2*((c*((d^2*e^3*(b - 20*a*c))/4 + 2*a*c*d^2*e^3))/d + (d*e^3*(a - b*c + 10
*a*c^2))/2 - (a*d*e^3*(4*c^2 + 4))/8) + x*((c*e^3*(6*a - 3*b*c + 20*a*c^2)
)/2 + ((4*c^2 + 4)*((d^2*e^3*(b - 20*a*c))/4 + 2*a*c*d^2*e^3))/(4*d^2) - (
2*c*((2*c*((d^2*e^3*(b - 20*a*c))/4 + 2*a*c*d^2*e^3))/d + d*e^3*(a - b*c +
 10*a*c^2) - (a*d*e^3*(4*c^2 + 4))/4))/d) + (a*d^3*e^3*x^4)/4 - (b*e^3*ata
n(((b*c*e^3*(c^2 + 1)*(c - 1)*(c + 1))/4 + (b*d*e^3*x*(c^2 + 1)*(c - 1)*(c
 + 1))/4)/((b*e^3)/4 - (b*c^4*e^3)/4))*(c^2 + 1)*(c - 1)*(c + 1))/(4*d)
```

3.2 $\int (ce+dex)^2(a+b\arctan(c+dx))\,dx$

3.2.1 Optimal result

Integrand size = 21, antiderivative size = 67

$$\int (ce+dex)^2(a+b\arctan(c+dx))\,dx=-\frac{be^2(c+dx)^2}{6d}+\frac{e^2(c+dx)^3(a+b\arctan(c+dx))}{3d}+\frac{be^2\log\left(1+(c+dx)^2\right)}{6d}$$

output
```
-1/6*b*e^2*(d*x+c)^2/d+1/3*e^2*(d*x+c)^3*(a+b*arctan(d*x+c))/d+1/6*b*e^2*l
n(1+(d*x+c)^2)/d
```

3.2.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int (ce+dex)^2(a+b\arctan(c+dx))\,dx=\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))-\frac{1}{6}b\left((c+dx)^2-\log\left(1+(c+dx)^2\right)\right)\right)}{d}$$

input
```
Integrate[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x]),x]
```

output
```
(e^2*(((c + d*x)^3*(a + b*ArcTan[c + d*x]))/3 - (b*((c + d*x)^2 - Log[1 +
(c + d*x)^2]))/6))/d
```

3.2.3 Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5566, 27, 5361, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^2(a+b\arctan(c+dx))\,dx$$

$$\Big\downarrow \text{5566}$$

$$\frac{\int e^2(c+dx)^2(a+b\arctan(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\int (c+dx)^2(a+b\arctan(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))-\frac{1}{3}b\int \frac{(c+dx)^3}{(c+dx)^2+1}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{243}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))-\frac{1}{6}b\int \frac{(c+dx)^2}{(c+dx)^2+1}d(c+dx)^2\right)}{d}$$

$$\Big\downarrow \text{49}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))-\frac{1}{6}b\int \left(1+\frac{1}{-c-dx-1}\right)d(c+dx)^2\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))-\frac{1}{6}b\left((c+dx)^2-\log(c+dx+1)\right)\right)}{d}$$

input
```
Int[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x]),x]
```

output
```
(e^2*(((c + d*x)^3*(a + b*ArcTan[c + d*x]))/3 - (b*((c + d*x)^2 - Log[1 +
c + d*x]))/6))/d
```

3.2.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5566
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.2.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{e^2a(dx+c)^3}{3}+e^2b\left(\frac{(dx+c)^3\arctan(dx+c)}{3}-\frac{(dx+c)^2}{6}+\frac{\ln\left(1+(dx+c)^2\right)}{6}\right)}{d}$
default	$\frac{\frac{e^2a(dx+c)^3}{3}+e^2b\left(\frac{(dx+c)^3\arctan(dx+c)}{3}-\frac{(dx+c)^2}{6}+\frac{\ln\left(1+(dx+c)^2\right)}{6}\right)}{d}$
parts	$\frac{e^2a(dx+c)^3}{3d}+\frac{e^2b\left(\frac{(dx+c)^3\arctan(dx+c)}{3}-\frac{(dx+c)^2}{6}+\frac{\ln\left(1+(dx+c)^2\right)}{6}\right)}{d}$
parallelrisch	$\frac{2d^4e^2b\arctan(dx+c)x^3+2x^3ad^4e^2+6d^3e^2cb\arctan(dx+c)x^2+6x^2acd^3e^2+6bc^2e^2\arctan(dx+c)xd^2-x^2bd^3e^2+6xac}{6d^2}$
risch	$-\frac{ie^2(dx+c)^3b\ln(1+i(dx+c))}{6d}+\frac{ie^2d^2bx^3\ln(1-i(dx+c))}{6}+\frac{ie^2dbcx^2\ln(1-i(dx+c))}{2}+\frac{ie^2bc^2x\ln(1-i(dx+c))}{2}+$

input
```
int((d*e*x+c*e)^2*(a+b*arctan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*e^2*a*(d*x+c)^3+e^2*b*(1/3*(d*x+c)^3*arctan(d*x+c)-1/6*(d*x+c)^2+
1/6*ln(1+(d*x+c)^2)))
```

3.2.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(61) = 122$.

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.93

$$\int (ce+dex)^2(a+b\arctan(c+dx))\,dx$$
$$=\frac{2\,ad^3e^2x^3+(6\,ac-b)d^2e^2x^2+2\,(3\,ac^2-bc)de^2x+be^2\log\left(d^2x^2+2\,cdx+c^2+1\right)+2\,(bd^3e^2x^3+3\,bcd^2e}{6\,d}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(2*a*d^3*e^2*x^3 + (6*a*c - b)*d^2*e^2*x^2 + 2*(3*a*c^2 - b*c)*d*e^2*x
 + b*e^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(b*d^3*e^2*x^3 + 3*b*c*d^2*e
^2*x^2 + 3*b*c^2*d*e^2*x + b*c^3*e^2)*arctan(d*x + c))/d
```

3.2.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.57 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.72

$$\int (ce+dex)^2(a+b\arctan(c+dx))\,dx$$
$$=\begin{cases} ac^2e^2x+acde^2x^2+\frac{ad^2e^2x^3}{3}+\frac{bc^3e^2\operatorname{atan}(c+dx)}{3d}+bc^2e^2x\operatorname{atan}(c+dx)+bcde^2x^2\operatorname{atan}(c+dx)-\frac{bce^2x}{3}+\frac{bd^2}{} \\ c^2e^2x(a+b\operatorname{atan}(c)) \end{cases}$$

input
```
integrate((d*e*x+c*e)**2*(a+b*atan(d*x+c)),x)
```

output
```
Piecewise((a*c**2*e**2*x + a*c*d*e**2*x**2 + a*d**2*e**2*x**3/3 + b*c**3*e
**2*atan(c + d*x)/(3*d) + b*c**2*e**2*x*atan(c + d*x) + b*c*d*e**2*x**2*at
an(c + d*x) - b*c*e**2*x/3 + b*d**2*e**2*x**3*atan(c + d*x)/3 - b*d*e**2*x
**2/6 + b*e**2*log(c/d + x - I/d)/(3*d) - I*b*e**2*atan(c + d*x)/(3*d), Ne
(d, 0)), (c**2*e**2*x*(a + b*atan(c)), True))
```

3.2.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(61) = 122.

Time = 0.31 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.55

$$\int (ce+dex)^2(a+b\arctan(c+dx))\,dx=\frac{1}{3}\,ad^2e^2x^3+acde^2x^2$$
$$+\left(x^2\arctan(dx+c)-d\left(\frac{x}{d^2}+\frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\right)\right)bcde^2$$
$$+\frac{1}{6}\left(2\,x^3\arctan(dx+c)-d\left(\frac{dx^2-4\,cx}{d^3}-\frac{2\,(c^3-3\,c)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^4}+\frac{(3\,c^2-1)\log\left(d^2x^2+2\,cdx\right.}{d^4}\right.\right.$$
$$+ac^2e^2x+\frac{\left(2\,(dx+c)\arctan(dx+c)-\log\left((dx+c)^2+1\right)\right)bc^2e^2}{2\,d}$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*a*d^2*e^2*x^3 + a*c*d*e^2*x^2 + (x^2*arctan(d*x + c) - d*(x/d^2 + (c^2
 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3
))*b*c*d*e^2 + 1/6*(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^
3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x +
 c^2 + 1)/d^4))*b*d^2*e^2 + a*c^2*e^2*x + 1/2*(2*(d*x + c)*arctan(d*x + c)
 - log((d*x + c)^2 + 1))*b*c^2*e^2/d
```

3.2.8 Giac [F]

$$\int (ce+dex)^2(a+b\arctan(c+dx))\,dx=\int (dex+ce)^2(b\arctan(dx+c)+a)\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

output
```
sage0*x
```

3.2.9 Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.15

$$\int (ce+dex)^2(a+b\arctan(c+dx))\,dx=\frac{a\,d^2\,e^2\,x^3}{3}-\frac{b\,c\,e^2\,x}{3}+\frac{b\,e^2\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{6\,d}+a\,c^2\,e^2\,x-\frac{b\,d\,e^2\,x^2}{6}+b\,c^2\,e^2\,x\,\mathrm{atan}(c+d\,x)+a\,c\,d\,e^2\,x^2+\frac{b\,c^3\,e^2\,\mathrm{atan}(c+d\,x)}{3\,d}+\frac{b\,d^2\,e^2\,x^3\,\mathrm{atan}(c+d\,x)}{3}+b\,c\,d\,e^2\,x^2\,\mathrm{atan}(c+d\,x)$$

input
```
int((c*e + d*e*x)^2*(a + b*atan(c + d*x)),x)
```

output
```
(a*d^2*e^2*x^3)/3 - (b*c*e^2*x)/3 + (b*e^2*log(c^2 + d^2*x^2 + 2*c*d*x + 1
))/(6*d) + a*c^2*e^2*x - (b*d*e^2*x^2)/6 + b*c^2*e^2*x*atan(c + d*x) + a*c
*d*e^2*x^2 + (b*c^3*e^2*atan(c + d*x))/(3*d) + (b*d^2*e^2*x^3*atan(c + d*x
))/3 + b*c*d*e^2*x^2*atan(c + d*x)
```

3.3 $\int (ce+dex)(a+b\arctan(c+dx))\,dx$

3.3.1 Optimal result

Integrand size = 19, antiderivative size = 48

$$\int (ce+dex)(a+b\arctan(c+dx))\,dx=-\frac{1}{2}bex+\frac{be\arctan(c+dx)}{2d}+\frac{e(c+dx)^2(a+b\arctan(c+dx))}{2d}$$

output
```
-1/2*b*e*x+1/2*b*e*arctan(d*x+c)/d+1/2*e*(d*x+c)^2*(a+b*arctan(d*x+c))/d
```

3.3.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int (ce+dex)(a+b\arctan(c+dx))\,dx=\frac{e(b(-dx+\arctan(c+dx))+(c+dx)^2(a+b\arctan(c+dx)))}{2d}$$

input
```
Integrate[(c*e + d*e*x)*(a + b*ArcTan[c + d*x]),x]
```

output
```
(e*(b*(-(d*x) + ArcTan[c + d*x]) + (c + d*x)^2*(a + b*ArcTan[c + d*x])))/(
2*d)
```

3.3.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5566, 27, 5361, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)(a+b\arctan(c+dx))\,dx$$

$$\Big\downarrow \text{5566}$$

$$\frac{\int e(c+dx)(a+b\arctan(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int (c+dx)(a+b\arctan(c+dx))d(c+dx)}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\arctan(c+dx))-\frac{1}{2}b\int \frac{(c+dx)^2}{(c+dx)^2+1}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{262}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\arctan(c+dx))-\frac{1}{2}b\left(-\int \frac{1}{(c+dx)^2+1}d(c+dx)+c+dx\right)\right)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{e(\frac{1}{2}(c+dx)^2(a+b\arctan(c+dx))-\frac{1}{2}b(-\arctan(c+dx)+c+dx))}{d}$$

input
```
Int[(c*e + d*e*x)*(a + b*ArcTan[c + d*x]),x]
```

output
```
(e*(-1/2*(b*(c + d*x - ArcTan[c + d*x])) + ((c + d*x)^2*(a + b*ArcTan[c + 
d*x]))/2))/d
```

3.3.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5566
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.3.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\frac{ea(dx+c)^2}{2}+be\left(\frac{(dx+c)^2\arctan(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}+\frac{\arctan(dx+c)}{2}\right)}{d}$
default	$\frac{\frac{ea(dx+c)^2}{2}+be\left(\frac{(dx+c)^2\arctan(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}+\frac{\arctan(dx+c)}{2}\right)}{d}$
parts	$ea\left(\frac{1}{2}d\,x^2+cx\right)+\frac{be\left(\frac{(dx+c)^2\arctan(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}+\frac{\arctan(dx+c)}{2}\right)}{d}$
parallelrisch	$\frac{d^3eb\arctan(dx+c)x^2+x^2a\,d^3e+2cbe\arctan(dx+c)x\,d^2+2xac\,d^2e+\arctan(dx+c)b\,c^2de-xb\,d^2e-5a\,c^2de+eb\arctan(dx-}{2d^2}$
risch	$-\frac{ieb(d\,x^2+2cx)\ln(1+i(dx+c))}{4}+\frac{iedb\,x^2\ln(1-i(dx+c))}{4}+\frac{iebcx\ln(1-i(dx+c))}{2}+\frac{ade\,x^2}{2}+\frac{e\arctan(dx+c)b\,c^2}{2d}$

input
```
int((d*e*x+c*e)*(a+b*arctan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*e*a*(d*x+c)^2+b*e*(1/2*(d*x+c)^2*arctan(d*x+c)-1/2*d*x-1/2*c+1/2*
arctan(d*x+c)))
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int (ce+dex)(a+b\arctan(c+dx))\,dx$$
$$=\frac{ad^2ex^2+(2\,ac-b)dex+(bd^2ex^2+2\,bcdex+(bc^2+b)e)\arctan{(dx+c)}}{2\,d}$$

input
```
integrate((d*e*x+c*e)*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a*d^2*e*x^2 + (2*a*c - b)*d*e*x + (b*d^2*e*x^2 + 2*b*c*d*e*x + (b*c^2
 + b)*e)*arctan(d*x + c))/d
```

3.3.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(41) = 82$.

Time = 6.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.98

$$\int (ce+dex)(a+b\arctan(c+dx))\,dx = \begin{cases} acex + \frac{adex^2}{2} + \frac{bc^2e\operatorname{atan}(c+dx)}{2d} + bcex\operatorname{atan}(c+dx) + \frac{bdex^2\operatorname{atan}(c+dx)}{2} - \frac{bex}{2} + \frac{be\operatorname{atan}(c+dx)}{2d} & \text{for } d\neq 0 \\ cex(a+b\operatorname{atan}(c)) & \text{otherwise} \end{cases}$$

input
```
integrate((d*e*x+c*e)*(a+b*atan(d*x+c)),x)
```

output
```
Piecewise((a*c*e*x + a*d*e*x**2/2 + b*c**2*e*atan(c + d*x)/(2*d) + b*c*e*x
*atan(c + d*x) + b*d*e*x**2*atan(c + d*x)/2 - b*e*x/2 + b*e*atan(c + d*x)/
(2*d), Ne(d, 0)), (c*e*x*(a + b*atan(c)), True))
```

3.3.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(42) = 84$.

Time = 0.31 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.50

$$\int (ce+dex)(a+b\arctan(c+dx))\,dx = \frac{1}{2}adex^2 + \frac{1}{2}\left(x^2\arctan(dx+c) - d\left(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log(d^2x^2+2\,cdx+c^2+1)}{d^3}\right)\right)bde + acex + \frac{\left(2\,(dx+c)\arctan(dx+c) - \log\left((dx+c)^2+1\right)\right)bce}{2\,d}$$

input
```
integrate((d*e*x+c*e)*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*a*d*e*x^2 + 1/2*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^
2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*d*e + a*c*e
*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b*c*e/d
```

3.3.8 Giac [F]

$$\int (ce + dex)(a + b\arctan(c + dx))\,dx = \int (dex + ce)(b\arctan(dx + c) + a)\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

output
```
sage0*x
```

3.3.9 Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int (ce + dex)(a + b\arctan(c + dx))\,dx = a\,c\,e\,x - \frac{b\,e\,x}{2} + \frac{b\,e\,\mathrm{atan}(c + d\,x)}{2\,d} + \frac{a\,d\,e\,x^2}{2} + \frac{b\,c^2\,e\,\mathrm{atan}(c + d\,x)}{2\,d} + b\,c\,e\,x\,\mathrm{atan}(c + d\,x) + \frac{b\,d\,e\,x^2\,\mathrm{atan}(c + d\,x)}{2}$$

input
```
int((c*e + d*e*x)*(a + b*atan(c + d*x)),x)
```

output
```
a*c*e*x - (b*e*x)/2 + (b*e*atan(c + d*x))/(2*d) + (a*d*e*x^2)/2 + (b*c^2*e
*atan(c + d*x))/(2*d) + b*c*e*x*atan(c + d*x) + (b*d*e*x^2*atan(c + d*x))/
2
```

3.4 $\int \frac{a+b\arctan(c+dx)}{ce+dex}\,dx$

3.4.1 Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \frac{a+b\arctan(c+dx)}{ce+dex}\,dx = \frac{a\log(c+dx)}{de} + \frac{ib\operatorname{PolyLog}(2,-i(c+dx))}{2de} - \frac{ib\operatorname{PolyLog}(2,i(c+dx))}{2de}$$

output
```
a*ln(d*x+c)/d/e+1/2*I*b*polylog(2,-I*(d*x+c))/d/e-1/2*I*b*polylog(2,I*(d*x
+c))/d/e
```

3.4.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{a+b\arctan(c+dx)}{ce+dex}\,dx = \frac{a\log(c+dx)+\frac{1}{2}ib\operatorname{PolyLog}(2,-i(c+dx))-\frac{1}{2}ib\operatorname{PolyLog}(2,i(c+dx))}{de}$$

input
```
Integrate[(a + b*ArcTan[c + d*x])/(c*e + d*e*x),x]
```

output
```
(a*Log[c + d*x] + (I/2)*b*PolyLog[2, (-I)*(c + d*x)] - (I/2)*b*PolyLog[2,
I*(c + d*x)])/(d*e)
```

3.4.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5566, 27, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(c+dx)}{ce+dex}\,dx$$

$$\Big\downarrow \text{ 5566}$$

$$\frac{\int \frac{a+b\arctan(c+dx)}{e(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{a+b\arctan(c+dx)}{c+dx}d(c+dx)}{de}$$

$$\Big\downarrow \text{ 5355}$$

$$\frac{\frac{1}{2}ib\int \frac{\log(1-i(c+dx))}{c+dx}d(c+dx)-\frac{1}{2}ib\int \frac{\log(i(c+dx)+1)}{c+dx}d(c+dx)+a\log(c+dx)}{de}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{a\log(c+dx)+\frac{1}{2}ib\,\text{PolyLog}(2,-i(c+dx))-\frac{1}{2}ib\,\text{PolyLog}(2,i(c+dx))}{de}$$

input
```
Int[(a + b*ArcTan[c + d*x])/(c*e + d*e*x),x]
```

output
```
(a*Log[c + d*x] + (I/2)*b*PolyLog[2, (-I)*(c + d*x)] - (I/2)*b*PolyLog[2,
I*(c + d*x)])/(d*e)
```

3.4.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5566
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.4.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{ib\,\mathrm{dilog}(-idx-ic+1)}{2ed}+\frac{a\ln(-idx-ic)}{ed}+\frac{ib\,\mathrm{dilog}(idx+ic+1)}{2ed}$
derivativedivides	$\frac{\frac{a\ln(dx+c)}{e}+\frac{b\left(\ln(dx+c)\arctan(dx+c)+\frac{i\ln(dx+c)\ln(1+i(dx+c))}{2}-\frac{i\ln(dx+c)\ln(1-i(dx+c))}{2}+\frac{i\,\mathrm{dilog}(1+i(dx+c))}{2}-\frac{i\,\mathrm{dilog}(1-i(dx+\right.}{e}}{d}$
default	$\frac{\frac{a\ln(dx+c)}{e}+\frac{b\left(\ln(dx+c)\arctan(dx+c)+\frac{i\ln(dx+c)\ln(1+i(dx+c))}{2}-\frac{i\ln(dx+c)\ln(1-i(dx+c))}{2}+\frac{i\,\mathrm{dilog}(1+i(dx+c))}{2}-\frac{i\,\mathrm{dilog}(1-i(dx+\right.}{e}}{d}$
parts	$\frac{a\ln(dx+c)}{de}+\frac{b\left(\ln(dx+c)\arctan(dx+c)+\frac{i\ln(dx+c)\ln(1+i(dx+c))}{2}-\frac{i\ln(dx+c)\ln(1-i(dx+c))}{2}+\frac{i\,\mathrm{dilog}(1+i(dx+c))}{2}-\frac{i\,\mathrm{dilog}(\right.}{ed}$

input
```
int((a+b*arctan(d*x+c))/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/e/d*b*dilog(-I*d*x-I*c+1)+1/e/d*a*ln(-I*d*x-I*c)+1/2*I*b/e/d*dilog(
I*d*x+I*c+1)
```

3.4.5 Fricas [F]

$$\int \frac{a+b\arctan(c+dx)}{ce+dex}\,dx=\int \frac{b\arctan(dx+c)+a}{dex+ce}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))/(d*e*x+c*e),x, algorithm="fricas")
```

output
```
integral((b*arctan(d*x + c) + a)/(d*e*x + c*e), x)
```

3.4.6 Sympy [F]

$$\int \frac{a+b\arctan(c+dx)}{ce+dex}\,dx=\frac{\int \frac{a}{c+dx}\,dx+\int \frac{b\,\mathrm{atan}\,(c+dx)}{c+dx}\,dx}{e}$$

input
```
integrate((a+b*atan(d*x+c))/(d*e*x+c*e),x)
```

output
```
(Integral(a/(c + d*x), x) + Integral(b*atan(c + d*x)/(c + d*x), x))/e
```

3.4.7 Maxima [F]

$$\int \frac{a+b\arctan(c+dx)}{ce+dex}\,dx=\int \frac{b\arctan(dx+c)+a}{dex+ce}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))/(d*e*x+c*e),x, algorithm="maxima")
```

output
```
2*b*integrate(1/2*arctan(d*x + c)/(d*e*x + c*e), x) + a*log(d*e*x + c*e)/(
d*e)
```

3.4.8 Giac [F]

$$\int \frac{a+b\arctan(c+dx)}{ce+dex}\,dx=\int \frac{b\arctan(dx+c)+a}{dex+ce}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))/(d*e*x+c*e),x, algorithm="giac")
```

output
```
sage0*x
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(c+dx)}{ce+dex}\,dx=\int \frac{a+b\,\mathrm{atan}(c+d\,x)}{c\,e+d\,e\,x}\,dx$$

input
```
int((a + b*atan(c + d*x))/(c*e + d*e*x),x)
```

output
```
int((a + b*atan(c + d*x))/(c*e + d*e*x), x)
```

3.5 $\int \frac{a+b\arctan(c+dx)}{(ce+dex)^2}\,dx$

3.5.1 Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^2}\,dx=-\frac{a+b\arctan(c+dx)}{de^2(c+dx)}+\frac{b\log(c+dx)}{de^2}-\frac{b\log\left(1+(c+dx)^2\right)}{2de^2}$$

output
```
(-a-b*arctan(d*x+c))/d/e^2/(d*x+c)+b*ln(d*x+c)/d/e^2-1/2*b*ln(1+(d*x+c)^2)
/d/e^2
```

3.5.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^2}\,dx=\frac{\frac{-a-b\arctan(c+dx)}{c+dx}+b\left(\log(c+dx)-\frac{1}{2}\log\left(1+(c+dx)^2\right)\right)}{de^2}$$

input
```
Integrate[(a + b*ArcTan[c + d*x])/(c*e + d*e*x)^2,x]
```

output
```
((-a - b*ArcTan[c + d*x])/(c + d*x) + b*(Log[c + d*x] - Log[1 + (c + d*x)^
2]/2))/(d*e^2)
```

3.5.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5566, 27, 5361, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^2}dx$$

$$\Big\downarrow \text{ 5566}$$

$$\frac{\int \frac{a+b\arctan(c+dx)}{e^2(c+dx)^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{a+b\arctan(c+dx)}{(c+dx)^2}d(c+dx)}{de^2}$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{b\int \frac{1}{(c+dx)((c+dx)^2+1)}d(c+dx)-\frac{a+b\arctan(c+dx)}{c+dx}}{de^2}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\frac{1}{2}b\int \frac{1}{(c+dx)^2((c+dx)^2+1)}d(c+dx)^2-\frac{a+b\arctan(c+dx)}{c+dx}}{de^2}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{\frac{1}{2}b\left(\int \frac{1}{(c+dx)^2}d(c+dx)^2-\int \frac{1}{(c+dx)^2+1}d(c+dx)^2\right)-\frac{a+b\arctan(c+dx)}{c+dx}}{de^2}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\frac{1}{2}b\left(\log\left((c+dx)^2\right)-\int \frac{1}{(c+dx)^2+1}d(c+dx)^2\right)-\frac{a+b\arctan(c+dx)}{c+dx}}{de^2}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\frac{1}{2}b\left(\log\left((c+dx)^2\right)-\log\left((c+dx)^2+1\right)\right)-\frac{a+b\arctan(c+dx)}{c+dx}}{de^2}$$

input
```
Int[(a + b*ArcTan[c + d*x])/(c*e + d*e*x)^2,x]
```

output
```
(-((a + b*ArcTan[c + d*x])/(c + d*x)) + (b*(Log[(c + d*x)^2] - Log[1 + (c
+ d*x)^2]))/2)/(d*e^2)
```

3.5.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5566
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.5.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-\frac{a}{e^2(dx+c)}+\frac{b\left(-\frac{\arctan(dx+c)}{dx+c}+\ln(dx+c)-\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{e^2}}{d}$
default	$\frac{-\frac{a}{e^2(dx+c)}+\frac{b\left(-\frac{\arctan(dx+c)}{dx+c}+\ln(dx+c)-\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{e^2}}{d}$
parts	$-\frac{a}{d\,e^2(dx+c)}+\frac{b\left(-\frac{\arctan(dx+c)}{dx+c}+\ln(dx+c)-\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{e^2 d}$
parallelrisch	$\frac{6\ln(dx+c)xbc\,d^2-3\ln\left(d^2x^2+2cdx+c^2+1\right)xbc\,d^2+6\ln(dx+c)b\,c^2d-3\ln\left(d^2x^2+2cdx+c^2+1\right)b\,c^2d+2xa\,d^2-6b\arctan(dx}{6(dx+c)c\,d^2e^2}$
risch	$\frac{ib\ln(1+i(dx+c))}{2d\,e^2(dx+c)}-\frac{-2\ln(-dx-c)bdx+\ln\left(-d^2x^2-2cdx-c^2-1\right)bdx-2\ln(-dx-c)bc+\ln\left(-d^2x^2-2cdx-c^2-1\right)bc+ib\,l}{2e^2(dx+c)d}$

input
```
int((a+b*arctan(d*x+c))/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a/e^2/(d*x+c)+b/e^2*(-1/(d*x+c)*arctan(d*x+c)+ln(d*x+c)-1/2*ln(1+(d*
x+c)^2)))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^2}\,dx = -\frac{2\,b\arctan\left(dx+c\right)+\left(bdx+bc\right)\log\left(d^2x^2+2\,cdx+c^2+1\right)-2\left(bdx+bc\right)\log\left(dx+c\right)+2\,a}{2\left(d^2e^2x+cde^2\right)}$$

input
```
integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*b*arctan(d*x + c) + (b*d*x + b*c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)
 - 2*(b*d*x + b*c)*log(d*x + c) + 2*a)/(d^2*e^2*x + c*d*e^2)
```

3.5.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.22 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.66

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^2}\,dx$$
$$=\begin{cases} -\frac{a}{cde^2+d^2e^2x}+\frac{bc\log\left(\frac{c}{d}+x\right)}{cde^2+d^2e^2x}-\frac{bc\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{cde^2+d^2e^2x}+\frac{ibc\,\mathrm{atan}\,(c+dx)}{cde^2+d^2e^2x}+\frac{bdx\log\left(\frac{c}{d}+x\right)}{cde^2+d^2e^2x}-\frac{bdx\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{cde^2+d^2e^2x}+\frac{ibdx\,\mathrm{atan}\,(c+dx)}{cde^2+d^2e^2x}- \\ \frac{x(a+b\,\mathrm{atan}\,(c))}{c^2e^2} \end{cases}$$

input
```
integrate((a+b*atan(d*x+c))/(d*e*x+c*e)**2,x)
```

output
```
Piecewise((-a/(c*d*e**2 + d**2*e**2*x) + b*c*log(c/d + x)/(c*d*e**2 + d**2
*e**2*x) - b*c*log(c/d + x - I/d)/(c*d*e**2 + d**2*e**2*x) + I*b*c*atan(c
+ d*x)/(c*d*e**2 + d**2*e**2*x) + b*d*x*log(c/d + x)/(c*d*e**2 + d**2*e**2
*x) - b*d*x*log(c/d + x - I/d)/(c*d*e**2 + d**2*e**2*x) + I*b*d*x*atan(c +
 d*x)/(c*d*e**2 + d**2*e**2*x) - b*atan(c + d*x)/(c*d*e**2 + d**2*e**2*x),
 Ne(d, 0)), (x*(a + b*atan(c))/(c**2*e**2), True))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.51

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^2}\,dx$$
$$= -\frac{1}{2}\left(d\left(\frac{\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^2e^2}-\frac{2\,\log\left(dx+c\right)}{d^2e^2}\right)+\frac{2\,\arctan\left(dx+c\right)}{d^2e^2x+cde^2}\right)b - \frac{a}{d^2e^2x+cde^2}$$

input
```
integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^2,x, algorithm="maxima")
```

output
```
-1/2*(d*(log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2) - 2*log(d*x + c)/(d^2*
e^2)) + 2*arctan(d*x + c)/(d^2*e^2*x + c*d*e^2))*b - a/(d^2*e^2*x + c*d*e^
2)
```

3.5.8 Giac [F]

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^2}\,dx = \int \frac{b\arctan(dx+c)+a}{(dex+ce)^2}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^2}\,dx = \frac{b\,\ln(c+d\,x)}{d\,e^2} - \frac{b\,\mathrm{atan}(c+d\,x)}{x\,d^2\,e^2+c\,d\,e^2} - \frac{b\,\ln(c^2+2\,c\,d\,x+d^2\,x^2+1)}{2\,d\,e^2} - \frac{a}{x\,d^2\,e^2+c\,d\,e^2}$$

input
```
int((a + b*atan(c + d*x))/(c*e + d*e*x)^2,x)
```

output
```
(b*log(c + d*x))/(d*e^2) - (b*atan(c + d*x))/(d^2*e^2*x + c*d*e^2) - (b*lo
g(c^2 + d^2*x^2 + 2*c*d*x + 1))/(2*d*e^2) - a/(d^2*e^2*x + c*d*e^2)
```

3.6 $\int \frac{a+b\arctan(c+dx)}{(ce+dex)^3}\,dx$

3.6.1 Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^3}\,dx=-\frac{b}{2de^3(c+dx)}-\frac{b\arctan(c+dx)}{2de^3}-\frac{a+b\arctan(c+dx)}{2de^3(c+dx)^2}$$

output
```
-1/2*b/d/e^3/(d*x+c)-1/2*b*arctan(d*x+c)/d/e^3+1/2*(-a-b*arctan(d*x+c))/d/
e^3/(d*x+c)^2
```

3.6.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^3}\,dx = -\frac{a+b\arctan(c+dx)+b(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-(c+dx)^2\right)}{2de^3(c+dx)^2}$$

input
```
Integrate[(a + b*ArcTan[c + d*x])/(c*e + d*e*x)^3,x]
```

output
```
-1/2*(a + b*ArcTan[c + d*x] + b*(c + d*x)*Hypergeometric2F1[-1/2, 1, 1/2,
-(c + d*x)^2])/(d*e^3*(c + d*x)^2)
```

3.6.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5566, 27, 5361, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^3}dx$$

$$\Big\downarrow \ 5566$$

$$\frac{\int \frac{a+b\arctan(c+dx)}{e^3(c+dx)^3}d(c+dx)}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{a+b\arctan(c+dx)}{(c+dx)^3}d(c+dx)}{de^3}$$

$$\Big\downarrow \ 5361$$

$$\frac{\frac{1}{2}b\int \frac{1}{(c+dx)^2((c+dx)^2+1)}d(c+dx)-\frac{a+b\arctan(c+dx)}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \ 264$$

$$\frac{\frac{1}{2}b\left(-\int \frac{1}{(c+dx)^2+1}d(c+dx)-\frac{1}{c+dx}\right)-\frac{a+b\arctan(c+dx)}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \ 216$$

$$\frac{\frac{1}{2}b\left(-\arctan(c+dx)-\frac{1}{c+dx}\right)-\frac{a+b\arctan(c+dx)}{2(c+dx)^2}}{de^3}$$

input
```
Int[(a + b*ArcTan[c + d*x])/(c*e + d*e*x)^3,x]
```

output
```
((b*(-(c + d*x)^(-1) - ArcTan[c + d*x]))/2 - (a + b*ArcTan[c + d*x])/(2*(c
 + d*x)^2))/(d*e^3)
```

3.6.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5566
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.6.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{a}{2e^3(dx+c)^2}+\frac{b\left(-\frac{\arctan(dx+c)}{2(dx+c)^2}-\frac{1}{2(dx+c)}-\frac{\arctan(dx+c)}{2}\right)}{e^3}}{d}$
default	$\frac{-\frac{a}{2e^3(dx+c)^2}+\frac{b\left(-\frac{\arctan(dx+c)}{2(dx+c)^2}-\frac{1}{2(dx+c)}-\frac{\arctan(dx+c)}{2}\right)}{e^3}}{d}$
parts	$-\frac{a}{2e^3(dx+c)^2 d}+\frac{b\left(-\frac{\arctan(dx+c)}{2(dx+c)^2}-\frac{1}{2(dx+c)}-\frac{\arctan(dx+c)}{2}\right)}{e^3 d}$
parallelrisch	$\frac{-4b\,d^4\arctan(dx+c)x^2c-8b\,c^2\arctan(dx+c)x\,d^3+b\,d^4x^2-4\arctan(dx+c)b\,c^3d^2-2xbc\,d^3-4b\arctan(dx+c)c\,d^2-3b\,c^2 d}{8(dx+c)^2e^3c\,d^3}$
risch	$\frac{ib\ln(1+i(dx+c))}{4d\,e^3(dx+c)^2}-\frac{-i\ln(-dx-c+i)b\,d^2x^2+i\ln(-dx-c-i)b\,d^2x^2-2i\ln(-dx-c+i)bcdx+2i\ln(-dx-c-i)bcdx-i\ln(}{4e^3(dx+c)^2d}$

input
```
int((a+b*arctan(d*x+c))/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*a/e^3/(d*x+c)^2+b/e^3*(-1/2/(d*x+c)^2*arctan(d*x+c)-1/2/(d*x+c)-
1/2*arctan(d*x+c)))
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^3}\,dx=-\frac{bdx+bc+(bd^2x^2+2\,bcdx+bc^2+b)\arctan{(dx+c)}+a}{2\,(d^3e^3x^2+2\,cd^2e^3x+c^2de^3)}$$

input
```
integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^3,x, algorithm="fricas")
```

output
```
-1/2*(b*d*x + b*c + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + b)*arctan(d*x + c) +
a)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

3.6.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(54) = 108$.

Time = 15.21 (sec) , antiderivative size = 314, normalized size of antiderivative = 4.98

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^3}\,dx$$
$$=\begin{cases}-\frac{a}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2}-\frac{bc^2\operatorname{atan}{(c+dx)}}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2}-\frac{2bcdx\operatorname{atan}{(c+dx)}}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2}-\frac{bc}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2}-\frac{bd^2x^2\operatorname{at}}{2c^2de^3+4cd^2}\\ \frac{x(a+b\operatorname{atan}{(c)})}{c^3e^3}\end{cases}$$

input
```
integrate((a+b*atan(d*x+c))/(d*e*x+c*e)**3,x)
```

output
```
Piecewise((-a/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*c**
2*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*b
*c*d*x*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2)
- b*c/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*d**2*x**2*a
tan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*d*x/
(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*atan(c + d*x)/(2*
c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2), Ne(d, 0)), (x*(a + b*at
an(c))/(c**3*e**3), True))
```

3.6.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(57) = 114$.

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.90

$$\int \frac{a + b\arctan(c + dx)}{(ce + dex)^3}\,dx$$
$$= -\frac{1}{2}\left(d\left(\frac{1}{d^3e^3x + cd^2e^3} + \frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{d^2e^3}\right) + \frac{\arctan(dx + c)}{d^3e^3x^2 + 2\,cd^2e^3x + c^2de^3}\right)b - \frac{a}{2\left(d^3e^3x^2 + 2\,cd^2e^3x + c^2de^3\right)}$$

input
```
integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^3,x, algorithm="maxima")
```

output
```
-1/2*(d*(1/(d^3*e^3*x + c*d^2*e^3) + arctan((d^2*x + c*d)/d)/(d^2*e^3)) +
arctan(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3))*b - 1/2*a/(d^3*
e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

3.6.8 Giac [F]

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^3}\,dx=\int \frac{b\arctan(dx+c)+a}{(dex+ce)^3}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.63

$$\int \frac{a+b\arctan(c+dx)}{(ce+dex)^3}\,dx=-\frac{\frac{a+b\,c}{d}+b\,x}{2\,c^2\,e^3+4\,c\,d\,e^3\,x+2\,d^2\,e^3\,x^2}-\frac{b\,\mathrm{atan}\left(\frac{b\,c+b\,d\,x}{b}\right)}{2\,d\,e^3}-\frac{b\,\mathrm{atan}(c+d\,x)}{2\,d^3\,e^3\,\left(x^2+\frac{c^2}{d^2}+\frac{2\,c\,x}{d}\right)}$$

input
```
int((a + b*atan(c + d*x))/(c*e + d*e*x)^3,x)
```

output
```
- ((a + b*c)/d + b*x)/(2*c^2*e^3 + 2*d^2*e^3*x^2 + 4*c*d*e^3*x) - (b*atan(
(b*c + b*d*x)/b))/(2*d*e^3) - (b*atan(c + d*x))/(2*d^3*e^3*(x^2 + c^2/d^2
+ (2*c*x)/d))
```

3.7 $\int (ce+dex)^3(a+b\arctan(c+dx))^2\,dx$

3.7.1 Optimal result

Integrand size = 23, antiderivative size = 157

$$\int (ce+dex)^3(a+b\arctan(c+dx))^2\,dx = \frac{1}{2}abe^3x+\frac{b^2e^3(c+dx)^2}{12d} + \frac{b^2e^3(c+dx)\arctan(c+dx)}{2d} - \frac{be^3(c+dx)^3(a+b\arctan(c+dx))}{6d} - \frac{e^3(a+b\arctan(c+dx))^2}{4d} + \frac{e^3(c+dx)^4(a+b\arctan(c+dx))^2}{4d} - \frac{b^2e^3\log\left(1+(c+dx)^2\right)}{3d}$$

output
```
1/2*a*b*e^3*x+1/12*b^2*e^3*(d*x+c)^2/d+1/2*b^2*e^3*(d*x+c)*arctan(d*x+c)/d
-1/6*b*e^3*(d*x+c)^3*(a+b*arctan(d*x+c))/d-1/4*e^3*(a+b*arctan(d*x+c))^2/d
+1/4*e^3*(d*x+c)^4*(a+b*arctan(d*x+c))^2/d-1/3*b^2*e^3*ln(1+(d*x+c)^2)/d
```

3.7.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.38

$$\int (ce+dex)^3(a+b\arctan(c+dx))^2\,dx$$
$$=\frac{e^3((c+dx)\left(b^2(c+dx)+3a^2(c+dx)^3-2ab(-3+c^2+2cdx+d^2x^2)\right)+2b(-b(-3c+c^3-3dx+3c^2dx}{}$$

input
```
Integrate[(c*e + d*e*x)^3*(a + b*ArcTan[c + d*x])^2,x]
```

output
```
(e^3*((c + d*x)*(b^2*(c + d*x) + 3*a^2*(c + d*x)^3 - 2*a*b*(-3 + c^2 + 2*c
*d*x + d^2*x^2)) + 2*b*(-(b*(-3*c + c^3 - 3*d*x + 3*c^2*d*x + 3*c*d^2*x^2
+ d^3*x^3)) + 3*a*(-1 + c^4 + 4*c^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^
4*x^4))*ArcTan[c + d*x] + 3*b^2*(-1 + c^4 + 4*c^3*d*x + 6*c^2*d^2*x^2 + 4*
c*d^3*x^3 + d^4*x^4)*ArcTan[c + d*x]^2 - 4*b^2*Log[1 + (c + d*x)^2]))/(12*
d)
```

3.7.3 Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.87, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5566, 27, 5361, 5451, 5361, 243, 49, 2009, 5451, 2009, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^3(a+b\arctan(c+dx))^2\,dx$$

$$\Big\downarrow \ 5566$$

$$\frac{\int e^3(c+dx)^3(a+b\arctan(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{e^3\int (c+dx)^3(a+b\arctan(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \ 5361$$

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\arctan(c+dx))^2-\frac{1}{2}b\int \frac{(c+dx)^4(a+b\arctan(c+dx))}{(c+dx)^2+1}d(c+dx)\right)}{d}$$

$\downarrow$ 5451

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\arctan(c+dx))^2-\frac{1}{2}b\left(\int(c+dx)^2(a+b\arctan(c+dx))d(c+dx)-\int\frac{(c+dx)^2(a+b\arctan(c+dx)}{(c+dx)^2+1}\right.\right.}{d}$$

$\downarrow$ 5361

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\arctan(c+dx))^2-\frac{1}{2}b\left(-\int\frac{(c+dx)^2(a+b\arctan(c+dx))}{(c+dx)^2+1}d(c+dx)-\frac{1}{3}b\int\frac{(c+dx)^3}{(c+dx)^2+1}d(c+dx)+\frac{1}{3}(c\right.\right.}{d}$$

$\downarrow$ 243

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\arctan(c+dx))^2-\frac{1}{2}b\left(-\int\frac{(c+dx)^2(a+b\arctan(c+dx))}{(c+dx)^2+1}d(c+dx)-\frac{1}{6}b\int\frac{(c+dx)^2}{(c+dx)^2+1}d(c+dx)^2+\frac{1}{3}(\right.\right.}{d}$$

$\downarrow$ 49

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\arctan(c+dx))^2-\frac{1}{2}b\left(-\int\frac{(c+dx)^2(a+b\arctan(c+dx))}{(c+dx)^2+1}d(c+dx)-\frac{1}{6}b\int\left(1+\frac{1}{-c-dx-1}\right)d(c+dx)\right.\right.}{d}$$

$\downarrow$ 2009

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\arctan(c+dx))^2-\frac{1}{2}b\left(-\int\frac{(c+dx)^2(a+b\arctan(c+dx))}{(c+dx)^2+1}d(c+dx)+\frac{1}{3}(c+dx)^3(a+b\arctan(c+d\right.\right.}{d}$$

$\downarrow$ 5451

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\arctan(c+dx))^2-\frac{1}{2}b\left(-\int(a+b\arctan(c+dx))d(c+dx)+\int\frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)+\frac{1}{3}(\right.\right.}{d}$$

$\downarrow$ 2009

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\arctan(c+dx))^2-\frac{1}{2}b\left(\int\frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)+\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))-a(c\right.\right.}{d}$$

$\downarrow$ 5419

$$\frac{e^3\left(\frac{1}{4}(c+dx)^4(a+b\arctan(c+dx))^2-\frac{1}{2}b\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))+\frac{(a+b\arctan(c+dx))^2}{2b}-a(c+dx)-b\right.\right.}{d}$$

input
```
Int[(c*e + d*e*x)^3*(a + b*ArcTan[c + d*x])^2,x]
```

3.7. $\int(ce+dex)^3(a+b\arctan(c+dx))^2\,dx$

output
```
(e^3*(((c + d*x)^4*(a + b*ArcTan[c + d*x])^2)/4 - (b*(-(a*(c + d*x)) - b*(
c + d*x)*ArcTan[c + d*x] + ((c + d*x)^3*(a + b*ArcTan[c + d*x]))/3 + (a +
b*ArcTan[c + d*x])^2/(2*b) - (b*((c + d*x)^2 - Log[1 + c + d*x]))/6 + (b*L
og[1 + (c + d*x)^2])/2))/2))/d
```

3.7.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5566
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.7.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{\frac{e^3a^2(dx+c)^4}{4}+e^3b^2\left(\frac{(dx+c)^4\arctan(dx+c)^2}{4}-\frac{(dx+c)^3\arctan(dx+c)}{6}+\frac{(dx+c)\arctan(dx+c)}{2}-\frac{\arctan(dx+c)^2}{4}+\frac{(dx+c)^2}{12}-\frac{\ln\left(1+\right.}{}\right.}{d}$
default	$\frac{\frac{e^3a^2(dx+c)^4}{4}+e^3b^2\left(\frac{(dx+c)^4\arctan(dx+c)^2}{4}-\frac{(dx+c)^3\arctan(dx+c)}{6}+\frac{(dx+c)\arctan(dx+c)}{2}-\frac{\arctan(dx+c)^2}{4}+\frac{(dx+c)^2}{12}-\frac{\ln\left(1+\right.}{}\right.}{d}$
parts	$\frac{e^3a^2(dx+c)^4}{4d}+\frac{e^3b^2\left(\frac{(dx+c)^4\arctan(dx+c)^2}{4}-\frac{(dx+c)^3\arctan(dx+c)}{6}+\frac{(dx+c)\arctan(dx+c)}{2}-\frac{\arctan(dx+c)^2}{4}+\frac{(dx+c)^2}{12}-\frac{\ln(}{}\right.}{d}$
parallelrisch	$-\frac{18e^3dc^2a^2+5e^3dc^2b^2+e^3b^2d-36x^2\arctan(dx+c)abc^2d^3e^3-24x^3\arctan(dx+c)abcd^4e^3-24x\arctan(dx+c)abc^3d^2e}{}$
risch	$ie^3d^2abcx^3\ln(1-i(dx+c))+\frac{3ie^3dabc^2x^2\ln(1-i(dx+c))}{2}-\frac{ie^3db^2cx^2\ln(1-i(dx+c))}{4}+ie^3abc^3x$

input
```
int((d*e*x+c*e)^3*(a+b*arctan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*e^3*a^2*(d*x+c)^4+e^3*b^2*(1/4*(d*x+c)^4*arctan(d*x+c)^2-1/6*(d*x
+c)^3*arctan(d*x+c)+1/2*(d*x+c)*arctan(d*x+c)-1/4*arctan(d*x+c)^2+1/12*(d*
x+c)^2-1/3*ln(1+(d*x+c)^2))+2*e^3*a*b*(1/4*(d*x+c)^4*arctan(d*x+c)-1/12*(d
*x+c)^3+1/4*d*x+1/4*c-1/4*arctan(d*x+c)))
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(143) = 286.

Time = 0.29 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.15

$$\int(ce+dex)^3(a+b\arctan(c+dx))^2\,dx$$
$$=\frac{3a^2d^4e^3x^4+2(6a^2c-ab)d^3e^3x^3+(18a^2c^2-6abc+b^2)d^2e^3x^2+2(6a^2c^3-3abc^2+b^2c+3ab)de^3x-}{}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/12*(3*a^2*d^4*e^3*x^4 + 2*(6*a^2*c - a*b)*d^3*e^3*x^3 + (18*a^2*c^2 - 6*
a*b*c + b^2)*d^2*e^3*x^2 + 2*(6*a^2*c^3 - 3*a*b*c^2 + b^2*c + 3*a*b)*d*e^3
*x - 4*b^2*e^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(b^2*d^4*e^3*x^4 + 4*b
^2*c*d^3*e^3*x^3 + 6*b^2*c^2*d^2*e^3*x^2 + 4*b^2*c^3*d*e^3*x + (b^2*c^4 -
b^2)*e^3)*arctan(d*x + c)^2 + 2*(3*a*b*d^4*e^3*x^4 + (12*a*b*c - b^2)*d^3*
e^3*x^3 + 3*(6*a*b*c^2 - b^2*c)*d^2*e^3*x^2 + 3*(4*a*b*c^3 - b^2*c^2 + b^2
)*d*e^3*x + (3*a*b*c^4 - b^2*c^3 + 3*b^2*c - 3*a*b)*e^3)*arctan(d*x + c))/
d
```

3.7.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 45.36 (sec) , antiderivative size = 583, normalized size of antiderivative = 3.71

$$\int (ce+dex)^3(a+b\arctan(c+dx))^2\,dx$$
$$= \begin{cases} a^2c^3e^3x + \frac{3a^2c^2de^3x^2}{2} + a^2cd^2e^3x^3 + \frac{a^2d^3e^3x^4}{4} + \frac{abc^4e^3\operatorname{atan}(c+dx)}{2d} + 2abc^3e^3x\operatorname{atan}(c+dx) + 3abc^2de^3x^2\operatorname{atan} \\ c^3e^3x(a+b\operatorname{atan}(c))^2 \end{cases}$$

input
```
integrate((d*e*x+c*e)**3*(a+b*atan(d*x+c))**2,x)
```

output
```
Piecewise((a**2*c**3*e**3*x + 3*a**2*c**2*d*e**3*x**2/2 + a**2*c*d**2*e**3
*x**3 + a**2*d**3*e**3*x**4/4 + a*b*c**4*e**3*atan(c + d*x)/(2*d) + 2*a*b*
c**3*e**3*x*atan(c + d*x) + 3*a*b*c**2*d*e**3*x**2*atan(c + d*x) - a*b*c**
2*e**3*x/2 + 2*a*b*c*d**2*e**3*x**3*atan(c + d*x) - a*b*c*d*e**3*x**2/2 +
a*b*d**3*e**3*x**4*atan(c + d*x)/2 - a*b*d**2*e**3*x**3/6 + a*b*e**3*x/2 -
 a*b*e**3*atan(c + d*x)/(2*d) + b**2*c**4*e**3*atan(c + d*x)**2/(4*d) + b*
*2*c**3*e**3*x*atan(c + d*x)**2 - b**2*c**3*e**3*atan(c + d*x)/(6*d) + 3*b
**2*c**2*d*e**3*x**2*atan(c + d*x)**2/2 - b**2*c**2*e**3*x*atan(c + d*x)/2
 + b**2*c*d**2*e**3*x**3*atan(c + d*x)**2 - b**2*c*d*e**3*x**2*atan(c + d*
x)/2 + b**2*c*e**3*x/6 + b**2*c*e**3*atan(c + d*x)/(2*d) + b**2*d**3*e**3*
x**4*atan(c + d*x)**2/4 - b**2*d**2*e**3*x**3*atan(c + d*x)/6 + b**2*d*e**
3*x**2/12 + b**2*e**3*x*atan(c + d*x)/2 - 2*b**2*e**3*log(c/d + x - I/d)/(
3*d) - b**2*e**3*atan(c + d*x)**2/(4*d) + 2*I*b**2*e**3*atan(c + d*x)/(3*d
), Ne(d, 0)), (c**3*e**3*x*(a + b*atan(c))**2, True))
```

3.7.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(143) = 286$.

Time = 1.06 (sec) , antiderivative size = 597, normalized size of antiderivative = 3.80

$$\int (ce+dex)^3(a+b\arctan(c+dx))^2\,dx = \frac{1}{4}a^2d^3e^3x^4 + a^2cd^2e^3x^3 + \frac{3}{2}a^2c^2de^3x^2$$
$$+3\left(x^2\arctan(dx+c) - d\left(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log(d^2x^2+2\,cdx+c^2+1)}{d^3}\right)\right)abc^2de^3$$
$$+\left(2\,x^3\arctan(dx+c) - d\left(\frac{dx^2-4\,cx}{d^3} - \frac{2\,(c^3-3\,c)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^4} + \frac{(3\,c^2-1)\log(d^2x^2+2\,cdx+c\right.\right.$$
$$+\frac{1}{6}\left(3\,x^4\arctan(dx+c) - d\left(\frac{d^2x^3-3\,cdx^2+3\,(3\,c^2-1)x}{d^4} + \frac{3\,(c^4-6\,c^2+1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^5} - \frac{6\,(c^3\right.\right.$$
$$+a^2c^3e^3x + \frac{\left(2\,(dx+c)\arctan(dx+c) - \log\left((dx+c)^2+1\right)\right)abc^3e^3}{d}$$
$$+\frac{b^2d^2e^3x^2+2\,b^2cde^3x-4\,b^2e^3\log(d^2x^2+2\,cdx+c^2+1)+3\,(b^2d^4e^3x^4+4\,b^2cd^3e^3x^3+6\,b^2c^2d^2e^3x^2+4}{}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/4*a^2*d^3*e^3*x^4 + a^2*c*d^2*e^3*x^3 + 3/2*a^2*c^2*d*e^3*x^2 + 3*(x^2*a
rctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(
d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*c^2*d*e^3 + (2*x^3*arctan(d*x + c)
- d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*
c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a*b*c*d^2*e^3 + 1/6*(3*x^4
*arctan(d*x + c) - d*((d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 - 1)*x)/d^4 + 3*(c^4
 - 6*c^2 + 1)*arctan((d^2*x + c*d)/d)/d^5 - 6*(c^3 - c)*log(d^2*x^2 + 2*c*
d*x + c^2 + 1)/d^5))*a*b*d^3*e^3 + a^2*c^3*e^3*x + (2*(d*x + c)*arctan(d*x
 + c) - log((d*x + c)^2 + 1))*a*b*c^3*e^3/d + 1/12*(b^2*d^2*e^3*x^2 + 2*b^
2*c*d*e^3*x - 4*b^2*e^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(b^2*d^4*e^3*
x^4 + 4*b^2*c*d^3*e^3*x^3 + 6*b^2*c^2*d^2*e^3*x^2 + 4*b^2*c^3*d*e^3*x + (b
^2*c^4 - b^2)*e^3)*arctan(d*x + c)^2 - 2*(b^2*d^3*e^3*x^3 + 3*b^2*c*d^2*e^
3*x^2 + 3*(b^2*c^2 - b^2)*d*e^3*x + (b^2*c^3 - 3*b^2*c)*e^3)*arctan(d*x +
c))/d
```

3.7.8 Giac [F]

$$\int (ce+dex)^3(a+b\arctan(c+dx))^2\,dx=\int (dex+ce)^3(b\arctan(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.7.9 Mupad [B] (verification not implemented)

Time = 3.68 (sec) , antiderivative size = 633, normalized size of antiderivative = 4.03

$$\int (ce+dex)^3(a+b\arctan(c+dx))^2\,dx$$
$$= x\left(\frac{c\,e^3\,(20\,a^2\,c^2+6\,a^2-6\,a\,b\,c+b^2)}{2}+\frac{(6\,c^2+6)\,\left(2\,a^2\,c\,d^2\,e^3+\frac{a\,d^2\,e^3\,(b-10\,a\,c)}{2}\right)}{6\,d^2}-\frac{2\,c\left(\frac{2\,c\left(2\,a^2\,c\,d^2\,e^3+\frac{a\,d^2\,e^3\,(b-10\,a\,c)}{2}\right)}{d}+\frac{d\,e^3\,(60\,a^2\,c^2+6\,a^2-12\,a\,b\,c+b^2)}{6}-\frac{a^2\,d\,e^3\,(6\,c^2+6)}{6}\right)}{d}\right)+x^2\left(\frac{c\left(2\,a^2\,c\,d^2\,e^3+\frac{a\,d^2\,e^3\,(b-10\,a\,c)}{2}\right)}{d}+\frac{d\,e^3\,(60\,a^2\,c^2+6\,a^2-12\,a\,b\,c+b^2)}{12}-\frac{a^2\,d\,e^3\,(6\,c^2+6)}{12}\right)-x^3\left(\frac{2\,a^2\,c\,d^2\,e^3}{3}+\frac{a\,d^2\,e^3\,(b-10\,a\,c)}{6}\right)+\mathrm{atan}(c+d\,x)^2\left(b^2\,c^3\,e^3\,x-\frac{b^2\,e^3-b^2\,c^4\,e^3}{4\,d}+\frac{b^2\,d^3\,e^3\,x^4}{4}+\frac{3\,b^2\,c^2\,d\,e^3\,x^2}{2}+b^2\,c\,d^2\,e^3\,x^3\right)-d^2\,\mathrm{atan}(c+d\,x)\left(x^3\left(\frac{b^2\,e^3}{6}-2\,a\,b\,c\,e^3\right)-\frac{x\,(-b^2\,c^2\,e^3+b^2\,e^3+4\,a\,b\,c^3\,e^3)}{2\,d^2}+\frac{x^2\,(b^2\,c\,e^3-6\,a\,b\,c^2\,e^3)}{2\,d}-\frac{a\,b\,d\,e^3\,x^4}{2}\right)+\frac{a^2\,d^3\,e^3\,x^4}{4}-\frac{b^2\,e^3\,\ln(c^2+2\,c\,d\,x+d^2\,x^2+1)}{3\,d}+\frac{b\,e^3\,\mathrm{atan}\left(\frac{\frac{b\,c\,e^3\,\left(-3\,a\,c^4+b\,c^3-3\,b\,c+3\,a\right)}{6}+\frac{b\,d\,e^3\,x\,\left(-3\,a\,c^4+b\,c^3-3\,b\,c+3\,a\right)}{6}}{-\frac{b^2\,c^3\,e^3}{6}+\frac{b^2\,c\,e^3}{2}+\frac{a\,b\,c^4\,e^3}{2}-\frac{a\,b\,e^3}{2}}\right)\,(-3\,a\,c^4+b\,c^3-3\,b\,c+3\,a)}{6\,d}$$

input
```
int((c*e + d*e*x)^3*(a + b*atan(c + d*x))^2,x)
```

output
```
x*((c*e^3*(6*a^2 + b^2 + 20*a^2*c^2 - 6*a*b*c))/2 + ((6*c^2 + 6)*(2*a^2*c*
d^2*e^3 + (a*d^2*e^3*(b - 10*a*c))/2))/(6*d^2) - (2*c*((2*c*(2*a^2*c*d^2*e
^3 + (a*d^2*e^3*(b - 10*a*c))/2))/d + (d*e^3*(6*a^2 + b^2 + 60*a^2*c^2 - 1
2*a*b*c))/6 - (a^2*d*e^3*(6*c^2 + 6))/6))/d) + x^2*((c*(2*a^2*c*d^2*e^3 +
(a*d^2*e^3*(b - 10*a*c))/2))/d + (d*e^3*(6*a^2 + b^2 + 60*a^2*c^2 - 12*a*b
*c))/12 - (a^2*d*e^3*(6*c^2 + 6))/12) - x^3*((2*a^2*c*d^2*e^3)/3 + (a*d^2*
e^3*(b - 10*a*c))/6) + atan(c + d*x)^2*(b^2*c^3*e^3*x - (b^2*e^3 - b^2*c^4
*e^3)/(4*d) + (b^2*d^3*e^3*x^4)/4 + (3*b^2*c^2*d*e^3*x^2)/2 + b^2*c*d^2*e^
3*x^3) - d^2*atan(c + d*x)*(x^3*((b^2*e^3)/6 - 2*a*b*c*e^3) - (x*(b^2*e^3
- b^2*c^2*e^3 + 4*a*b*c^3*e^3))/(2*d^2) + (x^2*(b^2*c*e^3 - 6*a*b*c^2*e^3)
)/(2*d) - (a*b*d*e^3*x^4)/2) + (a^2*d^3*e^3*x^4)/4 - (b^2*e^3*log(c^2 + d^
2*x^2 + 2*c*d*x + 1))/(3*d) + (b*e^3*atan(((b*c*e^3*(3*a - 3*b*c - 3*a*c^4
 + b*c^3))/6 + (b*d*e^3*x*(3*a - 3*b*c - 3*a*c^4 + b*c^3))/6)/((b^2*c*e^3)
/2 - (b^2*c^3*e^3)/6 - (a*b*e^3)/2 + (a*b*c^4*e^3)/2))*(3*a - 3*b*c - 3*a*
c^4 + b*c^3))/(6*d)
```

3.7. $\int (ce+dex)^3(a+b\arctan(c+dx))^2\,dx$

3.8 $\int (ce+dex)^2(a+b\arctan(c+dx))^2\,dx$

3.8.1 Optimal result

Integrand size = 23, antiderivative size = 183

$$\begin{aligned}\int (ce+dex)^2(a+b\arctan(c+dx))^2\,dx &= \frac{1}{3}b^2e^2x-\frac{b^2e^2\arctan(c+dx)}{3d}\\&\quad-\frac{be^2(c+dx)^2(a+b\arctan(c+dx))}{3d}\\&\quad-\frac{ie^2(a+b\arctan(c+dx))^2}{3d}\\&\quad+\frac{e^2(c+dx)^3(a+b\arctan(c+dx))^2}{3d}\\&\quad-\frac{2be^2(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{3d}\\&\quad-\frac{ib^2e^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{3d}\end{aligned}$$

output
```
1/3*b^2*e^2*x-1/3*b^2*e^2*arctan(d*x+c)/d-1/3*b*e^2*(d*x+c)^2*(a+b*arctan(
d*x+c))/d-1/3*I*e^2*(a+b*arctan(d*x+c))^2/d+1/3*e^2*(d*x+c)^3*(a+b*arctan(
d*x+c))^2/d-2/3*b*e^2*(a+b*arctan(d*x+c))*ln(2/(1+I*(d*x+c)))/d-1/3*I*b^2*
e^2*polylog(2,1-2/(1+I*(d*x+c)))/d
```

3.8.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.89

$$\int (ce+dex)^2(a+b\arctan(c+dx))^2\,dx$$
$$=\frac{e^2\left(a^2(c+dx)^3+ab(-(c+dx)^2+2(c+dx)^3\arctan(c+dx)+\log\left(1+(c+dx)^2\right))+b^2\left(c+dx-\arctan\right.\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x])^2,x]
```

output
```
(e^2*(a^2*(c + d*x)^3 + a*b*(-(c + d*x)^2 + 2*(c + d*x)^3*ArcTan[c + d*x]
+ Log[1 + (c + d*x)^2]) + b^2*(c + d*x - ArcTan[c + d*x] - (c + d*x)^2*Arc
Tan[c + d*x] + I*ArcTan[c + d*x]^2 + (c + d*x)^3*ArcTan[c + d*x]^2 - 2*Arc
Tan[c + d*x]*Log[1 + E^((2*I)*ArcTan[c + d*x])] + I*PolyLog[2, -E^((2*I)*A
rcTan[c + d*x])])))/(3*d)
```

3.8.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.82, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {5566, 27, 5361, 5451, 5361, 262, 216, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^2(a+b\arctan(c+dx))^2\,dx$$

$$\Big\downarrow \; 5566$$

$$\frac{\int e^2(c+dx)^2(a+b\arctan(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{e^2\int (c+dx)^2(a+b\arctan(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \; 5361$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))^2-\frac{2}{3}b\int \frac{(c+dx)^3(a+b\arctan(c+dx))}{(c+dx)^2+1}d(c+dx)\right)}{d}$$

$$\Big\downarrow \; 5451$$

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))^2-\frac{2}{3}b\left(\int(c+dx)(a+b\arctan(c+dx))d(c+dx)-\int\frac{(c+dx)(a+b\arctan(c+dx))}{(c+dx)^2+1}d\right.\right.}{d}$$

$\Big\downarrow$ 5361

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))^2-\frac{2}{3}b\left(-\int\frac{(c+dx)(a+b\arctan(c+dx))}{(c+dx)^2+1}d(c+dx)-\frac{1}{2}b\int\frac{(c+dx)^2}{(c+dx)^2+1}d(c+dx)+\frac{1}{2}(c\right.\right.}{d}$$

$\Big\downarrow$ 262

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))^2-\frac{2}{3}b\left(-\int\frac{(c+dx)(a+b\arctan(c+dx))}{(c+dx)^2+1}d(c+dx)-\frac{1}{2}b\left(-\int\frac{1}{(c+dx)^2+1}d(c+dx)+c\right.\right.\right.}{d}$$

$\Big\downarrow$ 216

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))^2-\frac{2}{3}b\left(-\int\frac{(c+dx)(a+b\arctan(c+dx))}{(c+dx)^2+1}d(c+dx)+\frac{1}{2}(c+dx)^2(a+b\arctan(c+dx\right.\right.}{d}$$

$\Big\downarrow$ 5455

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))^2-\frac{2}{3}b\left(\int\frac{a+b\arctan(c+dx)}{-c-dx+i}d(c+dx)+\frac{1}{2}(c+dx)^2(a+b\arctan(c+dx))+\frac{i(a+b}{}\right.\right.}{d}$$

$\Big\downarrow$ 5379

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))^2-\frac{2}{3}b\left(-b\int\frac{\log\left(\frac{2}{i(c+dx)+1}\right)}{(c+dx)^2+1}d(c+dx)+\frac{1}{2}(c+dx)^2(a+b\arctan(c+dx))+\frac{i(}{}\right.\right.}{d}$$

$\Big\downarrow$ 2849

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))^2-\frac{2}{3}b\left(ib\int\frac{\log\left(\frac{2}{i(c+dx)+1}\right)}{1-\frac{2}{i(c+dx)+1}}d\frac{1}{i(c+dx)+1}+\frac{1}{2}(c+dx)^2(a+b\arctan(c+dx))+\frac{i(}{}\right.\right.}{d}$$

$\Big\downarrow$ 2752

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))^2-\frac{2}{3}b\left(\frac{1}{2}(c+dx)^2(a+b\arctan(c+dx))+\frac{i(a+b\arctan(c+dx))^2}{2b}+\log\left(\frac{2}{1+i(c+dx}\right.\right.\right.}{d}$$

input
```
Int[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x])^2,x]
```

3.8. $\int(ce+dex)^2(a+b\arctan(c+dx))^2\,dx$

output
```
(e^2*(((c + d*x)^3*(a + b*ArcTan[c + d*x])^2)/3 - (2*b*(-1/2*(b*(c + d*x -
 ArcTan[c + d*x])) + ((c + d*x)^2*(a + b*ArcTan[c + d*x]))/2 + ((I/2)*(a +
 b*ArcTan[c + d*x])^2)/b + (a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))
] + (I/2)*b*PolyLog[2, 1 - 2/(1 + I*(c + d*x))]))/3))/d
```

3.8.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5566
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.8.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.51

method	result
derivativedivides	$\frac{\frac{e^2a^2(dx+c)^3}{3}+b^2e^2\left(\frac{(dx+c)^3\arctan(dx+c)^2}{3}-\frac{(dx+c)^2\arctan(dx+c)}{3}+\frac{\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{3}+\frac{dx}{3}+\frac{c}{3}-\frac{\arctan(dx+c)}{3}+\cdots\right.}{\cdots}$
default	$\frac{\frac{e^2a^2(dx+c)^3}{3}+b^2e^2\left(\frac{(dx+c)^3\arctan(dx+c)^2}{3}-\frac{(dx+c)^2\arctan(dx+c)}{3}+\frac{\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{3}+\frac{dx}{3}+\frac{c}{3}-\frac{\arctan(dx+c)}{3}+\cdots\right.}{\cdots}$
parts	$\frac{e^2a^2(dx+c)^3}{3d}+\frac{b^2e^2\left(\frac{(dx+c)^3\arctan(dx+c)^2}{3}-\frac{(dx+c)^2\arctan(dx+c)}{3}+\frac{\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{3}+\frac{dx}{3}+\frac{c}{3}-\frac{\arctan(dx+c)}{3}\cdots\right.}{\cdots}$
risch	Expression too large to display

input
```
int((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*e^2*a^2*(d*x+c)^3+b^2*e^2*(1/3*(d*x+c)^3*arctan(d*x+c)^2-1/3*(d*x
+c)^2*arctan(d*x+c)+1/3*arctan(d*x+c)*ln(1+(d*x+c)^2)+1/3*d*x+1/3*c-1/3*ar
ctan(d*x+c)+1/6*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog(-1/
2*I*(d*x+c+I))-ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))-1/6*I*(ln(d*x+c+I)*ln(1+(
d*x+c)^2)-1/2*ln(d*x+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I*(d
*x+c-I))))+2*e^2*a*b*(1/3*(d*x+c)^3*arctan(d*x+c)-1/6*(d*x+c)^2+1/6*ln(1+(
d*x+c)^2)))
```

3.8.5 Fricas [F]

$$\int (ce+dex)^2(a+b\arctan(c+dx))^2\,dx=\int (dex+ce)^2(b\arctan(dx+c)+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(a^2*d^2*e^2*x^2 + 2*a^2*c*d*e^2*x + a^2*c^2*e^2 + (b^2*d^2*e^2*x^
2 + 2*b^2*c*d*e^2*x + b^2*c^2*e^2)*arctan(d*x + c)^2 + 2*(a*b*d^2*e^2*x^2
+ 2*a*b*c*d*e^2*x + a*b*c^2*e^2)*arctan(d*x + c), x)
```

3.8.6 Sympy [F]

$$\begin{aligned}\int (ce+dex)^2(a+b\arctan(c+dx))^2\,dx\\ =e^2\left(\int a^2c^2\,dx+\int a^2d^2x^2\,dx+\int b^2c^2\operatorname{atan}^2(c+dx)\,dx+\int 2abc^2\operatorname{atan}(c+dx)\,dx\right.\\ +\int 2a^2cdx\,dx+\int b^2d^2x^2\operatorname{atan}^2(c+dx)\,dx+\int 2abd^2x^2\operatorname{atan}(c+dx)\,dx\\ \left.+\int 2b^2cdx\operatorname{atan}^2(c+dx)\,dx+\int 4abcdx\operatorname{atan}(c+dx)\,dx\right)\end{aligned}$$

input
```
integrate((d*e*x+c*e)**2*(a+b*atan(d*x+c))**2,x)
```

output
```
e**2*(Integral(a**2*c**2, x) + Integral(a**2*d**2*x**2, x) + Integral(b**2
*c**2*atan(c + d*x)**2, x) + Integral(2*a*b*c**2*atan(c + d*x), x) + Integ
ral(2*a**2*c*d*x, x) + Integral(b**2*d**2*x**2*atan(c + d*x)**2, x) + Inte
gral(2*a*b*d**2*x**2*atan(c + d*x), x) + Integral(2*b**2*c*d*x*atan(c + d*
x)**2, x) + Integral(4*a*b*c*d*x*atan(c + d*x), x))
```

3.8.7 Maxima [F]

$$\int (ce + dex)^2(a + b\arctan(c + dx))^2\,dx = \int (dex + ce)^2(b\arctan(dx + c) + a)^2\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

output
```
3/4*b^2*c^4*e^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 1/4*(3*arcta
n(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*b^2*
c^4*e^2 + 1/3*a^2*d^2*e^2*x^3 + 36*b^2*d^4*e^2*integrate(1/48*x^4*arctan(d
*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*d^4*e^2*integrate(1/48
*x^4*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
+ 144*b^2*c*d^3*e^2*integrate(1/48*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*
x + c^2 + 1), x) + 4*b^2*d^4*e^2*integrate(1/48*x^4*log(d^2*x^2 + 2*c*d*x
+ c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*c*d^3*e^2*integrate(
1/48*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1),
 x) + 216*b^2*c^2*d^2*e^2*integrate(1/48*x^2*arctan(d*x + c)^2/(d^2*x^2 +
2*c*d*x + c^2 + 1), x) + 16*b^2*c*d^3*e^2*integrate(1/48*x^3*log(d^2*x^2 +
 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 18*b^2*c^2*d^2*e^2
*integrate(1/48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x
+ c^2 + 1), x) + 144*b^2*c^3*d*e^2*integrate(1/48*x*arctan(d*x + c)^2/(d^2
*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^2*c^2*d^2*e^2*integrate(1/48*x^2*log(
d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*c^
3*d*e^2*integrate(1/48*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c
*d*x + c^2 + 1), x) + 12*b^2*c^3*d*e^2*integrate(1/48*x*log(d^2*x^2 + 2*c*
d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*c^4*e^2*integrate
(1/48*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1),...
```

3.8.8 Giac [F]

$$\int (ce + dex)^2 (a + b \arctan(c + dx))^2 \, dx = \int (dex + ce)^2 (b \arctan(dx + c) + a)^2 \, dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int (ce + dex)^2 (a + b \arctan(c + dx))^2 \, dx = \int (c\,e + d\,e\,x)^2 \, (a + b\,\mathrm{atan}(c + d\,x))^2 \, dx$$

input
```
int((c*e + d*e*x)^2*(a + b*atan(c + d*x))^2,x)
```

output
```
int((c*e + d*e*x)^2*(a + b*atan(c + d*x))^2, x)
```

3.9 $\int (ce+dex)(a+b\arctan(c+dx))^2\,dx$

3.9.1 Optimal result

Integrand size = 21, antiderivative size = 95

$$\int (ce+dex)(a+b\arctan(c+dx))^2\,dx = -abex-\frac{b^2e(c+dx)\arctan(c+dx)}{d}+\frac{e(a+b\arctan(c+dx))^2}{2d}+\frac{e(c+dx)^2(a+b\arctan(c+dx))^2}{2d}+\frac{b^2e\log\left(1+(c+dx)^2\right)}{2d}$$

output
```
-a*b*e*x-b^2*e*(d*x+c)*arctan(d*x+c)/d+1/2*e*(a+b*arctan(d*x+c))^2/d+1/2*e
*(d*x+c)^2*(a+b*arctan(d*x+c))^2/d+1/2*b^2*e*ln(1+(d*x+c)^2)/d
```

3.9.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int (ce+dex)(a+b\arctan(c+dx))^2\,dx = \frac{e(a(c+dx)(-2b+ac+adx)+2b(-b(c+dx)+a(1+c^2+2cdx+d^2x^2))\arctan(c+dx)+b^2(1+c^2+\cdots}{2d}$$

input
```
Integrate[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^2,x]
```

output
```
(e*(a*(c + d*x)*(-2*b + a*c + a*d*x) + 2*b*(-(b*(c + d*x)) + a*(1 + c^2 +
2*c*d*x + d^2*x^2))*ArcTan[c + d*x] + b^2*(1 + c^2 + 2*c*d*x + d^2*x^2)*Ar
cTan[c + d*x]^2 + b^2*Log[1 + (c + d*x)^2]))/(2*d)
```

3.9.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5566, 27, 5361, 5451, 2009, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)(a+b\arctan(c+dx))^2\,dx$$

$$\Big\downarrow \text{5566}$$

$$\frac{\int e(c+dx)(a+b\arctan(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int (c+dx)(a+b\arctan(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\arctan(c+dx))^2-b\int \frac{(c+dx)^2(a+b\arctan(c+dx))}{(c+dx)^2+1}d(c+dx)\right)}{d}$$

$$\Big\downarrow \text{5451}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\arctan(c+dx))^2-b\left(\int (a+b\arctan(c+dx))d(c+dx)-\int \frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)\right)\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\arctan(c+dx))^2-b\left(-\int \frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)+a(c+dx)+b(c+dx)\arctan(c+dx)-\right.\right.}{d}$$

$$\Big\downarrow \text{5419}$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\arctan(c+dx))^2-b\left(-\frac{(a+b\arctan(c+dx))^2}{2b}+a(c+dx)+b(c+dx)\arctan(c+dx)-\frac{1}{2}b\log\left((c\right.\right.\right.}{d}$$

input
```
Int[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^2,x]
```

output
```
(e*(((c + d*x)^2*(a + b*ArcTan[c + d*x])^2)/2 - b*(a*(c + d*x) + b*(c + d*
x)*ArcTan[c + d*x] - (a + b*ArcTan[c + d*x])^2/(2*b) - (b*Log[1 + (c + d*x
)^2])/2)))/d
```

3.9.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5566
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.9.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{\frac{e a^2 (dx+c)^2}{2}+e b^2\left(\frac{(dx+c)^2 \arctan(dx+c)^2}{2}+\frac{\arctan(dx+c)^2}{2}-(dx+c)\arctan(dx+c)+\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)+2eab\left(\frac{(dx+c)^2 \operatorname{arcta}}{2}\right.}{d}$
default	$\frac{\frac{e a^2 (dx+c)^2}{2}+e b^2\left(\frac{(dx+c)^2 \arctan(dx+c)^2}{2}+\frac{\arctan(dx+c)^2}{2}-(dx+c)\arctan(dx+c)+\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)+2eab\left(\frac{(dx+c)^2 \operatorname{arcta}}{2}\right.}{d}$
parts	$e a^2\left(\frac{1}{2}d x^2+cx\right)+\frac{e b^2\left(\frac{(dx+c)^2 \arctan(dx+c)^2}{2}+\frac{\arctan(dx+c)^2}{2}-(dx+c)\arctan(dx+c)+\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{d}+\frac{2ea}{}$
parallelrisch	$d^3 e b^2 \arctan(dx+c)^2 x^2+2x^2\arctan(dx+c)ab\,d^3e+2ce\,b^2\arctan(dx+c)^2 x\,d^2+x^2a^2d^3e+4x\arctan(dx+c)abc\,d^2e+\operatorname{arcta}$
risch	$-\frac{e b^2\left(d^2x^2+2cdx+c^2+1\right)\ln(1+i(dx+c))^2}{8d}+\frac{be\left(-2ia\,d^2x^2+b\,d^2x^2\ln(1-i(dx+c))-4iacdx+2bcdx\ln(1-i(dx+c))+2\right.}{4d}$

input
```
int((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*e*a^2*(d*x+c)^2+e*b^2*(1/2*(d*x+c)^2*arctan(d*x+c)^2+1/2*arctan(d
*x+c)^2-(d*x+c)*arctan(d*x+c)+1/2*ln(1+(d*x+c)^2))+2*e*a*b*(1/2*(d*x+c)^2*
arctan(d*x+c)-1/2*d*x-1/2*c+1/2*arctan(d*x+c)))
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.58

$$\int (ce+dex)(a+b\arctan(c+dx))^2\,dx$$
$$=\frac{a^2d^2ex^2+2\left(a^2c-ab\right)dex+b^2e\log\left(d^2x^2+2\,cdx+c^2+1\right)+\left(b^2d^2ex^2+2\,b^2cdex+\left(b^2c^2+b^2\right)e\right)\arctan\left(\right.}{2\,d}$$

input
```
integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(a^2*d^2*e*x^2 + 2*(a^2*c - a*b)*d*e*x + b^2*e*log(d^2*x^2 + 2*c*d*x +
 c^2 + 1) + (b^2*d^2*e*x^2 + 2*b^2*c*d*e*x + (b^2*c^2 + b^2)*e)*arctan(d*x
 + c)^2 + 2*(a*b*d^2*e*x^2 + (2*a*b*c - b^2)*d*e*x + (a*b*c^2 - b^2*c + a*
b)*e)*arctan(d*x + c))/d
```

3.9.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.31 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.53

$$\int (ce+dex)(a+b\arctan(c+dx))^2\,dx$$
$$=\begin{cases} a^2cex+\frac{a^2dex^2}{2}+\frac{abc^2e\,\mathrm{atan}\,(c+dx)}{d}+2abcex\,\mathrm{atan}\,(c+dx)+abdex^2\,\mathrm{atan}\,(c+dx)-abex+\frac{abe\,\mathrm{atan}\,(c+dx)}{d}+ \\ cex(a+b\,\mathrm{atan}\,(c))^2 \end{cases}$$

input
```
integrate((d*e*x+c*e)*(a+b*atan(d*x+c))**2,x)
```

output
```
Piecewise((a**2*c*e*x + a**2*d*e*x**2/2 + a*b*c**2*e*atan(c + d*x)/d + 2*a
*b*c*e*x*atan(c + d*x) + a*b*d*e*x**2*atan(c + d*x) - a*b*e*x + a*b*e*atan
(c + d*x)/d + b**2*c**2*e*atan(c + d*x)**2/(2*d) + b**2*c*e*x*atan(c + d*x
)**2 - b**2*c*e*atan(c + d*x)/d + b**2*d*e*x**2*atan(c + d*x)**2/2 - b**2*
e*x*atan(c + d*x) + b**2*e*log(c/d + x - I/d)/d + b**2*e*atan(c + d*x)**2/
(2*d) - I*b**2*e*atan(c + d*x)/d, Ne(d, 0)), (c*e*x*(a + b*atan(c))**2, Tr
ue))
```

3.9.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(89) = 178.

Time = 0.99 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.29

$$\int (ce+dex)(a+b\arctan(c+dx))^2\,dx=\frac{1}{2}a^2dex^2$$
$$+\left(x^2\arctan(dx+c)-d\left(\frac{x}{d^2}+\frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log(d^2x^2+2\,cdx+c^2+1)}{d^3}\right)\right)abde$$
$$+a^2cex+\frac{(2\,(dx+c)\arctan(dx+c)-\log\left((dx+c)^2+1\right))abce}{d}$$
$$+\frac{b^2e\log(d^2x^2+2\,cdx+c^2+1)+(b^2d^2ex^2+2\,b^2cdex+(b^2c^2+b^2)e)\arctan(dx+c)^2-2\,(b^2dex+b^2ce}{2\,d}$$

input
```
integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*d*e*x^2 + (x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*
x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*d*e + a^2*c
*e*x + (2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a*b*c*e/d + 1/
2*(b^2*e*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^2*e*x^2 + 2*b^2*c*d*e*x
 + (b^2*c^2 + b^2)*e)*arctan(d*x + c)^2 - 2*(b^2*d*e*x + b^2*c*e)*arctan(d
*x + c))/d
```

3.9.8 Giac [F]

$$\int (ce+dex)(a+b\arctan(c+dx))^2\,dx=\int (dex+ce)(b\arctan{(dx+c)}+a)^2\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.9.9 Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.27

$$\begin{aligned}&\int (ce+dex)(a+b\arctan(c+dx))^2\,dx\\&=\operatorname{atan}(c+d\,x)^2\left(\frac{e\,b^2\,c^2+e\,b^2}{2\,d}+b^2\,c\,e\,x+\frac{b^2\,d\,e\,x^2}{2}\right)-x\,\left(a\,e\,(b-3\,a\,c)+2\,a^2\,c\,e\right)\\&\quad-d^2\operatorname{atan}(c+d\,x)\,\left(\frac{x\,(b^2\,e-2\,a\,b\,c\,e)}{d^2}-\frac{a\,b\,e\,x^2}{d}\right)+\frac{b^2\,e\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d}\\&\quad+\frac{a^2\,d\,e\,x^2}{2}+\frac{b\,e\operatorname{atan}\left(\frac{b\,c\,e\,(a\,c^2-b\,c+a)+b\,d\,e\,x\,(a\,c^2-b\,c+a)}{-e\,b^2\,c+a\,e\,b\,c^2+a\,e\,b}\right)\,(a\,c^2-b\,c+a)}{d}\end{aligned}$$

input
```
int((c*e + d*e*x)*(a + b*atan(c + d*x))^2,x)
```

output
```
atan(c + d*x)^2*((b^2*e + b^2*c^2*e)/(2*d) + b^2*c*e*x + (b^2*d*e*x^2)/2)
- x*(a*e*(b - 3*a*c) + 2*a^2*c*e) - d^2*atan(c + d*x)*((x*(b^2*e - 2*a*b*c
*e))/d^2 - (a*b*e*x^2)/d) + (b^2*e*log(c^2 + d^2*x^2 + 2*c*d*x + 1))/(2*d)
 + (a^2*d*e*x^2)/2 + (b*e*atan((b*c*e*(a - b*c + a*c^2) + b*d*e*x*(a - b*c
 + a*c^2))/(a*b*e - b^2*c*e + a*b*c^2*e))*(a - b*c + a*c^2))/d
```

3.10 $\int \frac{(a+b\arctan(c+dx))^2}{ce+dex}\,dx$

3.10.1 Optimal result

Integrand size = 23, antiderivative size = 183

$$\int \frac{(a+b\arctan(c+dx))^2}{ce+dex}\,dx = \frac{2(a+b\arctan(c+dx))^2\operatorname{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{ib(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{de} + \frac{ib(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+i(c+dx)}\right)}{de} - \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2de} + \frac{b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+i(c+dx)}\right)}{2de}$$

output
```
-2*(a+b*arctan(d*x+c))^2*arctanh(-1+2/(1+I*(d*x+c)))/d/e-I*b*(a+b*arctan(d
*x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d/e+I*b*(a+b*arctan(d*x+c))*polylog(2,
-1+2/(1+I*(d*x+c)))/d/e-1/2*b^2*polylog(3,1-2/(1+I*(d*x+c)))/d/e+1/2*b^2*p
olylog(3,-1+2/(1+I*(d*x+c)))/d/e
```

3.10.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 381 vs. 2(183) = 366.

Time = 0.30 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.08

$$\int \frac{(a+b\arctan(c+dx))^2}{ce+dex}\,dx$$
$$= \frac{-6iab\pi^2 - ib^2\pi^3 + 24iab\pi\arctan(c+dx) - 48iab\arctan(c+dx)^2 + 16ib^2\arctan(c+dx)^3 - ab\pi\log(167}{}$$

input
```
Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x),x]
```

output
```
((-6*I)*a*b*Pi^2 - I*b^2*Pi^3 + (24*I)*a*b*Pi*ArcTan[c + d*x] - (48*I)*a*b
*ArcTan[c + d*x]^2 + (16*I)*b^2*ArcTan[c + d*x]^3 - a*b*Pi*Log[16777216] +
 24*b^2*ArcTan[c + d*x]^2*Log[1 - E^((-2*I)*ArcTan[c + d*x])] + 24*a*b*Pi*
Log[1 + E^((-2*I)*ArcTan[c + d*x])] - 48*a*b*ArcTan[c + d*x]*Log[1 + E^((-
2*I)*ArcTan[c + d*x])] + 48*a*b*ArcTan[c + d*x]*Log[1 - E^((2*I)*ArcTan[c
+ d*x])] - 24*b^2*ArcTan[c + d*x]^2*Log[1 + E^((2*I)*ArcTan[c + d*x])] + 2
4*a^2*Log[c + d*x] + 12*a*b*Pi*Log[1 + c^2 + 2*c*d*x + d^2*x^2] - (24*I)*a
*b*PolyLog[2, -E^((-2*I)*ArcTan[c + d*x])] + (24*I)*b^2*ArcTan[c + d*x]*Po
lyLog[2, E^((-2*I)*ArcTan[c + d*x])] + (24*I)*b^2*ArcTan[c + d*x]*PolyLog[
2, -E^((2*I)*ArcTan[c + d*x])] - (24*I)*a*b*PolyLog[2, E^((2*I)*ArcTan[c +
 d*x])] + 12*b^2*PolyLog[3, E^((-2*I)*ArcTan[c + d*x])] - 12*b^2*PolyLog[3
, -E^((2*I)*ArcTan[c + d*x])])/(24*d*e)
```

3.10.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5566, 27, 5357, 5523, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(c+dx))^2}{ce+dex}\,dx$$
$$\Big\downarrow \; 5566$$

$$\frac{\int \frac{(a+b\arctan(c+dx))^2}{e(c+dx)}d(c+dx)}{d}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(a+b\arctan(c+dx))^2}{c+dx}d(c+dx)}{de}$$

$\Big\downarrow$ 5357

$$\frac{2\text{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))^2-4b\int \frac{(a+b\arctan(c+dx))\text{arctanh}\left(1-\frac{2}{i(c+dx)+1}\right)}{(c+dx)^2+1}d(c+dx)}{de}$$

$\Big\downarrow$ 5523

$$\frac{2\text{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))^2-4b\left(\frac{1}{2}\int \frac{(a+b\arctan(c+dx))\log\left(2-\frac{2}{i(c+dx)+1}\right)}{(c+dx)^2+1}d(c+dx)-\frac{1}{2}\int \frac{(a+b\,\text{arc}}{}\right.}{de}$$

$\Big\downarrow$ 5529

$$2\text{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))^2-4b\left(\frac{1}{2}\left(\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)(a+b\arctan(c+dx))-\right.\right.$$

$\Big\downarrow$ 7164

$$2\text{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))^2-4b\left(\frac{1}{2}\left(\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)(a+b\arctan(c+dx))+\right.\right.$$

input
```
Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x),x]
```

output
```
(2*(a + b*ArcTan[c + d*x])^2*ArcTanh[1 - 2/(1 + I*(c + d*x))] - 4*b*(((I/2
)*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))] + (b*PolyLog
[3, 1 - 2/(1 + I*(c + d*x))])/4)/2 + ((-1/2*I)*(a + b*ArcTan[c + d*x])*Pol
yLog[2, -1 + 2/(1 + I*(c + d*x))] - (b*PolyLog[3, -1 + 2/(1 + I*(c + d*x))
])/4)/2))/(d*e)
```

3.10. $\int \frac{(a+b\arctan(c+dx))^2}{ce+dex}\,dx$

3.10.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5566
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.10.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.56 (sec) , antiderivative size = 1154, normalized size of antiderivative = 6.31

method	result	size
derivativedivides	Expression too large to display	1154
default	Expression too large to display	1154
parts	Expression too large to display	1159

input
```
int((a+b*arctan(d*x+c))^2/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2/e*ln(d*x+c)+b^2/e*(ln(d*x+c)*arctan(d*x+c)^2+I*arctan(d*x+c)*poly
log(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-1/2*polylog(3,-(1+I*(d*x+c))^2/(1+(d
*x+c)^2))-arctan(d*x+c)^2*ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)+arctan(d*x+c
)^2*ln(1+(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-2*I*arctan(d*x+c)*polylog(2,-(
1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+2*polylog(3,-(1+I*(d*x+c))/(1+(d*x+c)^2)
^(1/2))+arctan(d*x+c)^2*ln(1-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-2*I*arctan
(d*x+c)*polylog(2,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+2*polylog(3,(1+I*(d*x
+c))/(1+(d*x+c)^2)^(1/2))+1/2*I*Pi*(csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-
1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-
1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))-csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-
1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2+csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)
^2)-1))*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I*((1+I*(d*x+c))^2/
(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))-csgn(I*((1+I*(d*x+c))^
2/(1+(d*x+c)^2)-1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+
c))^2/(1+(d*x+c)^2)))^2-csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I*(
(1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2+csgn
(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^3-
csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))
)*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))
)^2+csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c...
```

3.10.5 Fricas [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{ce+dex}\,dx=\int \frac{(b\arctan(dx+c)+a)^2}{dex+ce}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e),x, algorithm="fricas")
```

output
```
integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(d*e*x + c*
e), x)
```

3.10.6 Sympy [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{ce+dex}\,dx=\frac{\int \frac{a^2}{c+dx}\,dx+\int \frac{b^2\operatorname{atan}^2(c+dx)}{c+dx}\,dx+\int \frac{2ab\operatorname{atan}(c+dx)}{c+dx}\,dx}{e}$$

input
```
integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e),x)
```

output
```
(Integral(a**2/(c + d*x), x) + Integral(b**2*atan(c + d*x)**2/(c + d*x), x
) + Integral(2*a*b*atan(c + d*x)/(c + d*x), x))/e
```

3.10.7 Maxima [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{ce+dex}\,dx=\int \frac{(b\arctan(dx+c)+a)^2}{dex+ce}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e),x, algorithm="maxima")
```

output
```
a^2*log(d*e*x + c*e)/(d*e) + integrate(1/16*(12*b^2*arctan(d*x + c)^2 + b^
2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 32*a*b*arctan(d*x + c))/(d*e*x + c*
e), x)
```

3.10.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{ce+dex}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e),x, algorithm="giac")
```

output
```
Timed out
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{ce+dex}\,dx = \int \frac{(a+b\,\mathrm{atan}(c+d\,x))^2}{c\,e+d\,e\,x}\,dx$$

input
```
int((a + b*atan(c + d*x))^2/(c*e + d*e*x),x)
```

output
```
int((a + b*atan(c + d*x))^2/(c*e + d*e*x), x)
```

3.11 $\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^2}\,dx$

3.11.1 Optimal result

Integrand size = 23, antiderivative size = 119

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^2}\,dx = -\frac{i(a+b\arctan(c+dx))^2}{de^2}-\frac{(a+b\arctan(c+dx))^2}{de^2(c+dx)} + \frac{2b(a+b\arctan(c+dx))\log\left(2-\frac{2}{1-i(c+dx)}\right)}{de^2} - \frac{ib^2\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{de^2}$$

output
```
-I*(a+b*arctan(d*x+c))^2/d/e^2-(a+b*arctan(d*x+c))^2/d/e^2/(d*x+c)+2*b*(a+
b*arctan(d*x+c))*ln(2-2/(1-I*(d*x+c)))/d/e^2-I*b^2*polylog(2,-1+2/(1-I*(d*
x+c)))/d/e^2
```

3.11.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.13

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^2}\,dx = \frac{-ib^2(-i+c+dx)\arctan(c+dx)^2+2b\arctan(c+dx)\left(-a+b(c+dx)\log\left(1-e^{2i\arctan(c+dx)}\right)\right)+a\Big(-a}{de^2(c+dx)}$$

input
```
Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^2,x]
```

output
```
((-I)*b^2*(-I + c + d*x)*ArcTan[c + d*x]^2 + 2*b*ArcTan[c + d*x]*(-a + b*(
c + d*x)*Log[1 - E^((2*I)*ArcTan[c + d*x])]) + a*(-a + 2*b*(c + d*x)*Log[(
c + d*x)/Sqrt[1 + (c + d*x)^2]]) - I*b^2*(c + d*x)*PolyLog[2, E^((2*I)*Arc
Tan[c + d*x])])/(d*e^2*(c + d*x))
```

3.11.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5566, 27, 5361, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^2}dx$$

$$\Big\downarrow \text{5566}$$

$$\frac{\int \frac{(a+b\arctan(c+dx))^2}{e^2(c+dx)^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\arctan(c+dx))^2}{(c+dx)^2}d(c+dx)}{de^2}$$

$$\Big\downarrow \text{5361}$$

$$\frac{2b\int \frac{a+b\arctan(c+dx)}{(c+dx)((c+dx)^2+1)}d(c+dx)-\frac{(a+b\arctan(c+dx))^2}{c+dx}}{de^2}$$

$$\Big\downarrow \text{5459}$$

$$\frac{-\frac{(a+b\arctan(c+dx))^2}{c+dx}+2b\left(i\int \frac{a+b\arctan(c+dx)}{(c+dx)(c+dx+i)}d(c+dx)-\frac{i(a+b\arctan(c+dx))^2}{2b}\right)}{de^2}$$

$$\Big\downarrow \text{5403}$$

$$\frac{-\frac{(a+b\arctan(c+dx))^2}{c+dx}+2b\left(i\left(ib\int \frac{\log\left(2-\frac{2}{1-i(c+dx)}\right)}{(c+dx)^2+1}d(c+dx)-i\log\left(2-\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))\right)-\frac{i(a+b}{}\right.}{de^2}$$

$$\Big\downarrow \; 2897$$

$$\frac{-\frac{(a+b\arctan(c+dx))^2}{c+dx}+2b\left(i\left(-i\log\left(2-\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-i(c+dx)}-1\right)\right)-\frac{i(a+}{}\right.}{de^2}$$

input
```
Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^2,x]
```

output
```
(-((a + b*ArcTan[c + d*x])^2/(c + d*x)) + 2*b*(((-1/2*I)*(a + b*ArcTan[c +
 d*x])^2)/b + I*((-I)*(a + b*ArcTan[c + d*x])*Log[2 - 2/(1 - I*(c + d*x))]
 - (b*PolyLog[2, -1 + 2/(1 - I*(c + d*x))])/2)))/(d*e^2)
```

3.11.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5566
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.11.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(115) = 230$.

Time = 1.05 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.73

method	result
derivativedivides	$-\frac{a^2}{e^2(dx+c)} + \frac{b^2\left(-\frac{\arctan(dx+c)^2}{dx+c}+2\ln(dx+c)\arctan(dx+c)-\arctan(dx+c)\ln\left(1+(dx+c)^2\right)-\frac{i\left(\ln(dx+c-i)\ln\left(1+(dx+c)^2\right)-\frac{\ln(dx}{}\right.}{}\right.}{}$
default	$-\frac{a^2}{e^2(dx+c)} + \frac{b^2\left(-\frac{\arctan(dx+c)^2}{dx+c}+2\ln(dx+c)\arctan(dx+c)-\arctan(dx+c)\ln\left(1+(dx+c)^2\right)-\frac{i\left(\ln(dx+c-i)\ln\left(1+(dx+c)^2\right)-\frac{\ln(dx}{}\right.}{}\right.}{}$
parts	$-\frac{a^2}{e^2(dx+c)d} + \frac{b^2\left(-\frac{\arctan(dx+c)^2}{dx+c}+2\ln(dx+c)\arctan(dx+c)-\arctan(dx+c)\ln\left(1+(dx+c)^2\right)-\frac{i\left(\ln(dx+c-i)\ln\left(1+(d\right.}{}\right.}{}$

input
```
int((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2/e^2/(d*x+c)+b^2/e^2*(-1/(d*x+c)*arctan(d*x+c)^2+2*ln(d*x+c)*arct
an(d*x+c)-arctan(d*x+c)*ln(1+(d*x+c)^2)-1/2*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)
-1/2*ln(d*x+c-I)^2-dilog(-1/2*I*(d*x+c+I))-ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I)
))+1/2*I*(ln(d*x+c+I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c+I)^2-dilog(1/2*I*(d*x+c
-I))-ln(d*x+c+I)*ln(1/2*I*(d*x+c-I)))+I*ln(d*x+c)*ln(1+I*(d*x+c))-I*ln(d*x
+c)*ln(1-I*(d*x+c))+I*dilog(1+I*(d*x+c))-I*dilog(1-I*(d*x+c)))+2*a*b/e^2*(
-1/(d*x+c)*arctan(d*x+c)+ln(d*x+c)-1/2*ln(1+(d*x+c)^2)))
```

3.11.5 Fricas [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^2}\,dx = \int \frac{\left(b\arctan\left(dx+c\right)+a\right)^2}{\left(dex+ce\right)^2}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(d^2*e^2*x^
2 + 2*c*d*e^2*x + c^2*e^2), x)
```

3.11.6 Sympy [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^2}\,dx = \frac{\int \frac{a^2}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^2\operatorname{atan}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{2ab\operatorname{atan}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

input
```
integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e)**2,x)
```

output
```
(Integral(a**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**2*atan(c + d
*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(2*a*b*atan(c + d*x)/(c*
*2 + 2*c*d*x + d**2*x**2), x))/e**2
```

3.11.7 Maxima [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^2}\,dx = \int \frac{\left(b\arctan\left(dx+c\right)+a\right)^2}{\left(dex+ce\right)^2}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="maxima")
```

output
```
-(d*(log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2) - 2*log(d*x + c)/(d^2*e^2)
) + 2*arctan(d*x + c)/(d^2*e^2*x + c*d*e^2))*a*b - 1/16*(4*arctan(d*x + c)
^2 - 16*(d^2*e^2*x + c*d*e^2)*integrate(1/16*(12*(d^2*x^2 + 2*c*d*x + c^2
+ 1)*arctan(d*x + c)^2 + (d^2*x^2 + 2*c*d*x + c^2 + 1)*log(d^2*x^2 + 2*c*d
*x + c^2 + 1)^2 + 8*(d*x + c)*arctan(d*x + c) - 4*(d^2*x^2 + 2*c*d*x + c^2
)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^4*e^2*x^4 + 4*c*d^3*e^2*x^3 + (6*c^
2 + 1)*d^2*e^2*x^2 + 2*(2*c^3 + c)*d*e^2*x + (c^4 + c^2)*e^2), x) - log(d^
2*x^2 + 2*c*d*x + c^2 + 1)^2)*b^2/(d^2*e^2*x + c*d*e^2) - a^2/(d^2*e^2*x +
 c*d*e^2)
```

3.11.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c+d\,x))^2}{(c\,e+d\,e\,x)^2}\,dx$$

input
```
int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^2,x)
```

output
```
int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^2, x)
```

3.12 $\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^3}\,dx$

3.12.1 Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^3}\,dx = -\frac{b(a+b\arctan(c+dx))}{de^3(c+dx)} - \frac{(a+b\arctan(c+dx))^2}{2de^3} - \frac{(a+b\arctan(c+dx))^2}{2de^3(c+dx)^2} + \frac{b^2\log(c+dx)}{de^3} - \frac{b^2\log\left(1+(c+dx)^2\right)}{2de^3}$$

output
```
-b*(a+b*arctan(d*x+c))/d/e^3/(d*x+c)-1/2*(a+b*arctan(d*x+c))^2/d/e^3-1/2*(
a+b*arctan(d*x+c))^2/d/e^3/(d*x+c)^2+b^2*ln(d*x+c)/d/e^3-1/2*b^2*ln(1+(d*x
+c)^2)/d/e^3
```

3.12.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.66

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^3}\,dx = -\frac{a^2+2abc+2abdx+2b(b(c+dx)+a(1+c^2+2cdx+d^2x^2))\arctan(c+dx)+b^2(1+c^2+2cdx+d^2x}{}$$

input
```
Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^3,x]
```

output
```
-1/2*(a^2 + 2*a*b*c + 2*a*b*d*x + 2*b*(b*(c + d*x) + a*(1 + c^2 + 2*c*d*x
+ d^2*x^2))*ArcTan[c + d*x] + b^2*(1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTan[c +
 d*x]^2 - 2*b^2*(c + d*x)^2*Log[c + d*x] + b^2*c^2*Log[1 + c^2 + 2*c*d*x +
 d^2*x^2] + 2*b^2*c*d*x*Log[1 + c^2 + 2*c*d*x + d^2*x^2] + b^2*d^2*x^2*Log
[1 + c^2 + 2*c*d*x + d^2*x^2])/(d*e^3*(c + d*x)^2)
```

3.12.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = $\{5566, 27, 5361, 5453, 5361, 243, 47, 14, 16, 5419\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^3}dx$$

$$\Big\downarrow \text{5566}$$

$$\frac{\int \frac{(a+b\arctan(c+dx))^2}{e^3(c+dx)^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\arctan(c+dx))^2}{(c+dx)^3}d(c+dx)}{de^3}$$

$$\Big\downarrow \text{5361}$$

$$\frac{b\int \frac{a+b\arctan(c+dx)}{(c+dx)^2((c+dx)^2+1)}d(c+dx)-\frac{(a+b\arctan(c+dx))^2}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{5453}$$

$$\frac{b\left(\int \frac{a+b\arctan(c+dx)}{(c+dx)^2}d(c+dx)-\int \frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)\right)-\frac{(a+b\arctan(c+dx))^2}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{5361}$$

$$\frac{b\left(-\int \frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)+b\int \frac{1}{(c+dx)((c+dx)^2+1)}d(c+dx)-\frac{a+b\arctan(c+dx)}{c+dx}\right)-\frac{(a+b\arctan(c+dx))^2}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{243}$$

$$\frac{b\left(-\int \frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)+\frac{1}{2}b\int \frac{1}{(c+dx)^2((c+dx)^2+1)}d(c+dx)^2-\frac{a+b\arctan(c+dx)}{c+dx}\right)-\frac{(a+b\arctan(c+dx))^2}{2(c+dx)^2}}{de^3}$$

$\Big\downarrow$ 47

$$\frac{b\left(-\int \frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)+\frac{1}{2}b\left(\int \frac{1}{(c+dx)^2}d(c+dx)^2-\int \frac{1}{(c+dx)^2+1}d(c+dx)^2\right)-\frac{a+b\arctan(c+dx)}{c+dx}\right)-\frac{(a+b\arcta}{2(c+}}{de^3}$$

$\Big\downarrow$ 14

$$\frac{b\left(-\int \frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)+\frac{1}{2}b\left(\log\left((c+dx)^2\right)-\int \frac{1}{(c+dx)^2+1}d(c+dx)^2\right)-\frac{a+b\arctan(c+dx)}{c+dx}\right)-\frac{(a+b\arctan(c+d}{2(c+dx)^2}}{de^3}$$

$\Big\downarrow$ 16

$$\frac{b\left(-\int \frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)-\frac{a+b\arctan(c+dx)}{c+dx}+\frac{1}{2}b\left(\log\left((c+dx)^2\right)-\log\left((c+dx)^2+1\right)\right)\right)-\frac{(a+b\arctan(c+dx))^2}{2(c+dx)^2}}{de^3}$$

$\Big\downarrow$ 5419

$$\frac{b\left(-\frac{(a+b\arctan(c+dx))^2}{2b}-\frac{a+b\arctan(c+dx)}{c+dx}+\frac{1}{2}b\left(\log\left((c+dx)^2\right)-\log\left((c+dx)^2+1\right)\right)\right)-\frac{(a+b\arctan(c+dx))^2}{2(c+dx)^2}}{de^3}$$

input
```
Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^3,x]
```

output
```
(-1/2*(a + b*ArcTan[c + d*x])^2/(c + d*x)^2 + b*(-((a + b*ArcTan[c + d*x])
/(c + d*x)) - (a + b*ArcTan[c + d*x])^2/(2*b) + (b*(Log[(c + d*x)^2] - Log
[1 + (c + d*x)^2]))/2))/(d*e^3)
```

3.12.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5566
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.12. $\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^3}\,dx$

3.12.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{-\frac{a^2}{2e^3(dx+c)^2}+\frac{b^2\left(-\frac{\arctan(dx+c)^2}{2(dx+c)^2}-\frac{\arctan(dx+c)}{dx+c}-\frac{\arctan(dx+c)^2}{2}+\ln(dx+c)-\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{e^3}+\frac{2ab\left(-\frac{\arctan(dx+c)}{2(dx+c)^2}-\frac{1}{2(dx \cdots}{e^3}}{d}$
default	$\frac{-\frac{a^2}{2e^3(dx+c)^2}+\frac{b^2\left(-\frac{\arctan(dx+c)^2}{2(dx+c)^2}-\frac{\arctan(dx+c)}{dx+c}-\frac{\arctan(dx+c)^2}{2}+\ln(dx+c)-\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{e^3}+\frac{2ab\left(-\frac{\arctan(dx+c)}{2(dx+c)^2}-\frac{1}{2(dx \cdots}{e^3}}{d}$
parts	$-\frac{a^2}{2e^3(dx+c)^2d}+\frac{b^2\left(-\frac{\arctan(dx+c)^2}{2(dx+c)^2}-\frac{\arctan(dx+c)}{dx+c}-\frac{\arctan(dx+c)^2}{2}+\ln(dx+c)-\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{e^3d}+\frac{2ab\left(-\frac{\arctan(d \cdots}{2(dx+c \cdots}$
parallelrisch	$\frac{-2a^2cd^2-8x\arctan(dx+c)abc^2d^3-4x^2\arctan(dx+c)abcd^4+abd^4x^2-2b^2\arctan(dx+c)^2cd^2-4\arctan(dx+c)b^2c^2d^2- \cdots}{\cdots}$
risch	$\frac{b^2(d^2x^2+2cdx+c^2+1)\ln(1+i(dx+c))^2}{8e^3(dx+c)^2d}-\frac{b\left(bd^2x^2\ln(1-i(dx+c))+2bcdx\ln(1-i(dx+c))-2ibdx+\ln(1-i(dx+c))bc^2- \cdots\right.}{4e^3(dx+c)^2d}$

input
```
int((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*a^2/e^3/(d*x+c)^2+b^2/e^3*(-1/2/(d*x+c)^2*arctan(d*x+c)^2-1/(d*x
+c)*arctan(d*x+c)-1/2*arctan(d*x+c)^2+ln(d*x+c)-1/2*ln(1+(d*x+c)^2))+2*a*b
/e^3*(-1/2/(d*x+c)^2*arctan(d*x+c)-1/2/(d*x+c)-1/2*arctan(d*x+c)))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.79

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^3}\,dx =$$
$$-\frac{2\,abdx+2\,abc+(b^2d^2x^2+2\,b^2cdx+b^2c^2+b^2)\arctan\left(dx+c\right)^2+a^2+2\left(abd^2x^2+abc^2+b^2c+(2\,abc \cdots\right.}{2\,(\cdots}$$

input
```
integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*a*b*d*x + 2*a*b*c + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + b^2)*ar
ctan(d*x + c)^2 + a^2 + 2*(a*b*d^2*x^2 + a*b*c^2 + b^2*c + (2*a*b*c + b^2)
*d*x + a*b)*arctan(d*x + c) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d^
2*x^2 + 2*c*d*x + c^2 + 1) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d
*x + c))/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

3.12.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 19.88 (sec) , antiderivative size = 1107, normalized size of antiderivative = 9.46

$$\int \frac{(a + b\arctan(c + dx))^2}{(ce + dex)^3} \, dx$$

$$= \begin{cases} -\frac{a^2}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{2abc^2 \operatorname{atan}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{4abcdx \operatorname{atan}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{2abc}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} - \frac{2abd^2x^2 \operatorname{a}}{2c^2de^3+4cd^2} \\ \frac{x(a+b\operatorname{atan}(c))^2}{c^3e^3} \end{cases}$$

input
```
integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e)**3,x)
```

output
```
Piecewise((-a**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*
a*b*c**2*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2
) - 4*a*b*c*d*x*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e*
*3*x**2) - 2*a*b*c/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) -
2*a*b*d**2*x**2*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e*
*3*x**2) - 2*a*b*d*x/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2)
- 2*a*b*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2)
 + 2*b**2*c**2*log(c/d + x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3
*x**2) - 2*b**2*c**2*log(c/d + x - I/d)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x +
 2*d**3*e**3*x**2) - b**2*c**2*atan(c + d*x)**2/(2*c**2*d*e**3 + 4*c*d**2*
e**3*x + 2*d**3*e**3*x**2) + 2*I*b**2*c**2*atan(c + d*x)/(2*c**2*d*e**3 +
4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 4*b**2*c*d*x*log(c/d + x)/(2*c**2*d*
e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 4*b**2*c*d*x*log(c/d + x - I/
d)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*b**2*c*d*x*ata
n(c + d*x)**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 4*I*b
**2*c*d*x*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**
2) - 2*b**2*c*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3
*x**2) + 2*b**2*d**2*x**2*log(c/d + x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x +
2*d**3*e**3*x**2) - 2*b**2*d**2*x**2*log(c/d + x - I/d)/(2*c**2*d*e**3 + 4
*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b**2*d**2*x**2*atan(c + d*x)**2/(2...
```

3.12.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(111) = 222$.

Time = 0.32 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.29

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^3}\,dx$$
$$= -\left(d\left(\frac{1}{d^3e^3x+cd^2e^3}+\frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{d^2e^3}\right)+\frac{\arctan(dx+c)}{d^3e^3x^2+2\,cd^2e^3x+c^2de^3}\right)ab$$
$$-\frac{1}{2}\left(2\,d\left(\frac{1}{d^3e^3x+cd^2e^3}+\frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{d^2e^3}\right)\arctan(dx+c)-\frac{\arctan(dx+c)^2-\log\left(d^2x^2+2\,cdx+\right.}{de^3}\right.$$
$$-\frac{b^2\arctan(dx+c)^2}{2\left(d^3e^3x^2+2\,cd^2e^3x+c^2de^3\right)}-\frac{a^2}{2\left(d^3e^3x^2+2\,cd^2e^3x+c^2de^3\right)}$$

input
```
integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="maxima")
```

output
```
-(d*(1/(d^3*e^3*x + c*d^2*e^3) + arctan((d^2*x + c*d)/d)/(d^2*e^3)) + arct
an(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3))*a*b - 1/2*(2*d*(1/(
d^3*e^3*x + c*d^2*e^3) + arctan((d^2*x + c*d)/d)/(d^2*e^3))*arctan(d*x + c
) - (arctan(d*x + c)^2 - log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*log(d*x + c)
)/(d*e^3))*b^2 - 1/2*b^2*arctan(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x +
c^2*d*e^3) - 1/2*a^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

3.12.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="giac")
```

output
```
Timed out
```

3.12.9 Mupad [B] (verification not implemented)

Time = 3.15 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.98

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^3}\,dx = \frac{b^2\ln(c+d\,x)}{d\,e^3} - \frac{\frac{a^2+2\,b\,c\,a}{2\,d}+a\,b\,x}{c^2\,e^3+2\,c\,d\,e^3\,x+d^2\,e^3\,x^2} - \frac{\operatorname{atan}(c+d\,x)\,\left(\frac{b^2\,c}{d^3\,e^3}+\frac{b^2\,x}{d^2\,e^3}+\frac{a\,b}{d^3\,e^3}\right)}{x^2+\frac{c^2}{d^2}+\frac{2\,c\,x}{d}} - \operatorname{atan}(c+d\,x)^2\left(\frac{b^2}{2\,d\,e^3}+\frac{b^2}{2\,d^3\,e^3\,\left(x^2+\frac{c^2}{d^2}+\frac{2\,c\,x}{d}\right)}\right) + \frac{\ln(c+d\,x-\mathrm{i})\,\left(-\frac{b^2}{2}+\frac{a\,b\,1\mathrm{i}}{2}\right)}{d\,e^3} - \frac{\ln(c+d\,x+1\mathrm{i})\,\left(\frac{b^2}{2}+\frac{1\mathrm{i}\,a\,b}{2}\right)}{d\,e^3}$$

input
```
int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^3,x)
```

output
```
(b^2*log(c + d*x))/(d*e^3) - ((a^2 + 2*a*b*c)/(2*d) + a*b*x)/(c^2*e^3 + d^
2*e^3*x^2 + 2*c*d*e^3*x) - (atan(c + d*x)*((b^2*c)/(d^3*e^3) + (b^2*x)/(d^
2*e^3) + (a*b)/(d^3*e^3)))/(x^2 + c^2/d^2 + (2*c*x)/d) - atan(c + d*x)^2*(
b^2/(2*d*e^3) + b^2/(2*d^3*e^3*(x^2 + c^2/d^2 + (2*c*x)/d))) + (log(c + d*
x - 1i)*((a*b*1i)/2 - b^2/2))/(d*e^3) - (log(c + d*x + 1i)*((a*b*1i)/2 + b
^2/2))/(d*e^3)
```

3.13 $\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^4}\,dx$

3.13.1 Optimal result

Integrand size = 23, antiderivative size = 194

$$\begin{aligned}\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^4}\,dx =& -\frac{b^2}{3de^4(c+dx)}-\frac{b^2\arctan(c+dx)}{3de^4}\\ &-\frac{b(a+b\arctan(c+dx))}{3de^4(c+dx)^2}\\ &+\frac{i(a+b\arctan(c+dx))^2}{3de^4}-\frac{(a+b\arctan(c+dx))^2}{3de^4(c+dx)^3}\\ &-\frac{2b(a+b\arctan(c+dx))\log\left(2-\frac{2}{1-i(c+dx)}\right)}{3de^4}\\ &+\frac{ib^2\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{3de^4}\end{aligned}$$

output
```
-1/3*b^2/d/e^4/(d*x+c)-1/3*b^2*arctan(d*x+c)/d/e^4-1/3*b*(a+b*arctan(d*x+c
))/d/e^4/(d*x+c)^2+1/3*I*(a+b*arctan(d*x+c))^2/d/e^4-1/3*(a+b*arctan(d*x+c
))^2/d/e^4/(d*x+c)^3-2/3*b*(a+b*arctan(d*x+c))*ln(2-2/(1-I*(d*x+c)))/d/e^4
+1/3*I*b^2*polylog(2,-1+2/(1-I*(d*x+c)))/d/e^4
```

3.13.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^4}\,dx =$$
$$-\frac{ab+\frac{a^2}{(c+dx)^3}+\frac{ab}{(c+dx)^2}+\frac{b^2}{c+dx}+b^2\left(-i+\frac{1}{(c+dx)^3}\right)\arctan(c+dx)^2+b\arctan(c+dx)\left(b+\frac{2a}{(c+dx)^3}+\frac{b}{(c+d}\right.}{3de^4}$$

input
```
Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^4,x]
```

output
```
-1/3*(a*b + a^2/(c + d*x)^3 + (a*b)/(c + d*x)^2 + b^2/(c + d*x) + b^2*(-I
+ (c + d*x)^(-3))*ArcTan[c + d*x]^2 + b*ArcTan[c + d*x]*(b + (2*a)/(c + d*
x)^3 + b/(c + d*x)^2 + 2*b*Log[1 - E^((2*I)*ArcTan[c + d*x])]) + 2*a*b*Log
[(c + d*x)/Sqrt[1 + (c + d*x)^2]] - I*b^2*PolyLog[2, E^((2*I)*ArcTan[c + d
*x])])/(d*e^4)
```

3.13.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.84, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5566, 27, 5361, 5453, 5361, 264, 216, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^4}\,dx$$
$$\Big\downarrow \; 5566$$
$$\frac{\int \frac{(a+b\arctan(c+dx))^2}{e^4(c+dx)^4}d(c+dx)}{d}$$
$$\Big\downarrow \; 27$$
$$\frac{\int \frac{(a+b\arctan(c+dx))^2}{(c+dx)^4}d(c+dx)}{de^4}$$
$$\Big\downarrow \; 5361$$
$$\frac{\frac{2}{3}b\int \frac{a+b\arctan(c+dx)}{(c+dx)^3((c+dx)^2+1)}d(c+dx)-\frac{(a+b\arctan(c+dx))^2}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \ 5453$$

$$\frac{\frac{2}{3}b\left(\int \frac{a+b\arctan(c+dx)}{(c+dx)^3}d(c+dx)-\int \frac{a+b\arctan(c+dx)}{(c+dx)((c+dx)^2+1)}d(c+dx)\right)-\frac{(a+b\arctan(c+dx))^2}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \ 5361$$

$$\frac{\frac{2}{3}b\left(-\int \frac{a+b\arctan(c+dx)}{(c+dx)((c+dx)^2+1)}d(c+dx)+\frac{1}{2}b\int \frac{1}{(c+dx)^2((c+dx)^2+1)}d(c+dx)-\frac{a+b\arctan(c+dx)}{2(c+dx)^2}\right)-\frac{(a+b\arctan(c+dx))^2}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \ 264$$

$$\frac{\frac{2}{3}b\left(-\int \frac{a+b\arctan(c+dx)}{(c+dx)((c+dx)^2+1)}d(c+dx)+\frac{1}{2}b\left(-\int \frac{1}{(c+dx)^2+1}d(c+dx)-\frac{1}{c+dx}\right)-\frac{a+b\arctan(c+dx)}{2(c+dx)^2}\right)-\frac{(a+b\arctan(c+dx))^2}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \ 216$$

$$\frac{\frac{2}{3}b\left(-\int \frac{a+b\arctan(c+dx)}{(c+dx)((c+dx)^2+1)}d(c+dx)-\frac{a+b\arctan(c+dx)}{2(c+dx)^2}+\frac{1}{2}b\left(-\arctan(c+dx)-\frac{1}{c+dx}\right)\right)-\frac{(a+b\arctan(c+dx))^2}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \ 5459$$

$$\frac{-\frac{(a+b\arctan(c+dx))^2}{3(c+dx)^3}+\frac{2}{3}b\left(-i\int \frac{a+b\arctan(c+dx)}{(c+dx)(c+dx+i)}d(c+dx)+\frac{i(a+b\arctan(c+dx))^2}{2b}-\frac{a+b\arctan(c+dx)}{2(c+dx)^2}+\frac{1}{2}b\left(-\arctan(c+ \right.\right.}{de^4}$$

$$\Big\downarrow \ 5403$$

$$\frac{-\frac{(a+b\arctan(c+dx))^2}{3(c+dx)^3}+\frac{2}{3}b\left(-i\left(ib\int \frac{\log\left(2-\frac{2}{1-i(c+dx)}\right)}{(c+dx)^2+1}d(c+dx)-i\log\left(2-\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))\right)+\frac{i(a-}{}\right.}{de^4}$$

$$\Big\downarrow \ 2897$$

$$\frac{-\frac{(a+b\arctan(c+dx))^2}{3(c+dx)^3}+\frac{2}{3}b\left(-i\left(-i\log\left(2-\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-i(c+dx)}-1\right)\right)+\frac{i(}{}\right.}{de^4}$$

input
```
Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^4,x]
```

3.13. $\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^4}\,dx$

output
```
(-1/3*(a + b*ArcTan[c + d*x])^2/(c + d*x)^3 + (2*b*((b*(-(c + d*x)^(-1) -
ArcTan[c + d*x]))/2 - (a + b*ArcTan[c + d*x])/(2*(c + d*x)^2) + ((I/2)*(a
+ b*ArcTan[c + d*x])^2)/b - I*((-I)*(a + b*ArcTan[c + d*x])*Log[2 - 2/(1 -
 I*(c + d*x))] - (b*PolyLog[2, -1 + 2/(1 - I*(c + d*x))])/2)))/3)/(d*e^4)
```

3.13.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5566
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.13.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(176) = 352$.

Time = 2.56 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.90

method	result
derivativedivides	$-\frac{a^2}{3e^4(dx+c)^3}+\frac{b^2\left(-\frac{\arctan(dx+c)^2}{3(dx+c)^3}-\frac{\arctan(dx+c)}{3(dx+c)^2}-\frac{2\ln(dx+c)\arctan(dx+c)}{3}+\frac{\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{3}+\frac{i\left(\ln(dx+c-i)\ln\left(\right.\right.}{}\right.}{}$
default	$-\frac{a^2}{3e^4(dx+c)^3}+\frac{b^2\left(-\frac{\arctan(dx+c)^2}{3(dx+c)^3}-\frac{\arctan(dx+c)}{3(dx+c)^2}-\frac{2\ln(dx+c)\arctan(dx+c)}{3}+\frac{\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{3}+\frac{i\left(\ln(dx+c-i)\ln\left(\right.\right.}{}\right.}{}$
parts	$-\frac{a^2}{3e^4(dx+c)^3d}+\frac{b^2\left(-\frac{\arctan(dx+c)^2}{3(dx+c)^3}-\frac{\arctan(dx+c)}{3(dx+c)^2}-\frac{2\ln(dx+c)\arctan(dx+c)}{3}+\frac{\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{3}+\frac{i\left(\ln(dx+c\right.}{}\right.}{}$

input
```
int((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*a^2/e^4/(d*x+c)^3+b^2/e^4*(-1/3/(d*x+c)^3*arctan(d*x+c)^2-1/3/(d
*x+c)^2*arctan(d*x+c)-2/3*ln(d*x+c)*arctan(d*x+c)+1/3*arctan(d*x+c)*ln(1+(
d*x+c)^2)+1/6*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog(-1/2*
I*(d*x+c+I))-ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))-1/6*I*(ln(d*x+c+I)*ln(1+(d*
x+c)^2)-1/2*ln(d*x+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I*(d*x
+c-I)))-1/3/(d*x+c)-1/3*arctan(d*x+c)-1/3*I*ln(d*x+c)*ln(1+I*(d*x+c))+1/3*
I*ln(d*x+c)*ln(1-I*(d*x+c))-1/3*I*dilog(1+I*(d*x+c))+1/3*I*dilog(1-I*(d*x+
c)))+2*a*b/e^4*(-1/3/(d*x+c)^3*arctan(d*x+c)-1/6/(d*x+c)^2-1/3*ln(d*x+c)+1
/6*ln(1+(d*x+c)^2)))
```

3.13.5 Fricas [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^4}\,dx = \int \frac{\left(b\arctan\left(dx+c\right)+a\right)^2}{\left(dex+ce\right)^4}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(d^4*e^4*x^
4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

3.13.6 Sympy [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^4}\,dx = \frac{\int \frac{a^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^2\operatorname{atan}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{2ab\operatorname{atan}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

input
```
integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e)**4,x)
```

output
```
(Integral(a**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**
4*x**4), x) + Integral(b**2*atan(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d
**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(2*a*b*atan(c + d*x)/(
c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**
4
```

3.13.7 Maxima [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^4}\,dx=\int \frac{(b\arctan(dx+c)+a)^2}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="maxima")
```

output
```
-1/3*(d*(1/(d^4*e^4*x^2 + 2*c*d^3*e^4*x + c^2*d^2*e^4) - log(d^2*x^2 + 2*c
*d*x + c^2 + 1)/(d^2*e^4) + 2*log(d*x + c)/(d^2*e^4)) + 2*arctan(d*x + c)/
(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4))*a*b - 1/48*
(4*arctan(d*x + c)^2 - 48*(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x
 + c^3*d*e^4)*integrate(1/48*(36*(d^2*x^2 + 2*c*d*x + c^2 + 1)*arctan(d*x
+ c)^2 + 3*(d^2*x^2 + 2*c*d*x + c^2 + 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^
2 + 8*(d*x + c)*arctan(d*x + c) - 4*(d^2*x^2 + 2*c*d*x + c^2)*log(d^2*x^2
+ 2*c*d*x + c^2 + 1))/(d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + (15*c^2 + 1)*d^4*e^
4*x^4 + 4*(5*c^3 + c)*d^3*e^4*x^3 + 3*(5*c^4 + 2*c^2)*d^2*e^4*x^2 + 2*(3*c
^5 + 2*c^3)*d*e^4*x + (c^6 + c^4)*e^4), x) - log(d^2*x^2 + 2*c*d*x + c^2 +
 1)^2)*b^2/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) -
 1/3*a^2/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4)
```

3.13.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^4}\,dx=\text{Timed out}$$

input
```
integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="giac")
```

output
```
Timed out
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^4}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c+d\,x)\right)^2}{\left(c\,e+d\,e\,x\right)^4}\,dx$$

input
```
int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^4,x)
```

output
```
int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^4, x)
```

3.14 $\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^5}\,dx$

3.14.1 Optimal result

Integrand size = 23, antiderivative size = 170

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^5}\,dx = -\frac{b^2}{12de^5(c+dx)^2} - \frac{b(a+b\arctan(c+dx))}{6de^5(c+dx)^3} + \frac{b(a+b\arctan(c+dx))}{2de^5(c+dx)} + \frac{(a+b\arctan(c+dx))^2}{4de^5} - \frac{(a+b\arctan(c+dx))^2}{4de^5(c+dx)^4} - \frac{2b^2\log(c+dx)}{3de^5} + \frac{b^2\log\left(1+(c+dx)^2\right)}{3de^5}$$

output

```
-1/12*b^2/d/e^5/(d*x+c)^2-1/6*b*(a+b*arctan(d*x+c))/d/e^5/(d*x+c)^3+1/2*b*
(a+b*arctan(d*x+c))/d/e^5/(d*x+c)+1/4*(a+b*arctan(d*x+c))^2/d/e^5-1/4*(a+b
*arctan(d*x+c))^2/d/e^5/(d*x+c)^4-2/3*b^2*ln(d*x+c)/d/e^5+1/3*b^2*ln(1+(d*
x+c)^2)/d/e^5
```

3.14.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.44

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^5}\,dx =$$
$$-\frac{3a^2+2ab(c+dx)+b^2(c+dx)^2-6ab(c+dx)^3-2b(b(-c+3c^3-dx+9c^2dx+9cd^2x^2+3d^3x^3)+3c}{}$$

input
```
Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^5,x]
```

output
```
-1/12*(3*a^2 + 2*a*b*(c + d*x) + b^2*(c + d*x)^2 - 6*a*b*(c + d*x)^3 - 2*b
*(b*(-c + 3*c^3 - d*x + 9*c^2*d*x + 9*c*d^2*x^2 + 3*d^3*x^3) + 3*a*(-1 + c
^4 + 4*c^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x^4))*ArcTan[c + d*x] -
 3*b^2*(-1 + c^4 + 4*c^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x^4)*ArcT
an[c + d*x]^2 + 8*b^2*(c + d*x)^4*Log[c + d*x] - 4*b^2*(c + d*x)^4*Log[1 +
 c^2 + 2*c*d*x + d^2*x^2])/(d*e^5*(c + d*x)^4)
```

3.14.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.92, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {5566, 27, 5361, 5453, 5361, 243, 54, 2009, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^5}dx$$

$$\Big\downarrow \text{ 5566}$$

$$\frac{\int \frac{(a+b\arctan(c+dx))^2}{e^5(c+dx)^5}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(a+b\arctan(c+dx))^2}{(c+dx)^5}d(c+dx)}{de^5}$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{\frac{1}{2}b\int \frac{a+b\arctan(c+dx)}{(c+dx)^4((c+dx)^2+1)}d(c+dx)-\frac{(a+b\arctan(c+dx))^2}{4(c+dx)^4}}{de^5}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\frac{1}{2}b\left(\int \frac{a+b\arctan(c+dx)}{(c+dx)^4}d(c+dx)-\int \frac{a+b\arctan(c+dx)}{(c+dx)^2((c+dx)^2+1)}d(c+dx)\right)-\frac{(a+b\arctan(c+dx))^2}{4(c+dx)^4}}{de^5}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{2}b\left(-\int \frac{a+b\arctan(c+dx)}{(c+dx)^2((c+dx)^2+1)}d(c+dx)+\frac{1}{3}b\int \frac{1}{(c+dx)^3((c+dx)^2+1)}d(c+dx)-\frac{a+b\arctan(c+dx)}{3(c+dx)^3}\right)-\frac{(a+b\arctan(c+dx))^2}{4(c+dx)^4}}{de^5}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{2}b\left(-\int \frac{a+b\arctan(c+dx)}{(c+dx)^2((c+dx)^2+1)}d(c+dx)+\frac{1}{6}b\int \frac{1}{(c+dx)^4((c+dx)^2+1)}d(c+dx)^2-\frac{a+b\arctan(c+dx)}{3(c+dx)^3}\right)-\frac{(a+b\arctan(c+dx))^2}{4(c+dx)^4}}{de^5}$$

$$\Big\downarrow \text{54}$$

$$\frac{\frac{1}{2}b\left(-\int \frac{a+b\arctan(c+dx)}{(c+dx)^2((c+dx)^2+1)}d(c+dx)+\frac{1}{6}b\int \left(-\frac{1}{(c+dx)^2}+\frac{1}{(c+dx)^4}+\frac{1}{(c+dx)^2+1}\right)d(c+dx)^2-\frac{a+b\arctan(c+dx)}{3(c+dx)^3}\right)-\frac{(a+b}{}}{de^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}b\left(-\int \frac{a+b\arctan(c+dx)}{(c+dx)^2((c+dx)^2+1)}d(c+dx)-\frac{a+b\arctan(c+dx)}{3(c+dx)^3}+\frac{1}{6}b\left(-\frac{1}{(c+dx)^2}-\log\left((c+dx)^2\right)+\log\left((c+dx)^2+1\right)\right)\right)-}{de^5}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\frac{1}{2}b\left(-\int \frac{a+b\arctan(c+dx)}{(c+dx)^2}d(c+dx)+\int \frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)-\frac{a+b\arctan(c+dx)}{3(c+dx)^3}+\frac{1}{6}b\left(-\frac{1}{(c+dx)^2}-\log\left((c+dx)^2\right)\right.\right.}{de^5}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{2}b\left(\int \frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)-b\int \frac{1}{(c+dx)((c+dx)^2+1)}d(c+dx)+\frac{a+b\arctan(c+dx)}{c+dx}-\frac{a+b\arctan(c+dx)}{3(c+dx)^3}+\frac{1}{6}b\left(-\frac{1}{(c+dx)^2}\right.\right.}{de^5}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{2}b\left(\int \frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)-\frac{1}{2}b\int \frac{1}{(c+dx)^2((c+dx)^2+1)}d(c+dx)^2+\frac{a+b\arctan(c+dx)}{c+dx}-\frac{a+b\arctan(c+dx)}{3(c+dx)^3}+\frac{1}{6}b\left(-\frac{1}{(c+d}\right.\right.}{de^5}$$

$$\Big\downarrow \text{47}$$

3.14. $\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^5}\,dx$

$$\frac{\frac{1}{2}b\left(\int \frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)-\frac{1}{2}b\left(\int \frac{1}{(c+dx)^2}d(c+dx)^2-\int \frac{1}{(c+dx)^2+1}d(c+dx)^2\right)+\frac{a+b\arctan(c+dx)}{c+dx}-\frac{a+b\arctan(c+dx)}{3(c+dx)^3}\right.}{de^5}$$

$\downarrow$ 14

$$\frac{\frac{1}{2}b\left(\int \frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)-\frac{1}{2}b\left(\log\left((c+dx)^2\right)-\int \frac{1}{(c+dx)^2+1}d(c+dx)^2\right)+\frac{a+b\arctan(c+dx)}{c+dx}-\frac{a+b\arctan(c+dx)}{3(c+dx)^3}\right.}{de^5}$$

$\downarrow$ 16

$$\frac{\frac{1}{2}b\left(\int \frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)+\frac{a+b\arctan(c+dx)}{c+dx}-\frac{a+b\arctan(c+dx)}{3(c+dx)^3}-\frac{1}{2}b\left(\log\left((c+dx)^2\right)-\log\left((c+dx)^2+1\right)\right)+\frac{1}{6}\right.}{de^5}$$

$\downarrow$ 5419

$$\frac{\frac{1}{2}b\left(\frac{(a+b\arctan(c+dx))^2}{2b}+\frac{a+b\arctan(c+dx)}{c+dx}-\frac{a+b\arctan(c+dx)}{3(c+dx)^3}-\frac{1}{2}b\left(\log\left((c+dx)^2\right)-\log\left((c+dx)^2+1\right)\right)+\frac{1}{6}b\left(-\frac{1}{(c+dx}\right.\right.}{de^5}$$

input
```
Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^5,x]
```

output
```
(-1/4*(a + b*ArcTan[c + d*x])^2/(c + d*x)^4 + (b*(-1/3*(a + b*ArcTan[c + d
*x])/(c + d*x)^3 + (a + b*ArcTan[c + d*x])/(c + d*x) + (a + b*ArcTan[c + d
*x])^2/(2*b) - (b*(Log[(c + d*x)^2] - Log[1 + (c + d*x)^2]))/2 + (b*(-(c +
 d*x)^(-2) - Log[(c + d*x)^2] + Log[1 + (c + d*x)^2]))/6))/2)/(d*e^5)
```

3.14.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5566
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.14. $\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^5}\,dx$

3.14.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{a^2}{4e^5(dx+c)^4}+\frac{b^2\left(-\frac{\arctan(dx+c)^2}{4(dx+c)^4}-\frac{\arctan(dx+c)}{6(dx+c)^3}+\frac{\arctan(dx+c)}{2dx+2c}+\frac{\arctan(dx+c)^2}{4}-\frac{1}{12(dx+c)^2}-\frac{2\ln(dx+c)}{3}+\frac{\ln\left(1+(dx+c)^2\right)}{3}\right)}{e^5}}{d}$
default	$\frac{-\frac{a^2}{4e^5(dx+c)^4}+\frac{b^2\left(-\frac{\arctan(dx+c)^2}{4(dx+c)^4}-\frac{\arctan(dx+c)}{6(dx+c)^3}+\frac{\arctan(dx+c)}{2dx+2c}+\frac{\arctan(dx+c)^2}{4}-\frac{1}{12(dx+c)^2}-\frac{2\ln(dx+c)}{3}+\frac{\ln\left(1+(dx+c)^2\right)}{3}\right)}{e^5}}{d}$
parts	$-\frac{a^2}{4e^5(dx+c)^4d}+\frac{b^2\left(-\frac{\arctan(dx+c)^2}{4(dx+c)^4}-\frac{\arctan(dx+c)}{6(dx+c)^3}+\frac{\arctan(dx+c)}{2dx+2c}+\frac{\arctan(dx+c)^2}{4}-\frac{1}{12(dx+c)^2}-\frac{2\ln(dx+c)}{3}+\frac{\ln\left(1+(dx\right.}{3}}{e^5d}$
parallelrisch	$\frac{6abc^3d^5-x^2b^2d^7-2xab\,d^6+6x^3ab\,d^8-2x\arctan(dx+c)b^2d^6+6x^3\arctan(dx+c)b^2d^8+3\arctan(dx+c)^2b^2c^4d^5+6\arctan}{}$
risch	Expression too large to display

input
```
int((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4*a^2/e^5/(d*x+c)^4+b^2/e^5*(-1/4/(d*x+c)^4*arctan(d*x+c)^2-1/6/(d
*x+c)^3*arctan(d*x+c)+1/2/(d*x+c)*arctan(d*x+c)+1/4*arctan(d*x+c)^2-1/12/(
d*x+c)^2-2/3*ln(d*x+c)+1/3*ln(1+(d*x+c)^2))+2*a*b/e^5*(-1/4/(d*x+c)^4*arct
an(d*x+c)-1/12/(d*x+c)^3+1/4/(d*x+c)+1/4*arctan(d*x+c)))
```

3.14.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(156) = 312$.

Time = 0.30 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.64

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^5}\,dx$$
$$= \frac{6\,abd^3x^3+6\,abc^3+(18\,abc-b^2)d^2x^2-b^2c^2-2\,abc+2\,(9\,abc^2-b^2c-ab)dx+3\,(b^2d^4x^4+4\,b^2cd^3x^3+6}{}$$

input
```
integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="fricas")
```

output
```
1/12*(6*a*b*d^3*x^3 + 6*a*b*c^3 + (18*a*b*c - b^2)*d^2*x^2 - b^2*c^2 - 2*a
*b*c + 2*(9*a*b*c^2 - b^2*c - a*b)*d*x + 3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3
+ 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 - b^2)*arctan(d*x + c)^2 - 3
*a^2 + 2*(3*a*b*d^4*x^4 + 3*(4*a*b*c + b^2)*d^3*x^3 + 3*a*b*c^4 + 3*b^2*c^
3 + 9*(2*a*b*c^2 + b^2*c)*d^2*x^2 - b^2*c + (12*a*b*c^3 + 9*b^2*c^2 - b^2)
*d*x - 3*a*b)*arctan(d*x + c) + 4*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c
^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 8
*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*
c^4)*log(d*x + c))/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*
c^3*d^2*e^5*x + c^4*d*e^5)
```

3.14.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^5}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e)**5,x)
```

output
```
Timed out
```

3.14.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(156) = 312$.

Time = 0.32 (sec) , antiderivative size = 534, normalized size of antiderivative = 3.14

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^5}\,dx$$
$$= \frac{1}{6}\left(d\left(\frac{3\,d^2x^2+6\,cdx+3\,c^2-1}{d^5e^5x^3+3\,cd^4e^5x^2+3\,c^2d^3e^5x+c^3d^2e^5}+\frac{3\,\arctan\left(\frac{d^2x+cd}{d}\right)}{d^2e^5}\right)-\frac{3\,\arctan\,(dx-}{d^5e^5x^4+4\,cd^4e^5x^3+6\,c^2d^3e^5x^2}\right.$$
$$+\frac{1}{12}\left(2\,d\left(\frac{3\,d^2x^2+6\,cdx+3\,c^2-1}{d^5e^5x^3+3\,cd^4e^5x^2+3\,c^2d^3e^5x+c^3d^2e^5}+\frac{3\,\arctan\left(\frac{d^2x+cd}{d}\right)}{d^2e^5}\right)\arctan\,(dx+c)-\frac{(3\,(d^2x^2+}{}\right.$$
$$-\frac{b^2\arctan\,(dx+c)^2}{4\,(d^5e^5x^4+4\,cd^4e^5x^3+6\,c^2d^3e^5x^2+4\,c^3d^2e^5x+c^4de^5)}$$
$$-\frac{a^2}{4\,(d^5e^5x^4+4\,cd^4e^5x^3+6\,c^2d^3e^5x^2+4\,c^3d^2e^5x+c^4de^5)}$$

input
```
integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="maxima")
```

output
```
1/6*(d*((3*d^2*x^2 + 6*c*d*x + 3*c^2 - 1)/(d^5*e^5*x^3 + 3*c*d^4*e^5*x^2 +
 3*c^2*d^3*e^5*x + c^3*d^2*e^5) + 3*arctan((d^2*x + c*d)/d)/(d^2*e^5)) - 3
*arctan(d*x + c)/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^
3*d^2*e^5*x + c^4*d*e^5))*a*b + 1/12*(2*d*((3*d^2*x^2 + 6*c*d*x + 3*c^2 -
1)/(d^5*e^5*x^3 + 3*c*d^4*e^5*x^2 + 3*c^2*d^3*e^5*x + c^3*d^2*e^5) + 3*arc
tan((d^2*x + c*d)/d)/(d^2*e^5))*arctan(d*x + c) - (3*(d^2*x^2 + 2*c*d*x +
c^2)*arctan(d*x + c)^2 - 4*(d^2*x^2 + 2*c*d*x + c^2)*log(d^2*x^2 + 2*c*d*x
 + c^2 + 1) + 8*(d^2*x^2 + 2*c*d*x + c^2)*log(d*x + c) + 1)*d^2/(d^5*e^5*x
^2 + 2*c*d^4*e^5*x + c^2*d^3*e^5))*b^2 - 1/4*b^2*arctan(d*x + c)^2/(d^5*e^
5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4*d*e^5)
 - 1/4*a^2/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*
e^5*x + c^4*d*e^5)
```

3.14.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^5}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="giac")
```

output
```
Timed out
```

3.14.9 Mupad [B] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.58

$$\int \frac{(a+b\arctan(c+dx))^2}{(ce+dex)^5}\,dx$$
$$= \operatorname{atan}(c+d\,x)^2\left(\frac{b^2}{4\,d\,e^5}-\frac{b^2}{4\,d^3\,e^5\,\left(\frac{c^4}{d^2}+6\,c^2\,x^2+d^2\,x^4+\frac{4\,c^3\,x}{d}+4\,c\,d\,x^3\right)}\right)$$
$$-\frac{x^2\left(\frac{b^2\,d}{2}-9\,a\,b\,c\,d\right)+x\,(b^2\,c-9\,a\,b\,c^2+a\,b)+\frac{3\,a^2-6\,a\,b\,c^3+2\,a\,b\,c+b^2\,c^2}{2\,d}-3\,a\,b\,d^2\,x^3}{6\,c^4\,e^5+24\,c^3\,d\,e^5\,x+36\,c^2\,d^2\,e^5\,x^2+24\,c\,d^3\,e^5\,x^3+6\,d^4\,e^5\,x^4}$$
$$+\frac{\operatorname{atan}(c+d\,x)\,\left(\frac{b^2\,x^3}{2\,e^5}-\frac{a\,b}{2\,d^3\,e^5}+\frac{b^2\,c\,\left(\frac{c^2-1}{3\,d^2}+\frac{2\,c^2}{3\,d^2}\right)}{2\,d\,e^5}+\frac{b^2\,x\,\left(d\,\left(\frac{c^2-1}{3\,d^2}+\frac{2\,c^2}{3\,d^2}\right)+\frac{2\,c^2}{d}\right)}{2\,d\,e^5}+\frac{3\,b^2\,c\,x^2}{2\,d\,e^5}\right)}{\frac{c^4}{d^2}+6\,c^2\,x^2+d^2\,x^4+\frac{4\,c^3\,x}{d}+4\,c\,d\,x^3}$$
$$-\frac{2\,b^2\,\ln(c+d\,x)}{3\,d\,e^5}-\frac{\ln(c+d\,x-\mathrm{i})\,\left(-\frac{b^2}{3}+\frac{a\,b\,1\mathrm{i}}{4}\right)}{d\,e^5}+\frac{\ln(c+d\,x+1\mathrm{i})\,\left(\frac{b^2}{3}+\frac{1\mathrm{i}\,a\,b}{4}\right)}{d\,e^5}$$

input
```
int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^5,x)
```

output
```
atan(c + d*x)^2*(b^2/(4*d*e^5) - b^2/(4*d^3*e^5*(c^4/d^2 + 6*c^2*x^2 + d^2
*x^4 + (4*c^3*x)/d + 4*c*d*x^3))) - (x^2*((b^2*d)/2 - 9*a*b*c*d) + x*(a*b
+ b^2*c - 9*a*b*c^2) + (3*a^2 + b^2*c^2 + 2*a*b*c - 6*a*b*c^3)/(2*d) - 3*a
*b*d^2*x^3)/(6*c^4*e^5 + 6*d^4*e^5*x^4 + 24*c*d^3*e^5*x^3 + 36*c^2*d^2*e^5
*x^2 + 24*c^3*d*e^5*x) + (atan(c + d*x)*((b^2*x^3)/(2*e^5) - (a*b)/(2*d^3*
e^5) + (b^2*c*((c^2 - 1)/(3*d^2) + (2*c^2)/(3*d^2)))/(2*d*e^5) + (b^2*x*(d
*((c^2 - 1)/(3*d^2) + (2*c^2)/(3*d^2)) + (2*c^2)/d))/(2*d*e^5) + (3*b^2*c*
x^2)/(2*d*e^5)))/(c^4/d^2 + 6*c^2*x^2 + d^2*x^4 + (4*c^3*x)/d + 4*c*d*x^3)
 - (2*b^2*log(c + d*x))/(3*d*e^5) - (log(c + d*x - 1i)*((a*b*1i)/4 - b^2/3
))/(d*e^5) + (log(c + d*x + 1i)*((a*b*1i)/4 + b^2/3))/(d*e^5)
```

3.15 $\int (ce+dex)^2(a+b\arctan(c+dx))^3\,dx$

3.15.1 Optimal result

Integrand size = 23, antiderivative size = 271

$$\begin{aligned}\int (ce+dex)^2(a+b\arctan(c+dx))^3\,dx &= ab^2e^2x+\frac{b^3e^2(c+dx)\arctan(c+dx)}{d}-\frac{be^2(a+b\arctan(c+dx))^2}{2d}\\ &-\frac{be^2(c+dx)^2(a+b\arctan(c+dx))^2}{2d}-\frac{ie^2(a+b\arctan(c+dx))^3}{3d}\\ &+\frac{e^2(c+dx)^3(a+b\arctan(c+dx))^3}{3d}-\frac{be^2(a+b\arctan(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d}\\ &-\frac{b^3e^2\log\left(1+(c+dx)^2\right)}{2d}-\frac{ib^2e^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d}\\ &-\frac{b^3e^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d}\end{aligned}$$

output
```
a*b^2*e^2*x+b^3*e^2*(d*x+c)*arctan(d*x+c)/d-1/2*b*e^2*(a+b*arctan(d*x+c))^
2/d-1/2*b*e^2*(d*x+c)^2*(a+b*arctan(d*x+c))^2/d-1/3*I*e^2*(a+b*arctan(d*x+
c))^3/d+1/3*e^2*(d*x+c)^3*(a+b*arctan(d*x+c))^3/d-b*e^2*(a+b*arctan(d*x+c)
)^2*ln(2/(1+I*(d*x+c)))/d-1/2*b^3*e^2*ln(1+(d*x+c)^2)/d-I*b^2*e^2*(a+b*arc
tan(d*x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d-1/2*b^3*e^2*polylog(3,1-2/(1+I*
(d*x+c)))/d
```

3.15.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.29

$$\int (ce+dex)^2(a+b\arctan(c+dx))^3\,dx = \frac{e^2\left(-3a^2b(c+dx)^2+2a^3(c+dx)^3+6a^2b(c+dx)^3\arctan(c+dx)+3a^2b\log\left(1+(c+dx)^2\right)+6ab^2(c+\right.}{}$$

input
```
Integrate[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x])^3,x]
```

output
```
(e^2*(-3*a^2*b*(c + d*x)^2 + 2*a^3*(c + d*x)^3 + 6*a^2*b*(c + d*x)^3*ArcTa
n[c + d*x] + 3*a^2*b*Log[1 + (c + d*x)^2] + 6*a*b^2*(c + d*x - ArcTan[c +
d*x] - (c + d*x)^2*ArcTan[c + d*x] + I*ArcTan[c + d*x]^2 + (c + d*x)^3*Arc
Tan[c + d*x]^2 - 2*ArcTan[c + d*x]*Log[1 + E^((2*I)*ArcTan[c + d*x])] + I*
PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]) + b^3*(6*(c + d*x)*ArcTan[c + d*x]
 - 3*(1 + (c + d*x)^2)*ArcTan[c + d*x]^2 + (2*I)*ArcTan[c + d*x]^3 - 2*(c
+ d*x)*ArcTan[c + d*x]^3 + 2*(c + d*x)*(1 + (c + d*x)^2)*ArcTan[c + d*x]^3
 - 6*ArcTan[c + d*x]^2*Log[1 + E^((2*I)*ArcTan[c + d*x])] + 6*Log[1/Sqrt[1
 + (c + d*x)^2]] + (6*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*ArcTan[c + d
*x])] - 3*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])))/(6*d)
```

3.15.3 Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.85, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {5566, 27, 5361, 5451, 5361, 5451, 2009, 5419, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^2(a+b\arctan(c+dx))^3\,dx$$

$$\Big\downarrow \; 5566$$

$$\frac{\int e^2(c+dx)^2(a+b\arctan(c+dx))^3d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{e^2\int (c+dx)^2(a+b\arctan(c+dx))^3d(c+dx)}{d}$$

$\downarrow$ 5361

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))^3-b\int\frac{(c+dx)^3(a+b\arctan(c+dx))^2}{(c+dx)^2+1}d(c+dx)\right)}{d}$$

$\downarrow$ 5451

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))^3-b\left(\int(c+dx)(a+b\arctan(c+dx))^2d(c+dx)-\int\frac{(c+dx)(a+b\arctan(c+dx))^2}{(c+dx)^2+1}d\right.\right.}{d}$$

$\downarrow$ 5361

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))^3-b\left(-b\int\frac{(c+dx)^2(a+b\arctan(c+dx))}{(c+dx)^2+1}d(c+dx)-\int\frac{(c+dx)(a+b\arctan(c+dx))^2}{(c+dx)^2+1}d(c+\right.\right.}{d}$$

$\downarrow$ 5451

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))^3-b\left(-b\left(\int(a+b\arctan(c+dx))d(c+dx)-\int\frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)\right)-\right.\right.}{d}$$

$\downarrow$ 2009

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))^3-b\left(-\int\frac{(c+dx)(a+b\arctan(c+dx))^2}{(c+dx)^2+1}d(c+dx)-b\left(-\int\frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)\right.\right.\right.}{d}$$

$\downarrow$ 5419

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))^3-b\left(-\int\frac{(c+dx)(a+b\arctan(c+dx))^2}{(c+dx)^2+1}d(c+dx)+\frac{1}{2}(c+dx)^2(a+b\arctan(c+dx\right.\right.}{d}$$

$\downarrow$ 5455

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))^3-b\left(\int\frac{(a+b\arctan(c+dx))^2}{-c-dx+i}d(c+dx)+\frac{i(a+b\arctan(c+dx))^3}{3b}+\frac{1}{2}(c+dx)^2(a+b\,\mathrm{ar}\right.\right.}{d}$$

$\downarrow$ 5379

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))^3-b\left(-2b\int\frac{(a+b\arctan(c+dx))\log\left(\frac{2}{i(c+dx)+1}\right)}{(c+dx)^2+1}d(c+dx)+\frac{i(a+b\arctan(c+dx))^3}{3b}+\frac{1}{2}\right.\right.}{}$$

$\downarrow$ 5529

3.15. $\int(ce+dex)^2(a+b\arctan(c+dx))^3\,dx$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))^3-b\left(-2b\left(\frac{1}{2}ib\int\frac{\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)}{(c+dx)^2+1}d(c+dx)-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{i(c+dx}\right.\right.\right.\right.$$

$$\Big\downarrow \text{7164}$$

$$e^2\left(\frac{1}{3}(c+dx)^3(a+b\arctan(c+dx))^3-b\left(-2b\left(-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)(a+b\arctan(c+dx))-\frac{1}{4}b\,\text{PolyL}\right.\right.\right.$$

input
```
Int[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x])^3,x]
```

output
```
(e^2*(((c + d*x)^3*(a + b*ArcTan[c + d*x])^3)/3 - b*(((c + d*x)^2*(a + b*A
rcTan[c + d*x])^2)/2 + ((I/3)*(a + b*ArcTan[c + d*x])^3)/b + (a + b*ArcTan
[c + d*x])^2*Log[2/(1 + I*(c + d*x))] - b*(a*(c + d*x) + b*(c + d*x)*ArcTa
n[c + d*x] - (a + b*ArcTan[c + d*x])^2/(2*b) - (b*Log[1 + (c + d*x)^2])/2)
 - 2*b*((-1/2*I)*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x)
)] - (b*PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/4))))/d
```

3.15.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5566
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.15.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.18 (sec) , antiderivative size = 1249, normalized size of antiderivative = 4.61

method	result	size
derivativedivides	Expression too large to display	1249
default	Expression too large to display	1249
parts	Expression too large to display	1257

input
```
int((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*e^2*a^3*(d*x+c)^3+e^2*b^3*(1/3*(d*x+c)^3*arctan(d*x+c)^3-1/2*(d*x
+c)^2*arctan(d*x+c)^2+1/2*arctan(d*x+c)^2*ln(1+(d*x+c)^2)-arctan(d*x+c)^2*
ln((1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+I*arctan(d*x+c)*polylog(2,-(1+I*(d*x
+c))^2/(1+(d*x+c)^2))-1/2*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+1/12*I
*arctan(d*x+c)*(3*csgn(I*(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))^2*csgn(I*(1+I*
(d*x+c))^2/(1+(d*x+c)^2))*Pi*arctan(d*x+c)-6*csgn(I*(1+I*(d*x+c))/(1+(d*x+
c)^2)^(1/2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2*Pi*arctan(d*x+c)+3*cs
gn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^3*Pi*arctan(d*x+c)-3*csgn(I*(1+I*(d*x+
c))^2/(1+(d*x+c)^2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))
^2/(1+(d*x+c)^2))^2)^2*Pi*arctan(d*x+c)+3*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c
)^2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2
))^2)*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)*Pi*arctan(d*x+c)-3*csgn(
I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c
)^2))^2)*Pi*arctan(d*x+c)+6*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn
(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^2*Pi*arctan(d*x+c)-3*csgn(I*(1+(1+
I*(d*x+c))^2/(1+(d*x+c)^2))^2)^3*Pi*arctan(d*x+c)+3*csgn(I*(1+I*(d*x+c))^2
/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^3*Pi*arctan(d*x+c)-3*c
sgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^2
*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)*Pi*arctan(d*x+c)+4*arctan(d*x
+c)^2+12*I*ln(2)*arctan(d*x+c)+6*I*arctan(d*x+c)-12-12*I*(d*x+c))+ln(1+...
```

3.15.5 Fricas [F]

$$\int (ce+dex)^2(a+b\arctan(c+dx))^3\,dx = \int (dex+ce)^2(b\arctan(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(a^3*d^2*e^2*x^2 + 2*a^3*c*d*e^2*x + a^3*c^2*e^2 + (b^3*d^2*e^2*x^
2 + 2*b^3*c*d*e^2*x + b^3*c^2*e^2)*arctan(d*x + c)^3 + 3*(a*b^2*d^2*e^2*x^
2 + 2*a*b^2*c*d*e^2*x + a*b^2*c^2*e^2)*arctan(d*x + c)^2 + 3*(a^2*b*d^2*e^
2*x^2 + 2*a^2*b*c*d*e^2*x + a^2*b*c^2*e^2)*arctan(d*x + c), x)
```

3.15.6 Sympy [F]

$$\int (ce+dex)^2(a+b\arctan(c+dx))^3\,dx$$
$$= e^2\left(\int a^3c^2\,dx + \int a^3d^2x^2\,dx + \int b^3c^2\,\mathrm{atan}^3\,(c+dx)\,dx + \int 3ab^2c^2\,\mathrm{atan}^2\,(c+dx)\,dx + \int 3a^2bc^2\,\mathrm{atan}\,(c+dx)\,dx + \int 2a^3cdx\,dx + \int b^3d^2x^2\,\mathrm{atan}^3\,(c+dx)\,dx + \int 3ab^2d^2x^2\,\mathrm{atan}^2\,(c+dx)\,dx + \int 3a^2bd^2x^2\,\mathrm{atan}\,(c+dx)\,dx + \int 2b^3cdx\,\mathrm{atan}^3\,(c+dx)\,dx + \int 6ab^2cdx\,\mathrm{atan}^2\,(c+dx)\,dx + \int 6a^2bcdx\,\mathrm{atan}\,(c+dx)\,dx\right)$$

input
```
integrate((d*e*x+c*e)**2*(a+b*atan(d*x+c))**3,x)
```

output
```
e**2*(Integral(a**3*c**2, x) + Integral(a**3*d**2*x**2, x) + Integral(b**3
*c**2*atan(c + d*x)**3, x) + Integral(3*a*b**2*c**2*atan(c + d*x)**2, x) +
 Integral(3*a**2*b*c**2*atan(c + d*x), x) + Integral(2*a**3*c*d*x, x) + In
tegral(b**3*d**2*x**2*atan(c + d*x)**3, x) + Integral(3*a*b**2*d**2*x**2*a
tan(c + d*x)**2, x) + Integral(3*a**2*b*d**2*x**2*atan(c + d*x), x) + Inte
gral(2*b**3*c*d*x*atan(c + d*x)**3, x) + Integral(6*a*b**2*c*d*x*atan(c +
d*x)**2, x) + Integral(6*a**2*b*c*d*x*atan(c + d*x), x))
```

3.15.7 Maxima [F]

$$\int (ce+dex)^2(a+b\arctan(c+dx))^3\,dx=\int (dex+ce)^2(b\arctan(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

output
```
7/8*b^3*c^4*e^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 3*a*b^2*c^4*
e^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - (3*arctan(d*x + c)*arcta
n((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^4*e^2 - 7/32
*(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arct
an((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^4*e^2 + 1/3*a
^3*d^2*e^2*x^3 + 7/8*b^3*c^2*e^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)
/d + 28*b^3*d^4*e^2*integrate(1/32*x^4*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*
x + c^2 + 1), x) + 3*b^3*d^4*e^2*integrate(1/32*x^4*arctan(d*x + c)*log(d^
2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*
d^4*e^2*integrate(1/32*x^4*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1)
, x) + 112*b^3*c*d^3*e^2*integrate(1/32*x^3*arctan(d*x + c)^3/(d^2*x^2 + 2
*c*d*x + c^2 + 1), x) + 4*b^3*d^4*e^2*integrate(1/32*x^4*arctan(d*x + c)*l
og(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3
*c*d^3*e^2*integrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2
+ 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*a*b^2*c*d^3*e^2*integrate(1
/32*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 168*b^3*c^2*
d^2*e^2*integrate(1/32*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1)
, x) + 16*b^3*c*d^3*e^2*integrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2
*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 18*b^3*c^2*d^2*e^2*i
ntegrate(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d...
```

3.15.8 Giac [F]

$$\int (ce+dex)^2(a+b\arctan(c+dx))^3\,dx=\int (dex+ce)^2(b\arctan(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int (ce+dex)^2(a+b\arctan(c+dx))^3\,dx=\int {\left(c\,e+d\,e\,x\right)}^2\,{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)}^3 \,d x$$

input
```
int((c*e + d*e*x)^2*(a + b*atan(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)^2*(a + b*atan(c + d*x))^3, x)
```

3.16 $\int (ce+dex)(a+b\arctan(c+dx))^3\,dx$

3.16.1 Optimal result

Integrand size = 21, antiderivative size = 164

$$\begin{aligned}\int (ce+dex)(a+b\arctan(c+dx))^3\,dx = &-\frac{3ibe(a+b\arctan(c+dx))^2}{2d}\\ &-\frac{3be(c+dx)(a+b\arctan(c+dx))^2}{2d}\\ &+\frac{e(a+b\arctan(c+dx))^3}{2d}\\ &+\frac{e(c+dx)^2(a+b\arctan(c+dx))^3}{2d}\\ &-\frac{3b^2e(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d}\\ &-\frac{3ib^3e\,\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{2d}\end{aligned}$$

output
```
-3/2*I*b*e*(a+b*arctan(d*x+c))^2/d-3/2*b*e*(d*x+c)*(a+b*arctan(d*x+c))^2/d
+1/2*e*(a+b*arctan(d*x+c))^3/d+1/2*e*(d*x+c)^2*(a+b*arctan(d*x+c))^3/d-3*b
^2*e*(a+b*arctan(d*x+c))*ln(2/(1+I*(d*x+c)))/d-3/2*I*b^3*e*polylog(2,1-2/(
1+I*(d*x+c)))/d
```

3.16.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.20

$$\int (ce+dex)(a+b\arctan(c+dx))^3\,dx$$
$$= \frac{e\left(3b^2(-i+c+dx)(-b+a(i+c+dx))\arctan(c+dx)^2+b^3(1+c^2+2cdx+d^2x^2)\arctan(c+dx)^3+3\right.}{}$$

input
```
Integrate[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^3,x]
```

output
```
(e*(3*b^2*(-I + c + d*x)*(-b + a*(I + c + d*x))*ArcTan[c + d*x]^2 + b^3*(1
 + c^2 + 2*c*d*x + d^2*x^2)*ArcTan[c + d*x]^3 + 3*b*ArcTan[c + d*x]*(a*(-2
*b*(c + d*x) + a*(1 + c^2 + 2*c*d*x + d^2*x^2)) - 2*b^2*Log[1 + E^((2*I)*A
rcTan[c + d*x])]) + a*(a*(c + d*x)*(-3*b + a*c + a*d*x) - 6*b^2*Log[1/Sqrt
[1 + (c + d*x)^2]]) + (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]))/(
2*d)
```

3.16.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5566, 27, 5361, 5451, 5345, 5419, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)(a+b\arctan(c+dx))^3\,dx$$

$$\Big\downarrow \ 5566$$

$$\frac{\int e(c+dx)(a+b\arctan(c+dx))^3d(c+dx)}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{e\int (c+dx)(a+b\arctan(c+dx))^3d(c+dx)}{d}$$

$$\Big\downarrow \ 5361$$

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\arctan(c+dx))^3-\frac{3}{2}b\int \frac{(c+dx)^2(a+b\arctan(c+dx))^2}{(c+dx)^2+1}d(c+dx)\right)}{d}$$

$\downarrow$ 5451

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\arctan(c+dx))^3-\frac{3}{2}b\left(\int(a+b\arctan(c+dx))^2d(c+dx)-\int\frac{(a+b\arctan(c+dx))^2}{(c+dx)^2+1}d(c+dx)\right)\right)}{d}$$

$\downarrow$ 5345

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\arctan(c+dx))^3-\frac{3}{2}b\left(-2b\int\frac{(c+dx)(a+b\arctan(c+dx))}{(c+dx)^2+1}d(c+dx)-\int\frac{(a+b\arctan(c+dx))^2}{(c+dx)^2+1}d(c+dx)-\right.\right.}{d}$$

$\downarrow$ 5419

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\arctan(c+dx))^3-\frac{3}{2}b\left(-2b\int\frac{(c+dx)(a+b\arctan(c+dx))}{(c+dx)^2+1}d(c+dx)-\frac{(a+b\arctan(c+dx))^3}{3b}+(c+dx)(a\right.\right.}{d}$$

$\downarrow$ 5455

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\arctan(c+dx))^3-\frac{3}{2}b\left(-2b\left(-\int\frac{a+b\arctan(c+dx)}{-c-dx+i}d(c+dx)-\frac{i(a+b\arctan(c+dx))^2}{2b}\right)-\frac{(a+b\arctan(c-}{3b}\right.\right.}{d}$$

$\downarrow$ 5379

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\arctan(c+dx))^3-\frac{3}{2}b\left(-2b\left(b\int\frac{\log\left(\frac{2}{i(c+dx)+1}\right)}{(c+dx)^2+1}d(c+dx)-\frac{i(a+b\arctan(c+dx))^2}{2b}-\log\left(\frac{2}{1+i(c+dx)}\right.\right.\right.\right.}{d}$$

$\downarrow$ 2849

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\arctan(c+dx))^3-\frac{3}{2}b\left(-2b\left(-ib\int\frac{\log\left(\frac{2}{i(c+dx)+1}\right)}{1-\frac{2}{i(c+dx)+1}}d\frac{1}{i(c+dx)+1}-\frac{i(a+b\arctan(c+dx))^2}{2b}-\log\left(\frac{2}{1+i(c+\right.\right.\right.\right.}{d}$$

$\downarrow$ 2752

$$\frac{e\left(\frac{1}{2}(c+dx)^2(a+b\arctan(c+dx))^3-\frac{3}{2}b\left(-2b\left(-\frac{i(a+b\arctan(c+dx))^2}{2b}-\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))-\frac{1}{2}\right.\right.\right.}{d}$$

input
```
Int[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^3,x]
```

3.16. $\int(ce+dex)(a+b\arctan(c+dx))^3\,dx$

output
```
(e*(((c + d*x)^2*(a + b*ArcTan[c + d*x])^3)/2 - (3*b*((c + d*x)*(a + b*Arc
Tan[c + d*x])^2 - (a + b*ArcTan[c + d*x])^3/(3*b) - 2*b*(((-1/2*I)*(a + b*
ArcTan[c + d*x])^2)/b - (a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))] -
 (I/2)*b*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])))/2))/d
```

3.16.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5566
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.16.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(150) = 300$.

Time = 0.55 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.96

method	result
derivativedivides	$\frac{\frac{e a^3 (dx+c)^2}{2} + e b^3 \left(\frac{(dx+c)^2 \arctan(dx+c)^3}{2} + \frac{\arctan(dx+c)^3}{2} - \frac{3(dx+c)\arctan(dx+c)^2}{2} + \frac{3\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{2} + \frac{3i\left(\ln(d \ldots\right.}{\ldots} \right.}{\ldots}$
default	$\frac{\frac{e a^3 (dx+c)^2}{2} + e b^3 \left(\frac{(dx+c)^2 \arctan(dx+c)^3}{2} + \frac{\arctan(dx+c)^3}{2} - \frac{3(dx+c)\arctan(dx+c)^2}{2} + \frac{3\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{2} + \frac{3i\left(\ln(d \ldots\right.}{\ldots} \right.}{\ldots}$
parts	$e a^3 \left(\frac{1}{2} d x^2 + cx\right) + \frac{e b^3 \left(\frac{(dx+c)^2 \arctan(dx+c)^3}{2} + \frac{\arctan(dx+c)^3}{2} - \frac{3(dx+c)\arctan(dx+c)^2}{2} + \frac{3\arctan(dx+c)\ln\left(1+(dx+ \ldots\right.}{2} \right.}{\ldots}$
risch	Expression too large to display

input
```
int((d*e*x+c*e)*(a+b*arctan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*e*a^3*(d*x+c)^2+e*b^3*(1/2*(d*x+c)^2*arctan(d*x+c)^3+1/2*arctan(d
*x+c)^3-3/2*(d*x+c)*arctan(d*x+c)^2+3/2*arctan(d*x+c)*ln(1+(d*x+c)^2)+3/4*
I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog(-1/2*I*(d*x+c+I))-l
n(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))-3/4*I*(ln(d*x+c+I)*ln(1+(d*x+c)^2)-1/2*ln
(d*x+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I*(d*x+c-I)))))+3*e*a
*b^2*(1/2*(d*x+c)^2*arctan(d*x+c)^2+1/2*arctan(d*x+c)^2-(d*x+c)*arctan(d*x
+c)+1/2*ln(1+(d*x+c)^2))+3*e*a^2*b*(1/2*(d*x+c)^2*arctan(d*x+c)-1/2*d*x-1/
2*c+1/2*arctan(d*x+c)))
```

3.16.5 Fricas [F]

$$\int (ce+dex)(a+b\arctan(c+dx))^3\,dx = \int (dex+ce)(b\arctan(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(a^3*d*e*x + a^3*c*e + (b^3*d*e*x + b^3*c*e)*arctan(d*x + c)^3 + 3
*(a*b^2*d*e*x + a*b^2*c*e)*arctan(d*x + c)^2 + 3*(a^2*b*d*e*x + a^2*b*c*e)
*arctan(d*x + c), x)
```

3.16.6 Sympy [F]

$$\begin{aligned}\int (ce+dex)(a+b\arctan(c+dx))^3\,dx = e\Bigg(&\int a^3c\,dx+\int a^3dx\,dx+\int b^3c\operatorname{atan}^3(c+dx)\,dx\\&+\int 3ab^2c\operatorname{atan}^2(c+dx)\,dx\\&+\int 3a^2bc\operatorname{atan}(c+dx)\,dx\\&+\int b^3dx\operatorname{atan}^3(c+dx)\,dx\\&+\int 3ab^2dx\operatorname{atan}^2(c+dx)\,dx\\&+\int 3a^2bdx\operatorname{atan}(c+dx)\,dx\Bigg)\end{aligned}$$

input
```
integrate((d*e*x+c*e)*(a+b*atan(d*x+c))**3,x)
```

output
```
e*(Integral(a**3*c, x) + Integral(a**3*d*x, x) + Integral(b**3*c*atan(c +
d*x)**3, x) + Integral(3*a*b**2*c*atan(c + d*x)**2, x) + Integral(3*a**2*b
*c*atan(c + d*x), x) + Integral(b**3*d*x*atan(c + d*x)**3, x) + Integral(3
*a*b**2*d*x*atan(c + d*x)**2, x) + Integral(3*a**2*b*d*x*atan(c + d*x), x)
)
```

3.16.7 Maxima [F]

$$\int (ce+dex)(a+b\arctan(c+dx))^3\,dx=\int (dex+ce)(b\arctan(dx+c)+a)^3\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*a^3*d*e*x^2 + 3/2*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((
d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*d*e +
 a^3*c*e*x + 3/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a^2*
b*c*e/d + 1/32*(8*(b^3*d^2*e*x^2 + 2*b^3*c*d*e*x + (b^3*c^2 + b^3)*e)*arct
an(d*x + c)^3 + 12*(a*b^2*d^2*e*x^2 + (2*a*b^2*c - b^3)*d*e*x)*arctan(d*x
+ c)^2 - 3*(a*b^2*d^2*e*x^2 + (2*a*b^2*c - b^3)*d*e*x)*log(d^2*x^2 + 2*c*d
*x + c^2 + 1)^2 + 4*(4*b^3*c^3*e*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)
/d + 18*a*b^2*c^3*e*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 6*(3*arc
tan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*
b^2*c^3*e - (6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*
x + c)*arctan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^3*
e - 3*b^3*c^2*e*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d + 4*b^3*c*e*ar
ctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 128*b^3*d^3*e*integrate(1/32*x
^3*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 576*a*b^2*d^3*e*i
ntegrate(1/32*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 38
4*b^3*c*d^2*e*integrate(1/32*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^
2 + 1), x) + 48*a*b^2*d^3*e*integrate(1/32*x^3*log(d^2*x^2 + 2*c*d*x + c^2
 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 1728*a*b^2*c*d^2*e*integrate(1
/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*b^3*c^2*
d*e*integrate(1/32*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x...
```

3.16.8 Giac [F]

$$\int (ce + dex)(a + b\arctan(c + dx))^3\,dx = \int (dex + ce)(b\arctan(dx + c) + a)^3\,dx$$

input
```
integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int (ce + dex)(a + b\arctan(c + dx))^3\,dx = \int (c\,e + d\,e\,x)\,(a + b\,\mathrm{atan}(c + d\,x))^3\,dx$$

input
```
int((c*e + d*e*x)*(a + b*atan(c + d*x))^3,x)
```

output
```
int((c*e + d*e*x)*(a + b*atan(c + d*x))^3, x)
```

3.17 $\int \frac{(a+b\arctan(c+dx))^3}{ce+dex}\,dx$

3.17.1 Optimal result

Integrand size = 23, antiderivative size = 279

$$\begin{aligned}\int \frac{(a+b\arctan(c+dx))^3}{ce+dex}\,dx &= \frac{2(a+b\arctan(c+dx))^3\operatorname{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)}{de}\\ &\quad-\frac{3ib(a+b\arctan(c+dx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{2de}\\ &\quad+\frac{3ib(a+b\arctan(c+dx))^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+i(c+dx)}\right)}{2de}\\ &\quad-\frac{3b^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2de}\\ &\quad+\frac{3b^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(3,-1+\frac{2}{1+i(c+dx)}\right)}{2de}\\ &\quad+\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+i(c+dx)}\right)}{4de}\\ &\quad-\frac{3ib^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+i(c+dx)}\right)}{4de}\end{aligned}$$

output
```
-2*(a+b*arctan(d*x+c))^3*arctanh(-1+2/(1+I*(d*x+c)))/d/e-3/2*I*b*(a+b*arct
an(d*x+c))^2*polylog(2,1-2/(1+I*(d*x+c)))/d/e+3/2*I*b*(a+b*arctan(d*x+c))^
2*polylog(2,-1+2/(1+I*(d*x+c)))/d/e-3/2*b^2*(a+b*arctan(d*x+c))*polylog(3,
1-2/(1+I*(d*x+c)))/d/e+3/2*b^2*(a+b*arctan(d*x+c))*polylog(3,-1+2/(1+I*(d*
x+c)))/d/e+3/4*I*b^3*polylog(4,1-2/(1+I*(d*x+c)))/d/e-3/4*I*b^3*polylog(4,
-1+2/(1+I*(d*x+c)))/d/e
```

3.17.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 562 vs. $2(279) = 558$.

Time = 0.55 (sec) , antiderivative size = 562, normalized size of antiderivative = 2.01

$$\int \frac{(a+b\arctan(c+dx))^3}{ce+dex}\,dx$$
$$= \frac{64a^3\log(c+dx) - 24ia^2b\big(\pi^2 - 4\pi\arctan(c+dx) + 8\arctan(c+dx)^2 - i\pi\log(16) + 4i\pi\log\left(1+e^{-2i\,\mathrm{arc}}\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x),x]
```

output
```
(64*a^3*Log[c + d*x] - (24*I)*a^2*b*(Pi^2 - 4*Pi*ArcTan[c + d*x] + 8*ArcTa
n[c + d*x]^2 - I*Pi*Log[16] + (4*I)*Pi*Log[1 + E^((-2*I)*ArcTan[c + d*x])]
 - (8*I)*ArcTan[c + d*x]*Log[1 + E^((-2*I)*ArcTan[c + d*x])] + (8*I)*ArcTa
n[c + d*x]*Log[1 - E^((2*I)*ArcTan[c + d*x])] + (2*I)*Pi*Log[1 + c^2 + 2*c
*d*x + d^2*x^2] + 4*PolyLog[2, -E^((-2*I)*ArcTan[c + d*x])] + 4*PolyLog[2,
 E^((2*I)*ArcTan[c + d*x])]) + 8*a*b^2*((-I)*Pi^3 + (16*I)*ArcTan[c + d*x]
^3 + 24*ArcTan[c + d*x]^2*Log[1 - E^((-2*I)*ArcTan[c + d*x])] - 24*ArcTan[
c + d*x]^2*Log[1 + E^((2*I)*ArcTan[c + d*x])] + (24*I)*ArcTan[c + d*x]*Pol
yLog[2, E^((-2*I)*ArcTan[c + d*x])] + (24*I)*ArcTan[c + d*x]*PolyLog[2, -E
^((2*I)*ArcTan[c + d*x])] + 12*PolyLog[3, E^((-2*I)*ArcTan[c + d*x])] - 12
*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])]) - I*b^3*(Pi^4 - 32*ArcTan[c + d*x
]^4 + (64*I)*ArcTan[c + d*x]^3*Log[1 - E^((-2*I)*ArcTan[c + d*x])] - (64*I
)*ArcTan[c + d*x]^3*Log[1 + E^((2*I)*ArcTan[c + d*x])] - 96*ArcTan[c + d*x
]^2*PolyLog[2, E^((-2*I)*ArcTan[c + d*x])] - 96*ArcTan[c + d*x]^2*PolyLog[
2, -E^((2*I)*ArcTan[c + d*x])] + (96*I)*ArcTan[c + d*x]*PolyLog[3, E^((-2*
I)*ArcTan[c + d*x])] - (96*I)*ArcTan[c + d*x]*PolyLog[3, -E^((2*I)*ArcTan[
c + d*x])] + 48*PolyLog[4, E^((-2*I)*ArcTan[c + d*x])] + 48*PolyLog[4, -E^
((2*I)*ArcTan[c + d*x])]))/(64*d*e)
```

3.17.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5566, 27, 5357, 5523, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(c+dx))^3}{ce+dex}dx$$

$$\Big\downarrow \text{5566}$$

$$\frac{\int \frac{(a+b\arctan(c+dx))^3}{e(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\arctan(c+dx))^3}{c+dx}d(c+dx)}{de}$$

$$\Big\downarrow \text{5357}$$

$$\frac{2\text{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))^3-6b\int \frac{(a+b\arctan(c+dx))^2\text{arctanh}\left(1-\frac{2}{i(c+dx)+1}\right)}{(c+dx)^2+1}d(c+dx)}{de}$$

$$\Big\downarrow \text{5523}$$

$$\frac{2\text{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))^3-6b\left(\frac{1}{2}\int \frac{(a+b\arctan(c+dx))^2\log\left(2-\frac{2}{i(c+dx)+1}\right)}{(c+dx)^2+1}d(c+dx)-\frac{1}{2}\int \frac{(a+b\text{ar}}{}\right.}{de}$$

$$\Big\downarrow \text{5529}$$

$$\frac{2\text{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))^3-6b\left(\frac{1}{2}\left(\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)(a+b\arctan(c+dx))^2-\right.\right.}{}$$

$$\Big\downarrow \text{5533}$$

$$\frac{2\text{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))^3-6b\left(\frac{1}{2}\left(\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)(a+b\arctan(c+dx))^2-\right.\right.}{}$$

$$\Big\downarrow \text{7164}$$

$$\frac{2\text{arctanh}\left(1-\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))^3-6b\left(\frac{1}{2}\left(\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)(a+b\arctan(c+dx))^2-\right.\right.}{}$$

input
```
Int[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x),x]
```

output
```
(2*(a + b*ArcTan[c + d*x])^3*ArcTanh[1 - 2/(1 + I*(c + d*x))] - 6*b*(((I/2
)*(a + b*ArcTan[c + d*x])^2*PolyLog[2, 1 - 2/(1 + I*(c + d*x))] - I*b*((I/
2)*(a + b*ArcTan[c + d*x])*PolyLog[3, 1 - 2/(1 + I*(c + d*x))] + (b*PolyLo
g[4, 1 - 2/(1 + I*(c + d*x))])/4))/2 + ((-1/2*I)*(a + b*ArcTan[c + d*x])^2
*PolyLog[2, -1 + 2/(1 + I*(c + d*x))] + I*b*((I/2)*(a + b*ArcTan[c + d*x])
*PolyLog[3, -1 + 2/(1 + I*(c + d*x))] + (b*PolyLog[4, -1 + 2/(1 + I*(c + d
*x))])/4))/2))/(d*e)
```

3.17.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5566
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.17.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.81 (sec) , antiderivative size = 2313, normalized size of antiderivative = 8.29

method	result	size
derivativedivides	Expression too large to display	2313
default	Expression too large to display	2313
parts	Expression too large to display	2321

input
```
int((a+b*arctan(d*x+c))^3/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3/e*ln(d*x+c)+b^3/e*(ln(d*x+c)*arctan(d*x+c)^3-arctan(d*x+c)^3*ln((
1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)+arctan(d*x+c)^3*ln(1-(1+I*(d*x+c))/(1+(d*x
+c)^2)^(1/2))-3*I*arctan(d*x+c)^2*polylog(2,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1
/2))+6*arctan(d*x+c)*polylog(3,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+6*I*poly
log(4,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+arctan(d*x+c)^3*ln(1+(1+I*(d*x+c)
)/(1+(d*x+c)^2)^(1/2))-3*I*arctan(d*x+c)^2*polylog(2,-(1+I*(d*x+c))/(1+(d*
x+c)^2)^(1/2))+6*arctan(d*x+c)*polylog(3,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2
))+6*I*polylog(4,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+1/2*I*Pi*(csgn(I*((1+
I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(((1+
I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))-csgn(((1+
I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2+csgn(I*
((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)
))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^
2)))-csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*csgn(I*((1+I*(d*x+c))^2/(1+
(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2-csgn(I/(1+(1+I*(d*x+c))
^2/(1+(d*x+c)^2)))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c
))^2/(1+(d*x+c)^2)))^2+csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d
*x+c))^2/(1+(d*x+c)^2)))^3-csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+
I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+
I*(d*x+c))^2/(1+(d*x+c)^2)))^2+csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(...
```

3.17.5 Fricas [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{ce+dex}\,dx=\int \frac{(b\arctan(dx+c)+a)^3}{dex+ce}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e),x, algorithm="fricas")
```

output
```
integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arct
an(d*x + c) + a^3)/(d*e*x + c*e), x)
```

3.17.6 Sympy [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{ce+dex}\,dx = \frac{\int \frac{a^3}{c+dx}\,dx + \int \frac{b^3\operatorname{atan}^3(c+dx)}{c+dx}\,dx + \int \frac{3ab^2\operatorname{atan}^2(c+dx)}{c+dx}\,dx + \int \frac{3a^2b\operatorname{atan}(c+dx)}{c+dx}\,dx}{e}$$

input
```
integrate((a+b*atan(d*x+c))**3/(d*e*x+c*e),x)
```

output
```
(Integral(a**3/(c + d*x), x) + Integral(b**3*atan(c + d*x)**3/(c + d*x), x
) + Integral(3*a*b**2*atan(c + d*x)**2/(c + d*x), x) + Integral(3*a**2*b*a
tan(c + d*x)/(c + d*x), x))/e
```

3.17.7 Maxima [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{ce+dex}\,dx = \int \frac{(b\arctan(dx+c)+a)^3}{dex+ce}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e),x, algorithm="maxima")
```

output
```
a^3*log(d*e*x + c*e)/(d*e) + integrate(1/32*(28*b^3*arctan(d*x + c)^3 + 3*
b^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 96*a*b^2*arctan(d
*x + c)^2 + 96*a^2*b*arctan(d*x + c))/(d*e*x + c*e), x)
```

3.17.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{ce+dex}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e),x, algorithm="giac")
```

output
```
Timed out
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{ce+dex}\,dx=\int \frac{{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)}^3}{c\,e+d\,e\,x}\,dx$$

input
```
int((a + b*atan(c + d*x))^3/(c*e + d*e*x),x)
```

output
```
int((a + b*atan(c + d*x))^3/(c*e + d*e*x), x)
```

3.18 $\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^2}\,dx$

3.18.1 Optimal result

Integrand size = 23, antiderivative size = 163

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^2}\,dx = -\frac{i(a+b\arctan(c+dx))^3}{de^2} - \frac{(a+b\arctan(c+dx))^3}{de^2(c+dx)} + \frac{3b(a+b\arctan(c+dx))^2\log\left(2-\frac{2}{1-i(c+dx)}\right)}{de^2} - \frac{3ib^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{de^2} + \frac{3b^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-i(c+dx)}\right)}{2de^2}$$

output
```
-I*(a+b*arctan(d*x+c))^3/d/e^2-(a+b*arctan(d*x+c))^3/d/e^2/(d*x+c)+3*b*(a+
b*arctan(d*x+c))^2*ln(2-2/(1-I*(d*x+c)))/d/e^2-3*I*b^2*(a+b*arctan(d*x+c))
*polylog(2,-1+2/(1-I*(d*x+c)))/d/e^2+3/2*b^3*polylog(3,-1+2/(1-I*(d*x+c)))
/d/e^2
```

3.18.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.61

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^2}\,dx$$
$$= \frac{-\frac{2a^3}{c+dx} - \frac{6a^2b\arctan(c+dx)}{c+dx} + 6a^2b\log(c+dx) - 3a^2b\log\left(1+c^2+2cdx+d^2x^2\right) + 6ab^2(\arctan(c+dx)\left((-i\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^2,x]
```

output
```
((-2*a^3)/(c + d*x) - (6*a^2*b*ArcTan[c + d*x])/(c + d*x) + 6*a^2*b*Log[c
+ d*x] - 3*a^2*b*Log[1 + c^2 + 2*c*d*x + d^2*x^2] + 6*a*b^2*(ArcTan[c + d*
x]*((-I - (c + d*x)^(-1))*ArcTan[c + d*x] + 2*Log[1 - E^((2*I)*ArcTan[c +
d*x])]) - I*PolyLog[2, E^((2*I)*ArcTan[c + d*x])]) + 2*b^3*((-1/8*I)*Pi^3
+ I*ArcTan[c + d*x]^3 - ArcTan[c + d*x]^3/(c + d*x) + 3*ArcTan[c + d*x]^2*
Log[1 - E^((-2*I)*ArcTan[c + d*x])] + (3*I)*ArcTan[c + d*x]*PolyLog[2, E^(
(-2*I)*ArcTan[c + d*x])] + (3*PolyLog[3, E^((-2*I)*ArcTan[c + d*x])])/2))/
(2*d*e^2)
```

3.18.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5566, 27, 5361, 5459, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^2}\,dx$$

$$\Big\downarrow \; 5566$$

$$\frac{\int \frac{(a+b\arctan(c+dx))^3}{e^2(c+dx)^2}d(c+dx)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(a+b\arctan(c+dx))^3}{(c+dx)^2}d(c+dx)}{de^2}$$

$$\Big\downarrow \text{5361}$$

$$\frac{3b\int \frac{(a+b\arctan(c+dx))^2}{(c+dx)((c+dx)^2+1)}d(c+dx)-\frac{(a+b\arctan(c+dx))^3}{c+dx}}{de^2}$$

$$\Big\downarrow \text{5459}$$

$$\frac{-\frac{(a+b\arctan(c+dx))^3}{c+dx}+3b\left(i\int \frac{(a+b\arctan(c+dx))^2}{(c+dx)(c+dx+i)}d(c+dx)-\frac{i(a+b\arctan(c+dx))^3}{3b}\right)}{de^2}$$

$$\Big\downarrow \text{5403}$$

$$\frac{-\frac{(a+b\arctan(c+dx))^3}{c+dx}+3b\left(i\left(2ib\int \frac{(a+b\arctan(c+dx))\log\left(2-\frac{2}{1-i(c+dx)}\right)}{(c+dx)^2+1}d(c+dx)-i\log\left(2-\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c\right.\right.}{de^2}$$

$$\Big\downarrow \text{5527}$$

$$\frac{-\frac{(a+b\arctan(c+dx))^3}{c+dx}+3b\left(i\left(2ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{2}{1-i(c+dx)}-1\right)(a+b\arctan(c+dx))-\frac{1}{2}ib\int \frac{\text{PolyLog}\left(2,\frac{2}{1-i(c+dx)}-1\right.}{(c+dx)^2+1}\right.\right.\right.}{de^2}$$

$$\Big\downarrow \text{7164}$$

$$\frac{-\frac{(a+b\arctan(c+dx))^3}{c+dx}+3b\left(i\left(2ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{2}{1-i(c+dx)}-1\right)(a+b\arctan(c+dx))-\frac{1}{4}b\,\text{PolyLog}\left(3,\frac{2}{1-i(c+dx)}-\right.\right.\right.\right.}{de^2}$$

input
```
Int[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^2,x]
```

output
```
(-((a + b*ArcTan[c + d*x])^3/(c + d*x)) + 3*b*(((-1/3*I)*(a + b*ArcTan[c +
 d*x])^3)/b + I*((-I)*(a + b*ArcTan[c + d*x])^2*Log[2 - 2/(1 - I*(c + d*x)
)] + (2*I)*b*((I/2)*(a + b*ArcTan[c + d*x])*PolyLog[2, -1 + 2/(1 - I*(c +
d*x))] - (b*PolyLog[3, -1 + 2/(1 - I*(c + d*x))])/4))))/(d*e^2)
```

3.18. $\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^2}\,dx$

3.18.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 5566
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.18. $\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^2}\,dx$

3.18.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.33 (sec) , antiderivative size = 2104, normalized size of antiderivative = 12.91

method	result	size
derivativedivides	Expression too large to display	2104
default	Expression too large to display	2104
parts	Expression too large to display	2112

input
```
int((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^3/e^2/(d*x+c)+b^3/e^2*(-1/(d*x+c)*arctan(d*x+c)^3+3*ln(d*x+c)*arct
an(d*x+c)^2-3/2*arctan(d*x+c)^2*ln(1+(d*x+c)^2)+3*arctan(d*x+c)^2*ln((1+I*
(d*x+c))/(1+(d*x+c)^2)^(1/2))-3*arctan(d*x+c)^2*ln((1+I*(d*x+c))^2/(1+(d*x
+c)^2)-1)-I*arctan(d*x+c)^3+3/4*(2*I*Pi*csgn(I*(1+I*(d*x+c))/(1+(d*x+c)^2)
^(1/2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2-2*I*Pi*csgn(((1+I*(d*x+c))
^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2+2*I*Pi+I*Pi*csgn(
I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(
1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^2+2*I*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+
c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^3-I*Pi*csgn(I*(1+I*(d*x+c))/(1
+(d*x+c)^2)^(1/2))^2*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))-I*Pi*csgn(I/(1+
(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*cs
gn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)-I*
Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^3-I*Pi*csgn(I*(1+I*(d*x+c))^2/(1+
(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^3+2*I*Pi*csgn(I*((1+I*(d*x
+c))^2/(1+(d*x+c)^2)-1))*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I*
((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))-2*I*P
i*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I*((1+I*(d*x+c))^2/(1+(d*
x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2-2*I*Pi*csgn(I*((1+I*(d*x+c
))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(((1+I*(d*x+c
))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2+I*Pi*csgn(I*...
```

3.18.5 Fricas [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^2}\,dx=\int \frac{\left(b\arctan\left(dx+c\right)+a\right)^3}{\left(dex+ce\right)^2}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arct
an(d*x + c) + a^3)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

3.18.6 Sympy [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^2}\,dx = \frac{\int \frac{a^3}{c^2+2cdx+d^2x^2}\,dx+\int \frac{b^3\operatorname{atan}^3\left(c+dx\right)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{3ab^2\operatorname{atan}^2\left(c+dx\right)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{3a^2b\operatorname{atan}\left(c+dx\right)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

input
```
integrate((a+b*atan(d*x+c))**3/(d*e*x+c*e)**2,x)
```

output
```
(Integral(a**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**3*atan(c + d
*x)**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a*b**2*atan(c + d*x)*
*2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a**2*b*atan(c + d*x)/(c**
2 + 2*c*d*x + d**2*x**2), x))/e**2
```

3.18.7 Maxima [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^2}\,dx=\int \frac{\left(b\arctan\left(dx+c\right)+a\right)^3}{\left(dex+ce\right)^2}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="maxima")
```

output
```
-3/2*(d*(log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2) - 2*log(d*x + c)/(d^2*
e^2)) + 2*arctan(d*x + c)/(d^2*e^2*x + c*d*e^2))*a^2*b - a^3/(d^2*e^2*x +
c*d*e^2) - 1/32*(4*b^3*arctan(d*x + c)^3 - 3*b^3*arctan(d*x + c)*log(d^2*x
^2 + 2*c*d*x + c^2 + 1)^2 - 32*(d^2*e^2*x + c*d*e^2)*integrate(1/32*(28*(b
^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2 + b^3)*arctan(d*x + c)^3 + 12*(8*a*b^2*
d^2*x^2 + 8*a*b^2*c^2 + b^3*c + 8*a*b^2 + (16*a*b^2*c + b^3)*d*x)*arctan(d
*x + c)^2 - 12*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*arctan(d*x + c)*log(d
^2*x^2 + 2*c*d*x + c^2 + 1) - 3*(b^3*d*x + b^3*c - (b^3*d^2*x^2 + 2*b^3*c*
d*x + b^3*c^2 + b^3)*arctan(d*x + c))*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2)/
(d^4*e^2*x^4 + 4*c*d^3*e^2*x^3 + (6*c^2 + 1)*d^2*e^2*x^2 + 2*(2*c^3 + c)*d
*e^2*x + (c^4 + c^2)*e^2), x))/(d^2*e^2*x + c*d*e^2)
```

3.18.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c+d\,x))^3}{(c\,e+d\,e\,x)^2}\,dx$$

input
```
int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^2,x)
```

output
```
int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^2, x)
```

3.19 $\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^3}\,dx$

3.19.1 Optimal result

Integrand size = 23, antiderivative size = 180

$$\begin{aligned}\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^3}\,dx = & -\frac{3ib(a+b\arctan(c+dx))^2}{2de^3}-\frac{3b(a+b\arctan(c+dx))^2}{2de^3(c+dx)} \\ & -\frac{(a+b\arctan(c+dx))^3}{2de^3}-\frac{(a+b\arctan(c+dx))^3}{2de^3(c+dx)^2} \\ & +\frac{3b^2(a+b\arctan(c+dx))\log\left(2-\frac{2}{1-i(c+dx)}\right)}{de^3} \\ & -\frac{3ib^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{2de^3}\end{aligned}$$

output
```
-3/2*I*b*(a+b*arctan(d*x+c))^2/d/e^3-3/2*b*(a+b*arctan(d*x+c))^2/d/e^3/(d*
x+c)-1/2*(a+b*arctan(d*x+c))^3/d/e^3-1/2*(a+b*arctan(d*x+c))^3/d/e^3/(d*x+
c)^2+3*b^2*(a+b*arctan(d*x+c))*ln(2-2/(1-I*(d*x+c)))/d/e^3-3/2*I*b^3*polyl
og(2,-1+2/(1-I*(d*x+c)))/d/e^3
```

3.19.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.25

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^3}\,dx =$$
$$-\frac{a^3+b^3(1+c^2+2cdx+d^2x^2)\arctan(c+dx)^3+3a^2b(c+dx+(1+(c+dx)^2)\arctan(c+dx))+3ab^2}{}$$

input
```
Integrate[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^3,x]
```

output
```
-1/2*(a^3 + b^3*(1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTan[c + d*x]^3 + 3*a^2*b*
(c + d*x + (1 + (c + d*x)^2)*ArcTan[c + d*x]) + 3*a*b^2*(2*(c + d*x)*ArcTa
n[c + d*x] + (1 + (c + d*x)^2)*ArcTan[c + d*x]^2 - 2*(c + d*x)^2*Log[(c +
d*x)/Sqrt[1 + (c + d*x)^2]]) + 3*b^3*(c + d*x)*(ArcTan[c + d*x]^2 - 2*(c +
 d*x)*ArcTan[c + d*x]*Log[1 - E^((2*I)*ArcTan[c + d*x])] + I*(c + d*x)*(Ar
cTan[c + d*x]^2 + PolyLog[2, E^((2*I)*ArcTan[c + d*x])])))/(d*e^3*(c + d*x
)^2)
```

3.19.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5566, 27, 5361, 5453, 5361, 5419, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^3}dx$$
$$\Big\downarrow \text{5566}$$
$$\frac{\int \frac{(a+b\arctan(c+dx))^3}{e^3(c+dx)^3}d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{(a+b\arctan(c+dx))^3}{(c+dx)^3}d(c+dx)}{de^3}$$
$$\Big\downarrow \text{5361}$$

$$\frac{\frac{3}{2}b\int \frac{(a+b\arctan(c+dx))^2}{(c+dx)^2((c+dx)^2+1)}d(c+dx)-\frac{(a+b\arctan(c+dx))^3}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\frac{3}{2}b\left(\int \frac{(a+b\arctan(c+dx))^2}{(c+dx)^2}d(c+dx)-\int \frac{(a+b\arctan(c+dx))^2}{(c+dx)^2+1}d(c+dx)\right)-\frac{(a+b\arctan(c+dx))^3}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{3}{2}b\left(2b\int \frac{a+b\arctan(c+dx)}{(c+dx)((c+dx)^2+1)}d(c+dx)-\int \frac{(a+b\arctan(c+dx))^2}{(c+dx)^2+1}d(c+dx)-\frac{(a+b\arctan(c+dx))^2}{c+dx}\right)-\frac{(a+b\arctan(c+dx))^3}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\frac{3}{2}b\left(2b\int \frac{a+b\arctan(c+dx)}{(c+dx)((c+dx)^2+1)}d(c+dx)-\frac{(a+b\arctan(c+dx))^3}{3b}-\frac{(a+b\arctan(c+dx))^2}{c+dx}\right)-\frac{(a+b\arctan(c+dx))^3}{2(c+dx)^2}}{de^3}$$

$$\Big\downarrow \text{5459}$$

$$\frac{-\frac{(a+b\arctan(c+dx))^3}{2(c+dx)^2}+\frac{3}{2}b\left(2b\left(i\int \frac{a+b\arctan(c+dx)}{(c+dx)(c+dx+i)}d(c+dx)-\frac{i(a+b\arctan(c+dx))^2}{2b}\right)-\frac{(a+b\arctan(c+dx))^3}{3b}-\frac{(a+b\arctan(c+}{c+dx}}{de^3}$$

$$\Big\downarrow \text{5403}$$

$$\frac{-\frac{(a+b\arctan(c+dx))^3}{2(c+dx)^2}+\frac{3}{2}b\left(2b\left(i\left(ib\int \frac{\log\left(2-\frac{2}{1-i(c+dx)}\right)}{(c+dx)^2+1}d(c+dx)-i\log\left(2-\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))\right)-\frac{i}{}\right.\right.}{de^3}$$

$$\Big\downarrow \text{2897}$$

$$\frac{-\frac{(a+b\arctan(c+dx))^3}{2(c+dx)^2}+\frac{3}{2}b\left(2b\left(i\left(-i\log\left(2-\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-i(c+dx)}-1\right)\right)-\right.\right.}{de^3}$$

input
```
Int[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^3,x]
```

output
```
(-1/2*(a + b*ArcTan[c + d*x])^3/(c + d*x)^2 + (3*b*(-((a + b*ArcTan[c + d*
x])^2/(c + d*x)) - (a + b*ArcTan[c + d*x])^3/(3*b) + 2*b*(((-1/2*I)*(a + b
*ArcTan[c + d*x])^2)/b + I*((-I)*(a + b*ArcTan[c + d*x])*Log[2 - 2/(1 - I*
(c + d*x))] - (b*PolyLog[2, -1 + 2/(1 - I*(c + d*x))])/2))))/2)/(d*e^3)
```

3.19. $\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^3}\,dx$

3.19.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5566
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.19.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(166) = 332$.

Time = 1.42 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.35

method	result
derivativedivides	$-\frac{a^3}{2e^3(dx+c)^2}+\frac{b^3\left(-\frac{\arctan(dx+c)^3}{2(dx+c)^2}-\frac{3\arctan(dx+c)^2}{2(dx+c)}-\frac{\arctan(dx+c)^3}{2}+3\ln(dx+c)\arctan(dx+c)-\frac{3\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{2}\right.}{}$
default	$-\frac{a^3}{2e^3(dx+c)^2}+\frac{b^3\left(-\frac{\arctan(dx+c)^3}{2(dx+c)^2}-\frac{3\arctan(dx+c)^2}{2(dx+c)}-\frac{\arctan(dx+c)^3}{2}+3\ln(dx+c)\arctan(dx+c)-\frac{3\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{2}\right.}{}$
parts	$-\frac{a^3}{2e^3(dx+c)^2d}+\frac{b^3\left(-\frac{\arctan(dx+c)^3}{2(dx+c)^2}-\frac{3\arctan(dx+c)^2}{2(dx+c)}-\frac{\arctan(dx+c)^3}{2}+3\ln(dx+c)\arctan(dx+c)-\frac{3\arctan(dx+c)\ln(1}{2}\right.}{}$

input
```
int((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*a^3/e^3/(d*x+c)^2+b^3/e^3*(-1/2/(d*x+c)^2*arctan(d*x+c)^3-3/2/(d
*x+c)*arctan(d*x+c)^2-1/2*arctan(d*x+c)^3+3*ln(d*x+c)*arctan(d*x+c)-3/2*ar
ctan(d*x+c)*ln(1+(d*x+c)^2)-3/4*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+
c-I)^2-dilog(-1/2*I*(d*x+c+I))-ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))+3/4*I*(ln
(d*x+c+I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+
c+I)*ln(1/2*I*(d*x+c-I)))+3/2*I*ln(d*x+c)*ln(1+I*(d*x+c))-3/2*I*ln(d*x+c)*
ln(1-I*(d*x+c))+3/2*I*dilog(1+I*(d*x+c))-3/2*I*dilog(1-I*(d*x+c)))+3*a*b^2
/e^3*(-1/2/(d*x+c)^2*arctan(d*x+c)^2-1/(d*x+c)*arctan(d*x+c)-1/2*arctan(d*
x+c)^2+ln(d*x+c)-1/2*ln(1+(d*x+c)^2))+3*a^2*b/e^3*(-1/2/(d*x+c)^2*arctan(d
*x+c)-1/2/(d*x+c)-1/2*arctan(d*x+c)))
```

3.19.5 Fricas [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^3}\,dx = \int \frac{(b\arctan(dx+c)+a)^3}{(dex+ce)^3}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arct
an(d*x + c) + a^3)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^
3), x)
```

3.19.6 Sympy [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^3}\,dx = \frac{\int \frac{a^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b^3\operatorname{atan}^3(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3ab^2\operatorname{atan}^2(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3a^2b\operatorname{atan}(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

input
```
integrate((a+b*atan(d*x+c))**3/(d*e*x+c*e)**3,x)
```

output
```
(Integral(a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integ
ral(b**3*atan(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
 x) + Integral(3*a*b**2*atan(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**
2 + d**3*x**3), x) + Integral(3*a**2*b*atan(c + d*x)/(c**3 + 3*c**2*d*x +
3*c*d**2*x**2 + d**3*x**3), x))/e**3
```

3.19.7 Maxima [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^3}\,dx = \int \frac{(b\arctan(dx+c)+a)^3}{(dex+ce)^3}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="maxima")
```

output
```
-3/2*(d*(1/(d^3*e^3*x + c*d^2*e^3) + arctan((d^2*x + c*d)/d)/(d^2*e^3)) +
arctan(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3))*a^2*b - 3/2*(2*
d*(1/(d^3*e^3*x + c*d^2*e^3) + arctan((d^2*x + c*d)/d)/(d^2*e^3))*arctan(d
*x + c) - (arctan(d*x + c)^2 - log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*log(d*
x + c))/(d*e^3))*a*b^2 - 3/2*a*b^2*arctan(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^
2*e^3*x + c^2*d*e^3) - 1/32*(8*(d^2*x^2 + 2*c*d*x + c^2 + 1)*arctan(d*x +
c)^3 + 12*(d*x + c)*arctan(d*x + c)^2 - 3*(d*x + c)*log(d^2*x^2 + 2*c*d*x
+ c^2 + 1)^2 - 32*(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)*integrate(1/32
*(16*(d^2*x^2 + 2*c*d*x + c^2 + 1)*arctan(d*x + c)^3 + 12*(d^3*x^3 + 3*c*d
^2*x^2 + c^3 + (3*c^2 + 1)*d*x + c)*arctan(d*x + c)^2 + 3*(d^3*x^3 + 3*c*d
^2*x^2 + c^3 + (3*c^2 + 1)*d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 2
4*(d^2*x^2 + 2*c*d*x + c^2)*arctan(d*x + c) - 12*(d^3*x^3 + 3*c*d^2*x^2 +
3*c^2*d*x + c^3)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^5*e^3*x^5 + 5*c*d^4*
e^3*x^4 + (10*c^2 + 1)*d^3*e^3*x^3 + (10*c^3 + 3*c)*d^2*e^3*x^2 + (5*c^4 +
 3*c^2)*d*e^3*x + (c^5 + c^3)*e^3), x))*b^3/(d^3*e^3*x^2 + 2*c*d^2*e^3*x +
 c^2*d*e^3) - 1/2*a^3/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

3.19.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="giac")
```

output
```
Timed out
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c+d\,x))^3}{(c\,e+d\,e\,x)^3}\,dx$$

input
```
int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^3,x)
```

output
```
int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^3, x)
```

3.20 $\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^4}\,dx$

3.20.1 Optimal result

Integrand size = 23, antiderivative size = 287

$$\begin{aligned}\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^4}\,dx = &-\frac{b^2(a+b\arctan(c+dx))}{de^4(c+dx)}-\frac{b(a+b\arctan(c+dx))^2}{2de^4}\\&-\frac{b(a+b\arctan(c+dx))^2}{2de^4(c+dx)^2}+\frac{i(a+b\arctan(c+dx))^3}{3de^4}\\&-\frac{(a+b\arctan(c+dx))^3}{3de^4(c+dx)^3}\\&+\frac{b^3\log(c+dx)}{de^4}-\frac{b^3\log\left(1+(c+dx)^2\right)}{2de^4}\\&-\frac{b(a+b\arctan(c+dx))^2\log\left(2-\frac{2}{1-i(c+dx)}\right)}{de^4}\\&+\frac{ib^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-i(c+dx)}\right)}{de^4}\\&-\frac{b^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-i(c+dx)}\right)}{2de^4}\end{aligned}$$

output
```
-b^2*(a+b*arctan(d*x+c))/d/e^4/(d*x+c)-1/2*b*(a+b*arctan(d*x+c))^2/d/e^4-1
/2*b*(a+b*arctan(d*x+c))^2/d/e^4/(d*x+c)^2+1/3*I*(a+b*arctan(d*x+c))^3/d/e
^4-1/3*(a+b*arctan(d*x+c))^3/d/e^4/(d*x+c)^3+b^3*ln(d*x+c)/d/e^4-1/2*b^3*l
n(1+(d*x+c)^2)/d/e^4-b*(a+b*arctan(d*x+c))^2*ln(2-2/(1-I*(d*x+c)))/d/e^4+I
*b^2*(a+b*arctan(d*x+c))*polylog(2,-1+2/(1-I*(d*x+c)))/d/e^4-1/2*b^3*polyl
og(3,-1+2/(1-I*(d*x+c)))/d/e^4
```

3.20.2 Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.29

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^4}\,dx$$
$$= \frac{-\frac{2a^3}{(c+dx)^3}-\frac{3a^2b}{(c+dx)^2}-\frac{6a^2b\arctan(c+dx)}{(c+dx)^3}-6a^2b\log(c+dx)+3a^2b\log\left(1+c^2+2cdx+d^2x^2\right)+6ab^2\left(-\frac{(c+dx)^2}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^4,x]
```

output
```
((-2*a^3)/(c + d*x)^3 - (3*a^2*b)/(c + d*x)^2 - (6*a^2*b*ArcTan[c + d*x])/
(c + d*x)^3 - 6*a^2*b*Log[c + d*x] + 3*a^2*b*Log[1 + c^2 + 2*c*d*x + d^2*x
^2] + 6*a*b^2*(-(((c + d*x)^2 + ArcTan[c + d*x]^2)/(c + d*x)^3) + ArcTan[c
 + d*x]*(-1 - (c + d*x)^(-2) + I*ArcTan[c + d*x] - 2*Log[1 - E^((2*I)*ArcT
an[c + d*x])]) + I*PolyLog[2, E^((2*I)*ArcTan[c + d*x])]) + 6*b^3*((I/24)*
Pi^3 - ArcTan[c + d*x]/(c + d*x) - ArcTan[c + d*x]^2/2 - ArcTan[c + d*x]^2
/(2*(c + d*x)^2) - (I/3)*ArcTan[c + d*x]^3 - ArcTan[c + d*x]^3/(3*(c + d*x
)^3) - ArcTan[c + d*x]^2*Log[1 - E^((-2*I)*ArcTan[c + d*x])] + Log[c + d*x
] + Log[1/Sqrt[1 + (c + d*x)^2]] - I*ArcTan[c + d*x]*PolyLog[2, E^((-2*I)*
ArcTan[c + d*x])] - PolyLog[3, E^((-2*I)*ArcTan[c + d*x])]/2))/(6*d*e^4)
```

3.20.3 Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.87, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.696, Rules used = {5566, 27, 5361, 5453, 5361, 5453, 5361, 243, 47, 14, 16, 5419, 5459, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^4}\,dx$$
$$\Big\downarrow \; 5566$$
$$\frac{\int \frac{(a+b\arctan(c+dx))^3}{e^4(c+dx)^4}d(c+dx)}{d}$$
$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(a+b\arctan(c+dx))^3}{(c+dx)^4}d(c+dx)}{de^4}$$

$$\Big\downarrow \; 5361$$

$$\frac{b\int \frac{(a+b\arctan(c+dx))^2}{(c+dx)^3((c+dx)^2+1)}d(c+dx)-\frac{(a+b\arctan(c+dx))^3}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \; 5453$$

$$\frac{b\left(\int \frac{(a+b\arctan(c+dx))^2}{(c+dx)^3}d(c+dx)-\int \frac{(a+b\arctan(c+dx))^2}{(c+dx)((c+dx)^2+1)}d(c+dx)\right)-\frac{(a+b\arctan(c+dx))^3}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \; 5361$$

$$\frac{b\left(b\int \frac{a+b\arctan(c+dx)}{(c+dx)^2((c+dx)^2+1)}d(c+dx)-\int \frac{(a+b\arctan(c+dx))^2}{(c+dx)((c+dx)^2+1)}d(c+dx)-\frac{(a+b\arctan(c+dx))^2}{2(c+dx)^2}\right)-\frac{(a+b\arctan(c+dx))^3}{3(c+dx)^3}}{de^4}$$

$$\Big\downarrow \; 5453$$

$$\frac{b\Bigg(b\Big(\int \frac{a+b\arctan(c+dx)}{(c+dx)^2}d(c+dx)-\int \frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)\Big)-\int \frac{(a+b\arctan(c+dx))^2}{(c+dx)((c+dx)^2+1)}d(c+dx)-\frac{(a+b\arctan(c+dx))^2}{2(c+dx)^2}\Bigg)\cdots}{de^4}$$

$$\Big\downarrow \; 5361$$

$$\frac{b\Bigg(b\Big(-\int \frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)+b\int \frac{1}{(c+dx)((c+dx)^2+1)}d(c+dx)-\frac{a+b\arctan(c+dx)}{c+dx}\Big)-\int \frac{(a+b\arctan(c+dx))^2}{(c+dx)((c+dx)^2+1)}d(c+d\cdots}{de^4}$$

$$\Big\downarrow \; 243$$

$$\frac{b\Bigg(b\Big(-\int \frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)+\frac{1}{2}b\int \frac{1}{(c+dx)^2((c+dx)^2+1)}d(c+dx)^2-\frac{a+b\arctan(c+dx)}{c+dx}\Big)-\int \frac{(a+b\arctan(c+dx))^2}{(c+dx)((c+dx)^2+1)}d(c\cdots}{de^4}$$

$$\Big\downarrow \; 47$$

$$\frac{b\Bigg(b\Big(-\int \frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)+\frac{1}{2}b\Big(\int \frac{1}{(c+dx)^2}d(c+dx)^2-\int \frac{1}{(c+dx)^2+1}d(c+dx)^2\Big)-\frac{a+b\arctan(c+dx)}{c+dx}\Big)-\int \frac{(a+b\cdots}{(c+\cdots}}{de^4}$$

$$\Big\downarrow \; 14$$

$$\frac{b\Bigg(-\int \frac{(a+b\arctan(c+dx))^2}{(c+dx)((c+dx)^2+1)}d(c+dx)+b\Big(-\int \frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)+\frac{1}{2}b\Big(\log\left((c+dx)^2\right)-\int \frac{1}{(c+dx)^2+1}d(c+dx)^2\cdots}{de^4}$$

$$\Big\downarrow \; 16$$

3.20. $\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^4}\,dx$

$$\frac{b\left(-\int \frac{(a+b\arctan(c+dx))^2}{(c+dx)((c+dx)^2+1)}d(c+dx)+b\left(-\int \frac{a+b\arctan(c+dx)}{(c+dx)^2+1}d(c+dx)-\frac{a+b\arctan(c+dx)}{c+dx}+\frac{1}{2}b(\log\left((c+dx)^2\right)-\log\left(\right.\right.\right.}{de^4}$$

$\downarrow$ 5419

$$\frac{b\left(-\int \frac{(a+b\arctan(c+dx))^2}{(c+dx)((c+dx)^2+1)}d(c+dx)-\frac{(a+b\arctan(c+dx))^2}{2(c+dx)^2}+b\left(-\frac{(a+b\arctan(c+dx))^2}{2b}-\frac{a+b\arctan(c+dx)}{c+dx}+\frac{1}{2}b(\log\left((c+dx\right.\right.\right.}{de^4}$$

$\downarrow$ 5459

$$\frac{-\frac{(a+b\arctan(c+dx))^3}{3(c+dx)^3}+b\left(-i\int \frac{(a+b\arctan(c+dx))^2}{(c+dx)(c+dx+i)}d(c+dx)+\frac{i(a+b\arctan(c+dx))^3}{3b}-\frac{(a+b\arctan(c+dx))^2}{2(c+dx)^2}+b\left(-\frac{(a+b\arctan(\right.\right.}{2b}}{de^4}$$

$\downarrow$ 5403

$$-\frac{(a+b\arctan(c+dx))^3}{3(c+dx)^3}+b\left(-i\left(2ib\int \frac{(a+b\arctan(c+dx))\log\left(2-\frac{2}{1-i(c+dx)}\right)}{(c+dx)^2+1}d(c+dx)-i\log\left(2-\frac{2}{1-i(c+dx)}\right)(a+b\arctan(\right.\right.$$

$\downarrow$ 5527

$$-\frac{(a+b\arctan(c+dx))^3}{3(c+dx)^3}+b\left(-i\left(2ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{2}{1-i(c+dx)}-1\right)(a+b\arctan(c+dx))-\frac{1}{2}ib\int \frac{\text{PolyLog}\left(2,\frac{2}{1-i(c+dx)}-\right.}{(c+dx)^2+1}\right.\right.\right.$$

$\downarrow$ 7164

$$-\frac{(a+b\arctan(c+dx))^3}{3(c+dx)^3}+b\left(-i\left(2ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{2}{1-i(c+dx)}-1\right)(a+b\arctan(c+dx))-\frac{1}{4}b\,\text{PolyLog}\left(3,\frac{2}{1-i(c+dx)}-\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^4,x]
```

output
```
(-1/3*(a + b*ArcTan[c + d*x])^3/(c + d*x)^3 + b*(-1/2*(a + b*ArcTan[c + d*
x])^2/(c + d*x)^2 + ((I/3)*(a + b*ArcTan[c + d*x])^3)/b + b*(-((a + b*ArcT
an[c + d*x])/(c + d*x)) - (a + b*ArcTan[c + d*x])^2/(2*b) + (b*(Log[(c + d
*x)^2] - Log[1 + (c + d*x)^2]))/2) - I*((-I)*(a + b*ArcTan[c + d*x])^2*Log
[2 - 2/(1 - I*(c + d*x))] + (2*I)*b*((I/2)*(a + b*ArcTan[c + d*x])*PolyLog
[2, -1 + 2/(1 - I*(c + d*x))] - (b*PolyLog[3, -1 + 2/(1 - I*(c + d*x))])/4
))))/(d*e^4)
```

3.20. $\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^4}\,dx$

3.20.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 5566
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.20.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.42 (sec) , antiderivative size = 2465, normalized size of antiderivative = 8.59

method	result	size
derivativedivides	Expression too large to display	2465
default	Expression too large to display	2465
parts	Expression too large to display	2473

input
```
int((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*a^3/e^4/(d*x+c)^3+b^3/e^4*(-1/3/(d*x+c)^3*arctan(d*x+c)^3-1/2/(d
*x+c)^2*arctan(d*x+c)^2-ln(d*x+c)*arctan(d*x+c)^2+1/2*arctan(d*x+c)^2*ln(1
+(d*x+c)^2)-arctan(d*x+c)^2*ln((1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+arctan(d
*x+c)^2*ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)-arctan(d*x+c)^2*ln(1+(1+I*(d*x
+c))/(1+(d*x+c)^2)^(1/2))+2*I*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))/(1+(d
*x+c)^2)^(1/2))-2*polylog(3,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-arctan(d*x
+c)^2*ln(1-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+2*I*arctan(d*x+c)*polylog(2,
(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-2*polylog(3,(1+I*(d*x+c))/(1+(d*x+c)^2)
^(1/2))+1/12*arctan(d*x+c)*(6*I*Pi*arctan(d*x+c)*csgn(I*(1+(1+I*(d*x+c))^2
/(1+(d*x+c)^2))^2)^2*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*(d*x+c)+4*I
*arctan(d*x+c)^2*(d*x+c)-3*I*Pi*arctan(d*x+c)*csgn(I*(1+I*(d*x+c))^2/(1+(d
*x+c)^2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+
c)^2))^2)^2*(d*x+c)-3*I*Pi*arctan(d*x+c)*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x
+c)^2))^2)*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2*(d*x+c)-6*I*Pi*arct
an(d*x+c)*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(
d*x+c)^2)))*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(
d*x+c)^2)))*(d*x+c)-6*I*Pi*arctan(d*x+c)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)
^2))^2*csgn(I*(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))*(d*x+c)-6*I*Pi*arctan(d*x
+c)*(d*x+c)+3*I*Pi*arctan(d*x+c)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*csg
n(I*(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))^2*(d*x+c)+3*I*Pi*arctan(d*x+c)*c...
```

3.20.5 Fricas [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^4}\,dx = \int \frac{(b\arctan(dx+c)+a)^3}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arct
an(d*x + c) + a^3)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*
c^3*d*e^4*x + c^4*e^4), x)
```

3.20.6 Sympy [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^4}\,dx = \frac{\int \frac{a^3}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^3\operatorname{atan}^3(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{3ab^2\operatorname{atan}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{}{c}}{e^4}$$

input
```
integrate((a+b*atan(d*x+c))**3/(d*e*x+c*e)**4,x)
```

output
```
(Integral(a**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**
4*x**4), x) + Integral(b**3*atan(c + d*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*d
**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(3*a*b**2*atan(c + d*x
)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x
) + Integral(3*a**2*b*atan(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2
+ 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

3.20.7 Maxima [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^4}\,dx = \int \frac{(b\arctan(dx+c)+a)^3}{(dex+ce)^4}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="maxima")
```

output
```
-1/2*(d*(1/(d^4*e^4*x^2 + 2*c*d^3*e^4*x + c^2*d^2*e^4) - log(d^2*x^2 + 2*c
*d*x + c^2 + 1)/(d^2*e^4) + 2*log(d*x + c)/(d^2*e^4)) + 2*arctan(d*x + c)/
(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4))*a^2*b - 1/3
*a^3/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/96*
(4*b^3*arctan(d*x + c)^3 - 3*b^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c
^2 + 1)^2 - 96*(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^
4)*integrate(1/32*(28*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2 + b^3)*arctan(d
*x + c)^3 + 4*(24*a*b^2*d^2*x^2 + 24*a*b^2*c^2 + b^3*c + 24*a*b^2 + (48*a*
b^2*c + b^3)*d*x)*arctan(d*x + c)^2 - 4*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c
^2)*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - (b^3*d*x + b^3*c -
3*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2 + b^3)*arctan(d*x + c))*log(d^2*x^2
 + 2*c*d*x + c^2 + 1)^2)/(d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + (15*c^2 + 1)*d^4
*e^4*x^4 + 4*(5*c^3 + c)*d^3*e^4*x^3 + 3*(5*c^4 + 2*c^2)*d^2*e^4*x^2 + 2*(
3*c^5 + 2*c^3)*d*e^4*x + (c^6 + c^4)*e^4), x))/(d^4*e^4*x^3 + 3*c*d^3*e^4*
x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4)
```

3.20.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="giac")
```

output
```
Timed out
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{(ce+dex)^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(c+d\,x))^3}{(c\,e+d\,e\,x)^4}\,dx$$

input
```
int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^4,x)
```

output
```
int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^4, x)
```

3.21 $\int \frac{\arctan(1+x)}{2+2x} dx$

3.21.1 Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{\arctan(1+x)}{2+2x} dx = \frac{1}{4} i \operatorname{PolyLog}(2,-i(1+x)) - \frac{1}{4} i \operatorname{PolyLog}(2,i(1+x))$$

output
```
1/4*I*polylog(2,-I*(1+x))-1/4*I*polylog(2,I*(1+x))
```

3.21.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(1+x)}{2+2x} dx = \frac{1}{4} i \operatorname{PolyLog}(2,-i(1+x)) - \frac{1}{4} i \operatorname{PolyLog}(2,i(1+x))$$

input
```
Integrate[ArcTan[1 + x]/(2 + 2*x),x]
```

output
```
(I/4)*PolyLog[2, (-I)*(1 + x)] - (I/4)*PolyLog[2, I*(1 + x)]
```

3.21.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5566, 27, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x+1)}{2x+2}\,dx$$

$$\Big\downarrow \text{ 5566}$$

$$\int \frac{\arctan(x+1)}{2(x+1)}d(x+1)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int \frac{\arctan(x+1)}{x+1}d(x+1)$$

$$\Big\downarrow \text{ 5355}$$

$$\frac{1}{2}\left(\frac{1}{2}i\int \frac{\log(1-i(x+1))}{x+1}d(x+1)-\frac{1}{2}i\int \frac{\log(i(x+1)+1)}{x+1}d(x+1)\right)$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{1}{2}\left(\frac{1}{2}i\operatorname{PolyLog}(2,-i(x+1))-\frac{1}{2}i\operatorname{PolyLog}(2,i(x+1))\right)$$

input
```
Int[ArcTan[1 + x]/(2 + 2*x),x]
```

output
```
((I/2)*PolyLog[2, (-I)*(1 + x)] - (I/2)*PolyLog[2, I*(1 + x)])/2
```

3.21.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5566
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.21.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{i \operatorname{dilog}(-ix-i+1)}{4}+\frac{i \operatorname{dilog}(ix+i+1)}{4}$
derivativedivides	$\frac{\ln(1+x)\arctan(1+x)}{2}+\frac{i\ln(1+x)\ln(1+i(1+x))}{4}-\frac{i\ln(1+x)\ln(1-i(1+x))}{4}+\frac{i \operatorname{dilog}(1+i(1+x))}{4}-\frac{i \operatorname{dilog}(1-i(1+$
default	$\frac{\ln(1+x)\arctan(1+x)}{2}+\frac{i\ln(1+x)\ln(1+i(1+x))}{4}-\frac{i\ln(1+x)\ln(1-i(1+x))}{4}+\frac{i \operatorname{dilog}(1+i(1+x))}{4}-\frac{i \operatorname{dilog}(1-i(1+$
parts	$\frac{\ln(1+x)\arctan(1+x)}{2}+\frac{i\ln(1+x)\ln(1+i(1+x))}{4}-\frac{i\ln(1+x)\ln(1-i(1+x))}{4}+\frac{i \operatorname{dilog}(1+i(1+x))}{4}-\frac{i \operatorname{dilog}(1-i(1+$

input
```
int(arctan(1+x)/(2+2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*dilog(-I*x+1-I)+1/4*I*dilog(I*x+1+I)
```

3.21.5 Fricas [F]

$$\int \frac{\arctan(1+x)}{2+2x}\,dx=\int \frac{\arctan(x+1)}{2(x+1)}\,dx$$

input
```
integrate(arctan(1+x)/(2+2*x),x, algorithm="fricas")
```

output
```
integral(1/2*arctan(x + 1)/(x + 1), x)
```

3.21.6 Sympy [F]

$$\int \frac{\arctan(1+x)}{2+2x}\,dx = \frac{\int \frac{\operatorname{atan}(x+1)}{x+1}\,dx}{2}$$

input
```
integrate(atan(1+x)/(2+2*x),x)
```

output
```
Integral(atan(x + 1)/(x + 1), x)/2
```

3.21.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(17) = 34$.

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{\arctan(1+x)}{2+2x}\,dx = -\frac{1}{4}\arctan(x+1,0)\log\left(x^2+2x+2\right) + \frac{1}{2}\arctan(x+1)\log(|x+1|) - \frac{1}{4}i\operatorname{Li}_2(ix+i+1) + \frac{1}{4}i\operatorname{Li}_2(-ix-i+1)$$

input
```
integrate(arctan(1+x)/(2+2*x),x, algorithm="maxima")
```

output
```
-1/4*arctan2(x + 1, 0)*log(x^2 + 2*x + 2) + 1/2*arctan(x + 1)*log(abs(x +
1)) - 1/4*I*dilog(I*x + I + 1) + 1/4*I*dilog(-I*x - I + 1)
```

3.21.8 Giac [F]

$$\int \frac{\arctan(1+x)}{2+2x}\,dx = \int \frac{\arctan(x+1)}{2(x+1)}\,dx$$

input
```
integrate(arctan(1+x)/(2+2*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.21.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{\arctan(1+x)}{2+2x}\,dx = -\frac{\mathrm{Li}_2(1 - x\,1\mathrm{i} - \mathrm{i})\ 1\mathrm{i}}{4} + \frac{\mathrm{Li}_2(x\,1\mathrm{i} + 1 + 1\mathrm{i})\ 1\mathrm{i}}{4}$$

input
```
int(atan(x + 1)/(2*x + 2),x)
```

output
```
(dilog(x*1i + (1 + 1i))*1i)/4 - (dilog((1 - 1i) - x*1i)*1i)/4
```

3.22 $\int \frac{\arctan(a+bx)}{\frac{ad}{b}+dx}\,dx$

3.22.1 Optimal result

Integrand size = 19, antiderivative size = 41

$$\int \frac{\arctan(a+bx)}{\frac{ad}{b}+dx}\,dx=\frac{i\,\text{PolyLog}(2,-i(a+bx))}{2d}-\frac{i\,\text{PolyLog}(2,i(a+bx))}{2d}$$

output
```
1/2*I*polylog(2,-I*(b*x+a))/d-1/2*I*polylog(2,I*(b*x+a))/d
```

3.22.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(a+bx)}{\frac{ad}{b}+dx}\,dx=\frac{i(\text{PolyLog}(2,-i(a+bx))-\text{PolyLog}(2,i(a+bx)))}{2d}$$

input
```
Integrate[ArcTan[a + b*x]/((a*d)/b + d*x),x]
```

output
```
((I/2)*(PolyLog[2, (-I)*(a + b*x)] - PolyLog[2, I*(a + b*x)]))/d
```

3.22.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5566, 27, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(a+bx)}{\frac{ad}{b}+dx} \, dx$$

$$\Big\downarrow \text{ 5566}$$

$$\frac{\int \frac{b\arctan(a+bx)}{d(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\arctan(a+bx)}{a+bx}d(a+bx)}{d}$$

$$\Big\downarrow \text{ 5355}$$

$$\frac{\frac{1}{2}i\int \frac{\log(1-i(a+bx))}{a+bx}d(a+bx)-\frac{1}{2}i\int \frac{\log(i(a+bx)+1)}{a+bx}d(a+bx)}{d}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\frac{1}{2}i\operatorname{PolyLog}(2,-i(a+bx))-\frac{1}{2}i\operatorname{PolyLog}(2,i(a+bx))}{d}$$

input
```
Int[ArcTan[a + b*x]/((a*d)/b + d*x),x]
```

output
```
((I/2)*PolyLog[2, (-I)*(a + b*x)] - (I/2)*PolyLog[2, I*(a + b*x)])/d
```

3.22.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5566
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
 x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
 IGtQ[p, 0]
```

3.22.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{i \operatorname{dilog}(-ibx-ia+1)}{2d}+\frac{i \operatorname{dilog}(ibx+ia+1)}{2d}$
parts	$\frac{\ln(bx+a)\arctan(bx+a)}{d}-\frac{-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\operatorname{dilog}(1+i(bx+a))}{2}+\frac{i\operatorname{dilog}(1-i(bx+a))}{2}}{d}$
derivativedivides	$\frac{\frac{b\ln(bx+a)\arctan(bx+a)}{d}-\frac{b\left(-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\operatorname{dilog}(1+i(bx+a))}{2}+\frac{i\operatorname{dilog}(1-i(bx+a))}{2}\right)}{d}}{b}$
default	$\frac{\frac{b\ln(bx+a)\arctan(bx+a)}{d}-\frac{b\left(-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\operatorname{dilog}(1+i(bx+a))}{2}+\frac{i\operatorname{dilog}(1-i(bx+a))}{2}\right)}{d}}{b}$

input
```
int(arctan(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/d*dilog(1-I*a-I*b*x)+1/2*I/d*dilog(1+I*a+I*b*x)
```

3.22.5 Fricas [F]

$$\int \frac{\arctan(a+bx)}{\frac{ad}{b}+dx}\,dx=\int \frac{\arctan(bx+a)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arctan(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

output
```
integral(b*arctan(b*x + a)/(b*d*x + a*d), x)
```

3.22.6 Sympy [F]

$$\int \frac{\arctan(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{b\int \frac{\operatorname{atan}(a+bx)}{a+bx}\,dx}{d}$$

input
```
integrate(atan(b*x+a)/(a*d/b+d*x),x)
```

output
```
b*Integral(atan(a + b*x)/(a + b*x), x)/d
```

3.22.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(29) = 58.

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 3.00

$$\int \frac{\arctan(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{\arctan(bx+a)\log\left(dx+\frac{ad}{b}\right)}{d} - \frac{\arctan\left(\frac{b^2x+ab}{b}\right)\log\left(dx+\frac{ad}{b}\right)}{d} - \frac{\arctan(bx+a,0)\log(b^2x^2+2abx+a^2+1) - 2\arctan(bx+a)\log(|bx+a|) + i\operatorname{Li}_2(ibx+ia+1) -}{2d}$$

input
```
integrate(arctan(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

output
```
arctan(b*x + a)*log(d*x + a*d/b)/d - arctan((b^2*x + a*b)/b)*log(d*x + a*d
/b)/d - 1/2*(arctan2(b*x + a, 0)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*arct
an(b*x + a)*log(abs(b*x + a)) + I*dilog(I*b*x + I*a + 1) - I*dilog(-I*b*x
- I*a + 1))/d
```

3.22.8 Giac [F]

$$\int \frac{\arctan(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\arctan(bx+a)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arctan(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\mathrm{atan}(a+b\,x)}{d\,x+\frac{a\,d}{b}}\,dx$$

input
```
int(atan(a + b*x)/(d*x + (a*d)/b),x)
```

output
```
int(atan(a + b*x)/(d*x + (a*d)/b), x)
```

3.23 $\int (a+bx)^2\sqrt{\arctan(a+bx)}\,dx$

3.23.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (a+bx)^2\sqrt{\arctan(a+bx)}\,dx = \text{Int}\left((a+bx)^2\sqrt{\arctan(a+bx)},x\right)$$

output
```
Unintegrable((b*x+a)^2*arctan(b*x+a)^(1/2),x)
```

3.23.2 Mathematica [N/A]

Not integrable

Time = 3.74 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (a+bx)^2\sqrt{\arctan(a+bx)}\,dx = \int (a+bx)^2\sqrt{\arctan(a+bx)}\,dx$$

input
```
Integrate[(a + b*x)^2*Sqrt[ArcTan[a + b*x]],x]
```

output
```
Integrate[(a + b*x)^2*Sqrt[ArcTan[a + b*x]], x]
```

3.23.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5572}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2\sqrt{\arctan(a+bx)}\,dx$$

$$\Big\downarrow \; 5572$$

$$\int (a+bx)^2\sqrt{\arctan(a+bx)}dx$$

input
```
Int[(a + b*x)^2*Sqrt[ArcTan[a + b*x]],x]
```

output
```
$Aborted
```

3.23.3.1 Defintions of rubi rules used

rule 5572
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_)*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Unintegrable[(e + f*x)^m*(a + b*ArcTan[c + d*x])^p, x] /;
FreeQ[{a, b, c, d, e, f, m, p}, x] && !IGtQ[p, 0]
```

3.23.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int (bx+a)^2\sqrt{\arctan(bx+a)}dx$$

input
```
int((b*x+a)^2*arctan(b*x+a)^(1/2),x)
```

output
```
int((b*x+a)^2*arctan(b*x+a)^(1/2),x)
```

3.23.5 Fricas [F(-2)]

Exception generated.

$$\int (a+bx)^2\sqrt{\arctan(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^2*arctan(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.23.6 Sympy [N/A]

Not integrable

Time = 3.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (a+bx)^2\sqrt{\arctan(a+bx)}\,dx = \int (a+bx)^2\sqrt{\operatorname{atan}(a+bx)}\,dx$$

input
```
integrate((b*x+a)**2*atan(b*x+a)**(1/2),x)
```

output
```
Integral((a + b*x)**2*sqrt(atan(a + b*x)), x)
```

3.23.7 Maxima [F(-2)]

Exception generated.

$$\int (a+bx)^2\sqrt{\arctan(a+bx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((b*x+a)^2*arctan(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.23.8 Giac [N/A]

Not integrable

Time = 127.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.17

$$\int (a+bx)^2\sqrt{\arctan(a+bx)}\,dx = \int (bx+a)^2\sqrt{\arctan\left(bx+a\right)}\,dx$$

input
```
integrate((b*x+a)^2*arctan(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.23.9 Mupad [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a+bx)^2\sqrt{\arctan(a+bx)}\,dx = \int \sqrt{\mathrm{atan}\left(a+b\,x\right)}\,{\left(a+b\,x\right)}^2\,dx$$

input
```
int(atan(a + b*x)^(1/2)*(a + b*x)^2,x)
```

output
```
int(atan(a + b*x)^(1/2)*(a + b*x)^2, x)
```

3.24 $\int (e+fx)^3(a+b\arctan(c+dx))\,dx$

3.24.1 Optimal result

Integrand size = 18, antiderivative size = 233

$$\begin{aligned}\int (e+fx)^3(a+b\arctan(c+dx))\,dx &= -\frac{bf(6d^2e^2-12cdef-(1-6c^2)f^2)x}{4d^3} - \frac{bf^2(de-cf)(c+dx)^2}{2d^4} - \frac{bf^3(c+dx)^3}{12d^4} \\ &\quad - \frac{b(d^4e^4-4cd^3e^3f-6(1-c^2)d^2e^2f^2+4c(3-c^2)def^3+(1-6c^2+c^4)f^4)\arctan(c+dx)}{4d^4f} \\ &\quad + \frac{(e+fx)^4(a+b\arctan(c+dx))}{4f} \\ &\quad - \frac{b(de-cf)(de+f-cf)(de-(1+c)f)\log(1+(c+dx)^2)}{2d^4}\end{aligned}$$

output
```
-1/4*b*f*(6*d^2*e^2-12*c*d*e*f-(-6*c^2+1)*f^2)*x/d^3-1/2*b*f^2*(-c*f+d*e)*
(d*x+c)^2/d^4-1/12*b*f^3*(d*x+c)^3/d^4-1/4*b*(d^4*e^4-4*c*d^3*e^3*f-6*(-c^
2+1)*d^2*e^2*f^2+4*c*(-c^2+3)*d*e*f^3+(c^4-6*c^2+1)*f^4)*arctan(d*x+c)/d^4
/f+1/4*(f*x+e)^4*(a+b*arctan(d*x+c))/f-1/2*b*(-c*f+d*e)*(-c*f+d*e+f)*(d*e-
(1+c)*f)*ln(1+(d*x+c)^2)/d^4
```

3.24.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.67

$$\int (e+fx)^3(a+b\arctan(c+dx))\,dx$$
$$= \frac{(e+fx)^4(a+b\arctan(c+dx)) - \frac{b(6df^2(6d^2e^2-12cdef+(-1+6c^2)f^2)x+12f^3(de-cf)(c+dx)^2+2f^4(c+dx)^3-3i(de-(-i+c)f)^4}{6d^4}}{4f}$$

input
```
Integrate[(e + f*x)^3*(a + b*ArcTan[c + d*x]),x]
```

output
```
((e + f*x)^4*(a + b*ArcTan[c + d*x]) - (b*(6*d*f^2*(6*d^2*e^2 - 12*c*d*e*f
 + (-1 + 6*c^2)*f^2)*x + 12*f^3*(d*e - c*f)*(c + d*x)^2 + 2*f^4*(c + d*x)^
3 - (3*I)*(d*e - (-I + c)*f)^4*Log[I - c - d*x] + (3*I)*(d*e - (I + c)*f)^
4*Log[I + c + d*x]))/(6*d^4))/(4*f)
```

3.24.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5570, 27, 5387, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3(a+b\arctan(c+dx))\,dx$$
$$\Big\downarrow \text{5570}$$
$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^3(a+b\arctan(c+dx))}{d^3}d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int (de-cf+f(c+dx))^3(a+b\arctan(c+dx))d(c+dx)}{d^4}$$
$$\Big\downarrow \text{5387}$$
$$\frac{\frac{(f(c+dx)-cf+de)^4(a+b\arctan(c+dx))}{4f}-\frac{b\int \frac{(de-cf+f(c+dx))^4}{(c+dx)^2+1}d(c+dx)}{4f}}{d^4}$$

$$\Big\downarrow \text{478}$$

$$\frac{\frac{(f(c+dx)-cf+de)^4(a+b\arctan(c+dx))}{4f}-\frac{b\int\left((c+dx)^2f^4+4(de-cf)(c+dx)f^3+\left(6d^2e^2-12cdfe-\left(1-6c^2\right)f^2\right)f^2+\frac{d^4e^4-4cd^3fe^3-6\left(1-c^2\right)d^2f^2e}{\ldots}\right.}{4f}}{d^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(f(c+dx)-cf+de)^4(a+b\arctan(c+dx))}{4f}-\frac{b\left(\arctan(c+dx)\left(-6\left(1-c^2\right)d^2e^2f^2+4c\left(3-c^2\right)def^3+\left(c^4-6c^2+1\right)f^4-4cd^3e^3f+d^4e^4\right)+f^2(c+dx)\left(-(1\ldots\right.}{\ldots}}{d^4}$$

input
```
Int[(e + f*x)^3*(a + b*ArcTan[c + d*x]),x]
```

output
```
(((d*e - c*f + f*(c + d*x))^4*(a + b*ArcTan[c + d*x]))/(4*f) - (b*(f^2*(6*
d^2*e^2 - 12*c*d*e*f - (1 - 6*c^2)*f^2)*(c + d*x) + 2*f^3*(d*e - c*f)*(c +
 d*x)^2 + (f^4*(c + d*x)^3)/3 + (d^4*e^4 - 4*c*d^3*e^3*f - 6*(1 - c^2)*d^2
*e^2*f^2 + 4*c*(3 - c^2)*d*e*f^3 + (1 - 6*c^2 + c^4)*f^4)*ArcTan[c + d*x]
+ 2*f*(d*e - c*f)*(d*e + f - c*f)*(d*e - (1 + c)*f)*Log[1 + (c + d*x)^2]))
/(4*f))/d^4
```

3.24.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

rule 5570
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.24.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(221) = 442$.

Time = 0.25 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.99

method	result
parts	$\frac{5b f^2 c^2 e}{2d^3} - \frac{3bfc e^2}{2d^2} + \frac{b f^2 \ln\left(1+(dx+c)^2\right) e}{2d^3} + \frac{b f^3 \ln\left(1+(dx+c)^2\right) c^3}{2d^4} - \frac{b f^3 \ln\left(1+(dx+c)^2\right) c}{2d^4} + b f^2 \arctan$
derivativedivides	$\frac{a(cf-de-f(dx+c))^4}{4d^3 f} - \frac{b\left(-\frac{f^3 \arctan(dx+c) c^4}{4} + f^2 \arctan(dx+c) c^3 de + f^3 \arctan(dx+c) c^3 (dx+c) - \frac{3f \arctan(dx+c) c^2 d^2 e^2}{2} - 3f^2 \mathrm{arc}\right.}{}$
default	$\frac{a(cf-de-f(dx+c))^4}{4d^3 f} - \frac{b\left(-\frac{f^3 \arctan(dx+c) c^4}{4} + f^2 \arctan(dx+c) c^3 de + f^3 \arctan(dx+c) c^3 (dx+c) - \frac{3f \arctan(dx+c) c^2 d^2 e^2}{2} - 3f^2 \mathrm{arc}\right.}{}$
parallelrisch	$-42b c^2 de f^2 + 36bc d^2 e^2 f + 3xbd f^3 - x^3 b d^3 f^3 + 6 \ln\left(d^2 x^2 + 2cdx + c^2 + 1\right) b c^3 f^3 - 6 \ln\left(d^2 x^2 + 2cdx + c^2 + 1\right) b d^3 e^3 - 6 \ln\left(d^2\right.$
risch	$\frac{x^4 f^3 a}{4} + xa e^3 - \frac{i(fx+e)^4 b \ln(1+i(dx+c))}{8f} + \frac{if^3 b x^4 \ln(1-i(dx+c))}{8} + \frac{ib e^3 x \ln(1-i(dx+c))}{2} + \frac{ib e^4 \ln\left(d^2 x^2 +\right.}{16}$

input
```
int((f*x+e)^3*(a+b*arctan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
5/2*b/d^3*f^2*c^2*e-3/2*b/d^2*f*c*e^2+1/2*b/d^3*f^2*ln(1+(d*x+c)^2)*e+1/2*
b/d^4*f^3*ln(1+(d*x+c)^2)*c^3-1/2*b/d^4*f^3*ln(1+(d*x+c)^2)*c+b*f^2*arctan
(d*x+c)*e*x^3+3/2*b*f*arctan(d*x+c)*e^2*x^2+b*arctan(d*x+c)*x*e^3-1/2*b/d*
ln(1+(d*x+c)^2)*e^3+1/4*b/d^4*f^3*c-13/12*b/d^4*f^3*c^3+1/4*b*f^3*arctan(d
*x+c)*x^4+3/2*b/d^2*f*ln(1+(d*x+c)^2)*c*e^2-3/2*b/d^3*f^2*ln(1+(d*x+c)^2)*
c^2*e-1/12/d*f^3*b*x^3+1/4/d^3*f^3*b*x-1/4/d^4*f^3*b*arctan(d*x+c)+1/d^3*f
^2*b*c^3*e*arctan(d*x+c)-3/d^3*f^2*b*c*e*arctan(d*x+c)-3/2/d^2*f*b*c^2*e^2
*arctan(d*x+c)+2/d^2*f^2*b*c*e*x+1/4*a*(f*x+e)^4/f+1/4/d^2*f^3*b*c*x^2-1/2
/d*f^2*b*e*x^2-3/4/d^3*f^3*b*c^2*x-3/2/d*f*b*e^2*x+1/d*b*c*e^3*arctan(d*x+
c)-1/4/d^4*f^3*b*c^4*arctan(d*x+c)+3/2/d^4*f^3*b*c^2*arctan(d*x+c)+3/2/d^2
*f*b*e^2*arctan(d*x+c)
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.36

$$\int (e+fx)^3(a+b\arctan(c+dx))\,dx = \frac{3\,ad^4f^3x^4+(12\,ad^4ef^2-bd^3f^3)x^3+3\,(6\,ad^4e^2f-2\,bd^3ef^2+bcd^2f^3)x^2+3\,(4\,ad^4e^3-6\,bd^3e^2f+8\,bcd^2}{}$$

input
```
integrate((f*x+e)^3*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(3*a*d^4*f^3*x^4 + (12*a*d^4*e*f^2 - b*d^3*f^3)*x^3 + 3*(6*a*d^4*e^2*
f - 2*b*d^3*e*f^2 + b*c*d^2*f^3)*x^2 + 3*(4*a*d^4*e^3 - 6*b*d^3*e^2*f + 8*
b*c*d^2*e*f^2 - (3*b*c^2 - b)*d*f^3)*x + 3*(b*d^4*f^3*x^4 + 4*b*d^4*e*f^2*
x^3 + 6*b*d^4*e^2*f*x^2 + 4*b*d^4*e^3*x + 4*b*c*d^3*e^3 - 6*(b*c^2 - b)*d^
2*e^2*f + 4*(b*c^3 - 3*b*c)*d*e*f^2 - (b*c^4 - 6*b*c^2 + b)*f^3)*arctan(d*
x + c) - 6*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + (3*b*c^2 - b)*d*e*f^2 - (b*c^3 -
 b*c)*f^3)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/d^4
```

3.24.6 Sympy [F(-1)]

Timed out.

$$\int (e+fx)^3(a+b\arctan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*(a+b*atan(d*x+c)),x)
```

output
```
Timed out
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.48

$$\int (e+fx)^3(a+b\arctan(c+dx))\,dx = \frac{1}{4}af^3x^4 + aef^2x^3 + \frac{3}{2}ae^2fx^2$$
$$+\frac{3}{2}\left(x^2\arctan\left(dx+c\right) - d\left(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\right)\right)be^2f$$
$$+\frac{1}{2}\left(2\,x^3\arctan\left(dx+c\right) - d\left(\frac{dx^2-4\,cx}{d^3} - \frac{2\,(c^3-3\,c)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^4} + \frac{(3\,c^2-1)\log\left(d^2x^2+2\,cdx\right.}{d^4}\right.\right.$$
$$+\frac{1}{12}\left(3\,x^4\arctan\left(dx+c\right) - d\left(\frac{d^2x^3-3\,cdx^2+3\,(3\,c^2-1)x}{d^4} + \frac{3\,(c^4-6\,c^2+1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^5} - 6\,(c\right.\right.$$
$$+ae^3x + \frac{\left(2\,(dx+c)\arctan\left(dx+c\right) - \log\left((dx+c)^2+1\right)\right)be^3}{2\,d}$$

input
```
integrate((f*x+e)^3*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*a*f^3*x^4 + a*e*f^2*x^3 + 3/2*a*e^2*f*x^2 + 3/2*(x^2*arctan(d*x + c) -
 d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*
x + c^2 + 1)/d^3))*b*e^2*f + 1/2*(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*
x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x
^2 + 2*c*d*x + c^2 + 1)/d^4))*b*e*f^2 + 1/12*(3*x^4*arctan(d*x + c) - d*((
d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 - 1)*x)/d^4 + 3*(c^4 - 6*c^2 + 1)*arctan((d
^2*x + c*d)/d)/d^5 - 6*(c^3 - c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^5))*b*
f^3 + a*e^3*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b
*e^3/d
```

3.24.8 Giac [F]

$$\int (e+fx)^3(a+b\arctan(c+dx))\,dx = \int (fx+e)^3\left(b\arctan\left(dx+c\right)+a\right)dx$$

input
```
integrate((f*x+e)^3*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

output
```
sage0*x
```

3.24.9 Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 787, normalized size of antiderivative = 3.38

$$\int (e+fx)^3(a+b\arctan(c+dx))\,dx = \mathrm{atan}(c+d\,x)\,\left(b\,e^3\,x+\frac{3\,b\,e^2\,f\,x^2}{2}+b\,e\,f^2\,x^3+\frac{b\,f^3\,x^4}{4}\right)$$
$$+x\left(\frac{e\,(6\,a\,c^2\,f^2+12\,a\,c\,d\,e\,f+2\,a\,d^2\,e^2-3\,b\,d\,e\,f+6\,a\,f^2)}{2\,d^2} - \frac{(4\,c^2+4)\,\left(\frac{f^2\,(8\,a\,c\,f-b\,f+12\,a\,d\,e)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{4\,d^2} + \frac{2\,c\,\left(\frac{2\,c\,\left(\frac{f^2\,(8\,a\,c\,f-b\,f+12\,a\,d\,e)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{d}-\frac{4\,a\,c^2\,f^3+24\,a\,c\,d\,e\,f^2+12\,a\,d^2\,e^2\,f-4\,b\,d\,e\,f^2+4\,a\,f^3}{4\,d^2}+\frac{a\,f^3\,(4\,c^2+4)}{4\,d^2}\right)}{d}\right)$$
$$-x^2\left(\frac{c\,\left(\frac{f^2\,(8\,a\,c\,f-b\,f+12\,a\,d\,e)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{d} - \frac{4\,a\,c^2\,f^3+24\,a\,c\,d\,e\,f^2+12\,a\,d^2\,e^2\,f-4\,b\,d\,e\,f^2+4\,a\,f^3}{8\,d^2}+\frac{a\,f^3\,(4\,c^2+4)}{8\,d^2}\right)$$
$$+x^3\left(\frac{f^2\,(8\,a\,c\,f-b\,f+12\,a\,d\,e)}{12\,d}-\frac{2\,a\,c\,f^3}{3\,d}\right)+\frac{a\,f^3\,x^4}{4}$$
$$-\frac{\ln(c^2+2\,c\,d\,x+d^2\,x^2+1)\,(-64\,b\,c^3\,d^4\,f^3+192\,b\,c^2\,d^5\,e\,f^2-192\,b\,c\,d^6\,e^2\,f+64\,b\,c\,d^4\,f^3+64\,b\,d^7\,e^3-}{128\,d^8}$$
$$-\frac{b\,\mathrm{atan}\left(\frac{4\,d^3\,\left(\frac{c\,\left(c^4\,f^3-4\,c^3\,d\,e\,f^2+6\,c^2\,d^2\,e^2\,f-6\,c^2\,f^3-4\,c\,d^3\,e^3+12\,c\,d\,e\,f^2-6\,d^2\,e^2\,f+f^3\right)}{4\,d^3}+\frac{x\,\left(c^4\,f^3-4\,c^3\,d\,e\,f^2+6\,c^2\,d^2\,e^2\,f-6\,c^2\,f^3-4\,c\,d^3\,e^3+12\right.}{4\,d^2}\right.}{c^4\,f^3-4\,c^3\,d\,e\,f^2+6\,c^2\,d^2\,e^2\,f-6\,c^2\,f^3-4\,c\,d^3\,e^3+12\,c\,d\,e\,f^2-6\,d^2\,e^2\,f+f^3}\right.}{4\,d}$$

input
```
int((e + f*x)^3*(a + b*atan(c + d*x)),x)
```

output
```
atan(c + d*x)*((b*f^3*x^4)/4 + b*e^3*x + (3*b*e^2*f*x^2)/2 + b*e*f^2*x^3)
+ x*((e*(6*a*f^2 + 6*a*c^2*f^2 + 2*a*d^2*e^2 - 3*b*d*e*f + 12*a*c*d*e*f))/
(2*d^2) - ((4*c^2 + 4)*((f^2*(8*a*c*f - b*f + 12*a*d*e))/(4*d) - (2*a*c*f^
3)/d))/(4*d^2) + (2*c*((2*c*((f^2*(8*a*c*f - b*f + 12*a*d*e))/(4*d) - (2*a
*c*f^3)/d))/d - (4*a*f^3 + 4*a*c^2*f^3 - 4*b*d*e*f^2 + 12*a*d^2*e^2*f + 24
*a*c*d*e*f^2)/(4*d^2) + (a*f^3*(4*c^2 + 4))/(4*d^2)))/d) - x^2*((c*((f^2*(
8*a*c*f - b*f + 12*a*d*e))/(4*d) - (2*a*c*f^3)/d))/d - (4*a*f^3 + 4*a*c^2*
f^3 - 4*b*d*e*f^2 + 12*a*d^2*e^2*f + 24*a*c*d*e*f^2)/(8*d^2) + (a*f^3*(4*c
^2 + 4))/(8*d^2)) + x^3*((f^2*(8*a*c*f - b*f + 12*a*d*e))/(12*d) - (2*a*c*
f^3)/(3*d)) + (a*f^3*x^4)/4 - (log(c^2 + d^2*x^2 + 2*c*d*x + 1)*(64*b*d^7*
e^3 - 64*b*c^3*d^4*f^3 + 64*b*c*d^4*f^3 - 64*b*d^5*e*f^2 - 192*b*c*d^6*e^2
*f + 192*b*c^2*d^5*e*f^2))/(128*d^8) - (b*atan((4*d^3*((c*(f^3 - 6*c^2*f^3
 + c^4*f^3 - 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*c^2*d^2*e^2*f + 12*c*d*e*f^2 -
4*c^3*d*e*f^2))/(4*d^3) + (x*(f^3 - 6*c^2*f^3 + c^4*f^3 - 4*c*d^3*e^3 - 6*
d^2*e^2*f + 6*c^2*d^2*e^2*f + 12*c*d*e*f^2 - 4*c^3*d*e*f^2))/(4*d^2)))/(f^
3 - 6*c^2*f^3 + c^4*f^3 - 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*c^2*d^2*e^2*f + 12
*c*d*e*f^2 - 4*c^3*d*e*f^2))*(f^3 - 6*c^2*f^3 + c^4*f^3 - 4*c*d^3*e^3 - 6*
d^2*e^2*f + 6*c^2*d^2*e^2*f + 12*c*d*e*f^2 - 4*c^3*d*e*f^2))/(4*d^4)
```

3.24. $\int (e+fx)^3(a+b\arctan(c+dx))\,dx$

3.25 $\int (e+fx)^2(a+b\arctan(c+dx))\,dx$

3.25.1 Optimal result

Integrand size = 18, antiderivative size = 155

$$\begin{aligned}&\int (e+fx)^2(a+b\arctan(c+dx))\,dx\\&=-\frac{bf(de-cf)x}{d^2}-\frac{bf^2(c+dx)^2}{6d^3}-\frac{b(de-cf)\left(d^2e^2-2cdef-(3-c^2)f^2\right)\arctan(c+dx)}{3d^3f}\\&\quad+\frac{(e+fx)^3(a+b\arctan(c+dx))}{3f}-\frac{b(3d^2e^2-6cdef-(1-3c^2)f^2)\log\left(1+(c+dx)^2\right)}{6d^3}\end{aligned}$$

output
```
-b*f*(-c*f+d*e)*x/d^2-1/6*b*f^2*(d*x+c)^2/d^3-1/3*b*(-c*f+d*e)*(d^2*e^2-2*
c*d*e*f-(-c^2+3)*f^2)*arctan(d*x+c)/d^3/f+1/3*(f*x+e)^3*(a+b*arctan(d*x+c)
)/f-1/6*b*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*ln(1+(d*x+c)^2)/d^3
```

3.25.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.76

$$\begin{aligned}&\int (e+fx)^2(a+b\arctan(c+dx))\,dx\\&=\frac{(e+fx)^3(a+b\arctan(c+dx))-\frac{b\left(6df^2(de-cf)x+f^3(c+dx)^2-i(de-(-i+c)f)^3\log(i-c-dx)+i(de-(i+c)f)^3\log(i+c+dx)\right)}{2d^3}}{3f}\end{aligned}$$

input
```
Integrate[(e + f*x)^2*(a + b*ArcTan[c + d*x]),x]
```

output
```
((e + f*x)^3*(a + b*ArcTan[c + d*x]) - (b*(6*d*f^2*(d*e - c*f)*x + f^3*(c
+ d*x)^2 - I*(d*e - (-I + c)*f)^3*Log[I - c - d*x] + I*(d*e - (I + c)*f)^3
*Log[I + c + d*x]))/(2*d^3))/(3*f)
```

3.25.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5570, 27, 5387, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2(a+b\arctan(c+dx))\,dx$$

$$\Big\downarrow \text{5570}$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^2(a+b\arctan(c+dx))}{d^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (de-cf+f(c+dx))^2(a+b\arctan(c+dx))d(c+dx)}{d^3}$$

$$\Big\downarrow \text{5387}$$

$$\frac{\frac{(f(c+dx)-cf+de)^3(a+b\arctan(c+dx))}{3f}-\frac{b\int \frac{(de-cf+f(c+dx))^3}{(c+dx)^2+1}d(c+dx)}{3f}}{d^3}$$

$$\Big\downarrow \text{478}$$

$$\frac{\frac{(f(c+dx)-cf+de)^3(a+b\arctan(c+dx))}{3f}-\frac{b\int \left((c+dx)f^3+3(de-cf)f^2+\frac{(de-cf)\left(d^2e^2-2cdfe+c^2f^2-3f^2\right)+f\left(3d^2e^2-6cdfe-\left(1-3c^2\right)f^2\right)(c+dx)}{(c+dx)^2+1}\right)d(}{3f}}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(f(c+dx)-cf+de)^3(a+b\arctan(c+dx))}{3f}-\frac{b\left(\arctan(c+dx)(de-cf)\left(-\left(3-c^2\right)f^2-2cdef+d^2e^2\right)+\frac{1}{2}f\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\log((c+dx)^2+\right.}{3f}}{d^3}$$

input
```
Int[(e + f*x)^2*(a + b*ArcTan[c + d*x]),x]
```

output
```
(((d*e - c*f + f*(c + d*x))^3*(a + b*ArcTan[c + d*x]))/(3*f) - (b*(3*f^2*(
d*e - c*f)*(c + d*x) + (f^3*(c + d*x)^2)/2 + (d*e - c*f)*(d^2*e^2 - 2*c*d*
e*f - (3 - c^2)*f^2)*ArcTan[c + d*x] + (f*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*
c^2)*f^2)*Log[1 + (c + d*x)^2])/2))/(3*f))/d^3
```

3.25.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

rule 5570
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.25.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.71

method	result
parts	$\frac{a(fx+e)^3}{3f} - \frac{f^2bc\arctan(dx+c)}{d^3} + \frac{fbe\arctan(dx+c)}{d^2} + \frac{b\,f^2\arctan(dx+c)x^3}{3} + b\arctan(dx+c)\,x\,e^2 -$
derivativedivides	$\frac{-\frac{a(cf-de-f(dx+c))^3}{3d^2f} + \frac{b\,f^2\arctan(dx+c)c^2(dx+c)}{d^2} - \frac{2bf\arctan(dx+c)ce(dx+c)}{d} - \frac{b\,f^2\arctan(dx+c)c(dx+c)^2}{d^2} + b\arctan(dx+c)e}{}$
default	$\frac{-\frac{a(cf-de-f(dx+c))^3}{3d^2f} + \frac{b\,f^2\arctan(dx+c)c^2(dx+c)}{d^2} - \frac{2bf\arctan(dx+c)ce(dx+c)}{d} - \frac{b\,f^2\arctan(dx+c)c(dx+c)^2}{d^2} + b\arctan(dx+c)e}{}$
parallelrisch	$-\frac{-\ln(d^2x^2+2cdx+c^2+1)b\,f^2+6\arctan(dx+c)b\,c^2def-6\ln(d^2x^2+2cdx+c^2+1)bcdef-6x^2\arctan(dx+c)b\,d^3ef-6x}{}$
risch	$-\frac{i(fx+e)^3b\ln(1+i(dx+c))}{6f} + \frac{ib\,e^3\ln(d^2x^2+2cdx+c^2+1)}{12f} + \frac{fbce\ln(d^2x^2+2cdx+c^2+1)}{d^2} + \frac{if^2b\,x^3\ln(1-i(dx+c))}{6}$

input
```
int((f*x+e)^2*(a+b*arctan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*(f*x+e)^3/f-f^2/d^3*b*c*arctan(d*x+c)+f/d^2*b*e*arctan(d*x+c)+1/3*b*
f^2*arctan(d*x+c)*x^3+b*arctan(d*x+c)*x*e^2-b/d^2*f*c*e+1/3*f^2/d^3*b*c^3*
arctan(d*x+c)+2/3*f^2/d^2*b*c*x-f/d*b*e*x-1/2*b/d^3*f^2*ln(1+(d*x+c)^2)*c^
2-1/2*b*e^2*ln(1+(d*x+c)^2)/d+b*f*arctan(d*x+c)*e*x^2+b/d^2*f*ln(1+(d*x+c)
^2)*c*e+5/6*b/d^3*f^2*c^2-f/d^2*b*c^2*e*arctan(d*x+c)+1/6*b/d^3*f^2*ln(1+(
d*x+c)^2)+1/d*b*c*e^2*arctan(d*x+c)-1/6*f^2/d*b*x^2
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.28

$$\int (e+fx)^2(a+b\arctan(c+dx))\,dx$$
$$= \frac{2\,ad^3f^2x^3 + (6\,ad^3ef - bd^2f^2)x^2 + 2\,(3\,ad^3e^2 - 3\,bd^2ef + 2\,bcdf^2)x + 2\,(bd^3f^2x^3 + 3\,bd^3efx^2 + 3\,bd^3e^2x}{}$$

input
```
integrate((f*x+e)^2*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(2*a*d^3*f^2*x^3 + (6*a*d^3*e*f - b*d^2*f^2)*x^2 + 2*(3*a*d^3*e^2 - 3*
b*d^2*e*f + 2*b*c*d*f^2)*x + 2*(b*d^3*f^2*x^3 + 3*b*d^3*e*f*x^2 + 3*b*d^3*
e^2*x + 3*b*c*d^2*e^2 - 3*(b*c^2 - b)*d*e*f + (b*c^3 - 3*b*c)*f^2)*arctan(
d*x + c) - (3*b*d^2*e^2 - 6*b*c*d*e*f + (3*b*c^2 - b)*f^2)*log(d^2*x^2 + 2
*c*d*x + c^2 + 1))/d^3
```

3.25.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 131.97 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.43

$$\int (e+fx)^2(a+b\arctan(c+dx))\,dx$$
$$= \begin{cases} ae^2x + aefx^2 + \frac{af^2x^3}{3} + \frac{bc^3f^2\operatorname{atan}(c+dx)}{3d^3} - \frac{bc^2ef\operatorname{atan}(c+dx)}{d^2} - \frac{bc^2f^2\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^3} + \frac{ibc^2f^2\operatorname{atan}(c+dx)}{d^3} + \frac{bce^2\operatorname{atan}(c+ \\ (a+b\operatorname{atan}(c))\left(e^2x+efx^2+\frac{f^2x^3}{3}\right) \end{cases}$$

input
```
integrate((f*x+e)**2*(a+b*atan(d*x+c)),x)
```

output
```
Piecewise((a*e**2*x + a*e*f*x**2 + a*f**2*x**3/3 + b*c**3*f**2*atan(c + d*
x)/(3*d**3) - b*c**2*e*f*atan(c + d*x)/d**2 - b*c**2*f**2*log(c/d + x - I/
d)/d**3 + I*b*c**2*f**2*atan(c + d*x)/d**3 + b*c*e**2*atan(c + d*x)/d + 2*
b*c*e*f*log(c/d + x - I/d)/d**2 - 2*I*b*c*e*f*atan(c + d*x)/d**2 + 2*b*c*f
**2*x/(3*d**2) - b*c*f**2*atan(c + d*x)/d**3 + b*e**2*x*atan(c + d*x) + b*
e*f*x**2*atan(c + d*x) + b*f**2*x**3*atan(c + d*x)/3 - b*e**2*log(c/d + x
- I/d)/d + I*b*e**2*atan(c + d*x)/d - b*e*f*x/d - b*f**2*x**2/(6*d) + b*e*
f*atan(c + d*x)/d**2 + b*f**2*log(c/d + x - I/d)/(3*d**3) - I*b*f**2*atan(
c + d*x)/(3*d**3), Ne(d, 0)), ((a + b*atan(c))*(e**2*x + e*f*x**2 + f**2*x
**3/3), True))
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.42

$$\int (e+fx)^2(a+b\arctan(c+dx))\,dx = \frac{1}{3}af^2x^3 + aefx^2$$
$$+ \left(x^2\arctan(dx+c) - d\left(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log(d^2x^2+2cdx+c^2+1)}{d^3}\right)\right)bef$$
$$+\frac{1}{6}\left(2x^3\arctan(dx+c) - d\left(\frac{dx^2-4cx}{d^3} - \frac{2(c^3-3c)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^4} + \frac{(3c^2-1)\log(d^2x^2+2cdx-}{d^4}\right.\right.$$
$$+ ae^2x + \frac{\left(2(dx+c)\arctan(dx+c) - \log\left((dx+c)^2+1\right)\right)be^2}{2d}$$

input
```
integrate((f*x+e)^2*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*a*f^2*x^3 + a*e*f*x^2 + (x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*ar
ctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*e*f
 + 1/6*(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arc
tan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^
4))*b*f^2 + a*e^2*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 +
 1))*b*e^2/d
```

3.25.8 Giac [F]

$$\int (e+fx)^2(a+b\arctan(c+dx))\,dx=\int (fx+e)^2(b\arctan(dx+c)+a)\,dx$$

input
```
integrate((f*x+e)^2*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

output
```
sage0*x
```

3.25.9 Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.65

$$\int (e+fx)^2(a+b\arctan(c+dx))\,dx=x^2\left(\frac{f\,(6\,a\,c\,f-b\,f+6\,a\,d\,e)}{6\,d}-\frac{a\,c\,f^2}{d}\right)-x\left(\frac{2\,c\left(\frac{f\,(6\,a\,c\,f-b\,f+6\,a\,d\,e)}{3\,d}-\frac{2\,a\,c\,f^2}{d}\right)}{d}-\frac{3\,a\,c^2\,f^2+12\,a\,c\,d\,e\,f+3\,a\,d^2\,e^2-3\,b\,d\,e\,f+3\,a\,f^2}{3\,d^2}+\frac{a\,f^2\,(3\,c^2+3)}{3\,d^2}\right)+\mathrm{atan}(c+d\,x)\left(b\,e^2\,x+b\,e\,f\,x^2+\frac{b\,f^2\,x^3}{3}\right)+\frac{a\,f^2\,x^3}{3}-\frac{\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)\,\left(36\,b\,c^2\,d^3\,f^2-72\,b\,c\,d^4\,e\,f+36\,b\,d^5\,e^2-12\,b\,d^3\,f^2\right)}{72\,d^6}+\frac{b\,\mathrm{atan}\left(\frac{3\,d^2\left(\frac{c\,\left(c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f\right)}{3\,d^2}+\frac{x\,\left(c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f\right)}{3\,d}\right)}{c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f}\right)\,\left(c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\right.}{3\,d^3}$$

input
```
int((e + f*x)^2*(a + b*atan(c + d*x)),x)
```

output
```
x^2*((f*(6*a*c*f - b*f + 6*a*d*e))/(6*d) - (a*c*f^2)/d) - x*((2*c*((f*(6*a
*c*f - b*f + 6*a*d*e))/(3*d) - (2*a*c*f^2)/d))/d - (3*a*f^2 + 3*a*c^2*f^2
+ 3*a*d^2*e^2 - 3*b*d*e*f + 12*a*c*d*e*f)/(3*d^2) + (a*f^2*(3*c^2 + 3))/(3
*d^2)) + atan(c + d*x)*((b*f^2*x^3)/3 + b*e^2*x + b*e*f*x^2) + (a*f^2*x^3)
/3 - (log(c^2 + d^2*x^2 + 2*c*d*x + 1)*(36*b*d^5*e^2 - 12*b*d^3*f^2 + 36*b
*c^2*d^3*f^2 - 72*b*c*d^4*e*f))/(72*d^6) + (b*atan((3*d^2*((c*(c^3*f^2 - 3
*c*f^2 + 3*c*d^2*e^2 + 3*d*e*f - 3*c^2*d*e*f))/(3*d^2) + (x*(c^3*f^2 - 3*c
*f^2 + 3*c*d^2*e^2 + 3*d*e*f - 3*c^2*d*e*f))/(3*d)))/(c^3*f^2 - 3*c*f^2 +
3*c*d^2*e^2 + 3*d*e*f - 3*c^2*d*e*f))*(c^3*f^2 - 3*c*f^2 + 3*c*d^2*e^2 + 3
*d*e*f - 3*c^2*d*e*f))/(3*d^3)
```

3.25. $\int (e+fx)^2(a+b\arctan(c+dx))\,dx$

3.26 $\int (e+fx)(a+b\arctan(c+dx))\,dx$

3.26.1 Optimal result

Integrand size = 16, antiderivative size = 97

$$\int (e+fx)(a+b\arctan(c+dx))\,dx = -\frac{bfx}{2d} - \frac{b(de+f-cf)(de-(1+c)f)\arctan(c+dx)}{2d^2f} + \frac{(e+fx)^2(a+b\arctan(c+dx))}{2f} - \frac{b(de-cf)\log\left(1+(c+dx)^2\right)}{2d^2}$$

output

```
-1/2*b*f*x/d-1/2*b*(-c*f+d*e+f)*(d*e-(1+c)*f)*arctan(d*x+c)/d^2/f+1/2*(f*x
+e)^2*(a+b*arctan(d*x+c))/f-1/2*b*(-c*f+d*e)*ln(1+(d*x+c)^2)/d^2
```

3.26.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.68

$$\int (e+fx)(a+b\arctan(c+dx))\,dx = aex+\frac{1}{2}afx^2+bex\arctan(c+dx) + \frac{bf\left(\frac{1}{2}d\left(-\frac{c}{d}+\frac{c+dx}{d}\right)^2\arctan(c+dx)-\frac{1}{2}d\left(\frac{x}{d}-\frac{i(i-c)^2\log(i-c-dx)}{2d^2}+\frac{i(i+c)^2\log(i+c+dx)}{2d^2}\right)\right)}{d} - \frac{be\left(-2c\arctan(c+dx)+\log\left(1+c^2+2cdx+d^2x^2\right)\right)}{2d}$$

input
```
Integrate[(e + f*x)*(a + b*ArcTan[c + d*x]),x]
```

output
```
a*e*x + (a*f*x^2)/2 + b*e*x*ArcTan[c + d*x] + (b*f*((d*(-(c/d) + (c + d*x)
/d)^2*ArcTan[c + d*x])/2 - (d*(x/d - ((I/2)*(I - c)^2*Log[I - c - d*x])/d^
2 + ((I/2)*(I + c)^2*Log[I + c + d*x])/d^2))/2))/d - (b*e*(-2*c*ArcTan[c +
 d*x] + Log[1 + c^2 + 2*c*d*x + d^2*x^2]))/(2*d)
```

3.26.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5570, 27, 5387, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)(a+b\arctan(c+dx))\,dx$$

$$\Big\downarrow \text{ 5570}$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)(a+b\arctan(c+dx))}{d}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int (de-cf+f(c+dx))(a+b\arctan(c+dx))d(c+dx)}{d^2}$$

$$\Big\downarrow \text{ 5387}$$

$$\frac{\frac{(f(c+dx)-cf+de)^2(a+b\arctan(c+dx))}{2f}-\frac{b\int \frac{(de-cf+f(c+dx))^2}{(c+dx)^2+1}d(c+dx)}{2f}}{d^2}$$

$$\Big\downarrow \text{ 478}$$

$$\frac{\frac{(f(c+dx)-cf+de)^2(a+b\arctan(c+dx))}{2f}-\frac{b\int \left(f^2+\frac{(de-cf-f)(de-cf+f)+2f(de-cf)(c+dx)}{(c+dx)^2+1}\right)d(c+dx)}{2f}}{d^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{(f(c+dx)-cf+de)^2(a+b\arctan(c+dx))}{2f}-\frac{b\left(\arctan(c+dx)(-cf+de+f)(de-(c+1)f)+f(de-cf)\log\left((c+dx)^2+1\right)+f^2(c+dx)\right)}{2f}}{d^2}$$

input
```
Int[(e + f*x)*(a + b*ArcTan[c + d*x]),x]
```

output
```
(((d*e - c*f + f*(c + d*x))^2*(a + b*ArcTan[c + d*x]))/(2*f) - (b*(f^2*(c
+ d*x) + (d*e + f - c*f)*(d*e - (1 + c)*f)*ArcTan[c + d*x] + f*(d*e - c*f)
*Log[1 + (c + d*x)^2]))/(2*f))/d^2
```

3.26.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

rule 5570
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.26.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.16

method	result
parts	$a\left(\frac{1}{2}f\,x^2+ex\right)+\frac{b\left(\frac{\arctan(dx+c)(dx+c)^2 f}{2d}-\frac{\arctan(dx+c)cf(dx+c)}{d}+\arctan(dx+c)e(dx+c)-\frac{f(dx+c)+\frac{(-2cf+2de)\ln}{}}{}\right.}{d}$
derivativedivides	$\frac{-\frac{a\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b\left(\arctan(dx+c)fc(dx+c)-\arctan(dx+c)ed(dx+c)-\frac{\arctan(dx+c)f(dx+c)^2}{2}+\frac{f(dx+c)}{2}-\right.}{d}}{d}$
default	$\frac{-\frac{a\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b\left(\arctan(dx+c)fc(dx+c)-\arctan(dx+c)ed(dx+c)-\frac{\arctan(dx+c)f(dx+c)^2}{2}+\frac{f(dx+c)}{2}-\right.}{d}}{d}$
parallelrisch	$\frac{\arctan(dx+c)b\,d^2f\,x^2+a\,d^2f\,x^2+2x\arctan(dx+c)b\,d^2e+2a\,d^2ex-\arctan(dx+c)b\,c^2f+2bcde\arctan(dx+c)+bcf\ln(d^2x}{2d^2}$
risch	$-\frac{ib(f\,x^2+2ex)\ln(1+i(dx+c))}{4}+\frac{ibf\,x^2\ln(1-i(dx+c))}{4}+\frac{ibex\ln(1-i(dx+c))}{2}+\frac{af\,x^2}{2}-\frac{\arctan(dx+c)b\,c^2f}{2d^2}+$

input
```
int((f*x+e)*(a+b*arctan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*f*x^2+e*x)+b/d*(1/2/d*arctan(d*x+c)*(d*x+c)^2*f-1/d*arctan(d*x+c)*c
*f*(d*x+c)+arctan(d*x+c)*e*(d*x+c)-1/2/d*(f*(d*x+c)+1/2*(-2*c*f+2*d*e)*ln(
1+(d*x+c)^2)-f*arctan(d*x+c)))
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06

$$\int(e+fx)(a+b\arctan(c+dx))\,dx$$
$$=\frac{ad^2fx^2+(2\,ad^2e-bdf)x+(bd^2fx^2+2\,bd^2ex+2\,bcde-(bc^2-b)f)\arctan(dx+c)-(bde-bcf)\log(d}{2\,d^2}$$

input
```
integrate((f*x+e)*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a*d^2*f*x^2 + (2*a*d^2*e - b*d*f)*x + (b*d^2*f*x^2 + 2*b*d^2*e*x + 2*
b*c*d*e - (b*c^2 - b)*f)*arctan(d*x + c) - (b*d*e - b*c*f)*log(d^2*x^2 + 2
*c*d*x + c^2 + 1))/d^2
```

3.26.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.21 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.82

$$\int (e+fx)(a+b\arctan(c+dx))\,dx$$
$$=\begin{cases} aex+\frac{afx^2}{2}-\frac{bc^2f\operatorname{atan}(c+dx)}{2d^2}+\frac{bce\operatorname{atan}(c+dx)}{d}+\frac{bcf\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^2}-\frac{ibcf\operatorname{atan}(c+dx)}{d^2}+bex\operatorname{atan}(c+dx)+\frac{bfx^2\operatorname{atan}}{2} \\ (a+b\operatorname{atan}(c))\left(ex+\frac{fx^2}{2}\right) \end{cases}$$

input
```
integrate((f*x+e)*(a+b*atan(d*x+c)),x)
```

output
```
Piecewise((a*e*x + a*f*x**2/2 - b*c**2*f*atan(c + d*x)/(2*d**2) + b*c*e*at
an(c + d*x)/d + b*c*f*log(c/d + x - I/d)/d**2 - I*b*c*f*atan(c + d*x)/d**2
 + b*e*x*atan(c + d*x) + b*f*x**2*atan(c + d*x)/2 - b*e*log(c/d + x - I/d)
/d + I*b*e*atan(c + d*x)/d - b*f*x/(2*d) + b*f*atan(c + d*x)/(2*d**2), Ne(
d, 0)), ((a + b*atan(c))*(e*x + f*x**2/2), True))
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.20

$$\int (e+fx)(a+b\arctan(c+dx))\,dx = \frac{1}{2}afx^2 + \frac{1}{2}\left(x^2\arctan(dx+c)-d\left(\frac{x}{d^2}+\frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log(d^2x^2+2cdx+c^2+1)}{d^3}\right)\right)bf + aex + \frac{\left(2(dx+c)\arctan(dx+c)-\log\left((dx+c)^2+1\right)\right)be}{2d}$$

input
```
integrate((f*x+e)*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*a*f*x^2 + 1/2*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*
x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*f + a*e*x + 1
/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b*e/d
```

3.26.8 Giac [F]

$$\int (e+fx)(a+b\arctan(c+dx))\,dx = \int (fx+e)(b\arctan(dx+c)+a)\,dx$$

input
```
integrate((f*x+e)*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

output
```
sage0*x
```

3.26.9 Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.40

$$\int (e+fx)(a+b\arctan(c+dx))\,dx = a\,e\,x + \frac{a\,f\,x^2}{2} - \frac{b\,e\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d} + \frac{b\,f\,\mathrm{atan}(c+d\,x)}{2\,d^2} + \frac{b\,f\,x^2\,\mathrm{atan}(c+d\,x)}{2} - \frac{b\,f\,x}{2\,d} + b\,e\,x\,\mathrm{atan}(c+d\,x) - \frac{b\,c^2\,f\,\mathrm{atan}(c+d\,x)}{2\,d^2} + \frac{b\,c\,f\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d^2} + \frac{b\,c\,e\,\mathrm{atan}(c+d\,x)}{d}$$

input
```
int((e + f*x)*(a + b*atan(c + d*x)),x)
```

output
```
a*e*x + (a*f*x^2)/2 - (b*e*log(c^2 + d^2*x^2 + 2*c*d*x + 1))/(2*d) + (b*f*
atan(c + d*x))/(2*d^2) + (b*f*x^2*atan(c + d*x))/2 - (b*f*x)/(2*d) + b*e*x
*atan(c + d*x) - (b*c^2*f*atan(c + d*x))/(2*d^2) + (b*c*f*log(c^2 + d^2*x^
2 + 2*c*d*x + 1))/(2*d^2) + (b*c*e*atan(c + d*x))/d
```

3.27 $\int (a + b \arctan(c + dx))\,dx$

3.27.1 Optimal result

Integrand size = 10, antiderivative size = 38

$$\int (a + b \arctan(c + dx))\,dx = ax + \frac{b(c + dx)\arctan(c + dx)}{d} - \frac{b\log\left(1 + (c + dx)^2\right)}{2d}$$

output
```
a*x+b*(d*x+c)*arctan(d*x+c)/d-1/2*b*ln(1+(d*x+c)^2)/d
```

3.27.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int (a + b \arctan(c + dx))\,dx = ax + bx\arctan(c + dx) - \frac{b(-2c\arctan(c + dx) + \log\left(1 + c^2 + 2cdx + d^2x^2\right))}{2d}$$

input
```
Integrate[a + b*ArcTan[c + d*x],x]
```

output
```
a*x + b*x*ArcTan[c + d*x] - (b*(-2*c*ArcTan[c + d*x] + Log[1 + c^2 + 2*c*d
*x + d^2*x^2]))/(2*d)
```

3.27.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arctan(c+dx))\,dx$$

$$\Big\downarrow \; 2009$$

$$ax+\frac{b(c+dx)\arctan(c+dx)}{d}-\frac{b\log\left((c+dx)^2+1\right)}{2d}$$

input
```
Int[a + b*ArcTan[c + d*x],x]
```

output
```
a*x + (b*(c + d*x)*ArcTan[c + d*x])/d - (b*Log[1 + (c + d*x)^2])/(2*d)
```

3.27.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.27.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
default	$ax+\frac{b\left((dx+c)\arctan(dx+c)-\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{d}$	35
parts	$ax+\frac{b\left((dx+c)\arctan(dx+c)-\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{d}$	35
derivativedivides	$\frac{(dx+c)a+b\left((dx+c)\arctan(dx+c)-\frac{\ln\left(1+(dx+c)^2\right)}{2}\right)}{d}$	40
parallelrisch	$-\frac{b\left(-2\arctan(dx+c)x\,d^2-2c\arctan(dx+c)d+\ln(d^2x^2+2cdx+c^2+1)d\right)}{2d^2}+ax$	54
risch	$ax-\frac{ibx\ln(1+i(dx+c))}{2}+\frac{ibx\ln(1-i(dx+c))}{2}+\frac{bc\arctan(dx+c)}{d}-\frac{b\ln\left(d^2x^2+2cdx+c^2+1\right)}{2d}$	73

input
```
int(a+b*arctan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*x+b/d*((d*x+c)*arctan(d*x+c)-1/2*ln(1+(d*x+c)^2))
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int (a+b\arctan(c+dx))\,dx = \frac{2\,adx+2\,(bdx+bc)\arctan\left(dx+c\right)-b\log\left(d^2x^2+2\,cdx+c^2+1\right)}{2\,d}$$

input
```
integrate(a+b*arctan(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(2*a*d*x + 2*(b*d*x + b*c)*arctan(d*x + c) - b*log(d^2*x^2 + 2*c*d*x +
 c^2 + 1))/d
```

3.27.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int (a+b\arctan(c+dx))\,dx = ax+b\left(\begin{cases} \frac{c\operatorname{atan}(c+dx)}{d}+x\operatorname{atan}(c+dx)-\frac{\log(c^2+2cdx+d^2x^2+1)}{2d} & \text{for } d\neq 0 \\ x\operatorname{atan}(c) & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*atan(d*x+c),x)
```

output
```
a*x + b*Piecewise((c*atan(c + d*x)/d + x*atan(c + d*x) - log(c**2 + 2*c*d*
x + d**2*x**2 + 1)/(2*d), Ne(d, 0)), (x*atan(c), True))
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int (a + b\arctan(c + dx))\,dx = ax + \frac{\left(2\,(dx + c)\arctan\left(dx + c\right) - \log\left((dx + c)^2 + 1\right)\right)b}{2\,d}$$

input
```
integrate(a+b*arctan(d*x+c),x, algorithm="maxima")
```

output
```
a*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b/d
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int (a + b\arctan(c + dx))\,dx = ax + \frac{\left(2\,(dx + c)\arctan\left(dx + c\right) - \log\left((dx + c)^2 + 1\right)\right)b}{2\,d}$$

input
```
integrate(a+b*arctan(d*x+c),x, algorithm="giac")
```

output
```
a*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b/d
```

3.27.9 Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int (a + b\arctan(c + dx))\,dx = a\,x + b\,x\,\mathrm{atan}(c + d\,x) - \frac{b\,\ln\left(c^2 + 2\,c\,d\,x + d^2\,x^2 + 1\right)}{2\,d} + \frac{b\,c\,\mathrm{atan}(c + d\,x)}{d}$$

input
```
int(a + b*atan(c + d*x),x)
```

output
```
a*x + b*x*atan(c + d*x) - (b*log(c^2 + d^2*x^2 + 2*c*d*x + 1))/(2*d) + (b*
c*atan(c + d*x))/d
```

3.28 $\int \frac{a+b\arctan(c+dx)}{e+fx}\,dx$

3.28.1 Optimal result

Integrand size = 18, antiderivative size = 162

$$\int \frac{a+b\arctan(c+dx)}{e+fx}\,dx = -\frac{(a+b\arctan(c+dx))\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{(a+b\arctan(c+dx))\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} + \frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f} - \frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f}$$

output
```
-(a+b*arctan(d*x+c))*ln(2/(1-I*(d*x+c)))/f+(a+b*arctan(d*x+c))*ln(2*d*(f*x
+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f+1/2*I*b*polylog(2,1-2/(1-I*(d*x+c)))/f-
1/2*I*b*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f
```

3.28.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.99

$$\int \frac{a+b\arctan(c+dx)}{e+fx}\,dx$$
$$= \frac{2a\log(d(e+fx))+ib\log\left(\frac{d(e+fx)}{de-(i+c)f}\right)\log(1-i(c+dx))-ib\log\left(\frac{d(e+fx)}{de+if-cf}\right)\log(1+i(c+dx))-ib\,\text{PolyL}}{2f}$$

input
```
Integrate[(a + b*ArcTan[c + d*x])/(e + f*x),x]
```

output
```
(2*a*Log[d*(e + f*x)] + I*b*Log[(d*(e + f*x))/(d*e - (I + c)*f)]*Log[1 - I
*(c + d*x)] - I*b*Log[(d*(e + f*x))/(d*e + I*f - c*f)]*Log[1 + I*(c + d*x)
] - I*b*PolyLog[2, (f*(-I + c + d*x))/(-(d*e) + (-I + c)*f)] + I*b*PolyLog
[2, (f*(I + c + d*x))/(-(d*e) + (I + c)*f)])/(2*f)
```

3.28.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5570, 27, 5381, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(c+dx)}{e+fx}\,dx$$
$$\Big\downarrow \text{5570}$$
$$\frac{\int \frac{d(a+b\arctan(c+dx))}{d\left(e-\frac{cf}{d}\right)+f(c+dx)}d(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$\int \frac{a+b\arctan(c+dx)}{f(c+dx)-cf+de}d(c+dx)$$
$$\Big\downarrow \text{5381}$$

$$-\frac{b\int \frac{\log\left(\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{(c+dx)^2+1}d(c+dx)}{f}+\frac{b\int \frac{\log\left(\frac{2}{1-i(c+dx)}\right)}{(c+dx)^2+1}d(c+dx)}{f}+\frac{(a+b\arctan(c+dx))\log\left(\frac{2(f(c+dx)-cf+de)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))}{f}$$

$$\Big\downarrow \text{2849}$$

$$-\frac{b\int \frac{\log\left(\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{(c+dx)^2+1}d(c+dx)}{f}+\frac{ib\int \frac{\log\left(\frac{2}{1-i(c+dx)}\right)}{1-\frac{2}{1-i(c+dx)}}d\frac{1}{1-i(c+dx)}}{f}+\frac{(a+b\arctan(c+dx))\log\left(\frac{2(f(c+dx)-cf+de)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))}{f}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{b\int \frac{\log\left(\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{(c+dx)^2+1}d(c+dx)}{f}+\frac{(a+b\arctan(c+dx))\log\left(\frac{2(f(c+dx)-cf+de)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))}{f}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f}$$

$$\Big\downarrow \text{2897}$$

$$\frac{(a+b\arctan(c+dx))\log\left(\frac{2(f(c+dx)-cf+de)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))}{f}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{2f}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f}$$

input
```
Int[(a + b*ArcTan[c + d*x])/(e + f*x),x]
```

output
```
-(((a + b*ArcTan[c + d*x])*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcTan[c
 + d*x])*Log[(2*(d*e - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(c +
d*x)))])/f + ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/f - ((I/2)*b*Po
lyLog[2, 1 - (2*(d*e - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(c +
d*x)))])/f
```

3.28. $\int \frac{a+b\arctan(c+dx)}{e+fx}\,dx$

3.28.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5381
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[b*(c/e)
   Int[Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Simp[b*(c/e)   Int[Log[2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x]) /; FreeQ[{a
, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

rule 5570
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.28.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.22

method	result
parts	$\frac{a\ln(fx+e)}{f}+\frac{b\left(\frac{d\ln(f(dx+c)-cf+de)\arctan(dx+c)}{f}-d\left(-\frac{i\ln(f(dx+c)-cf+de)\left(\ln\left(\frac{if-f(dx+c)}{-cf+de+if}\right)-\ln\left(\frac{if+f(dx+c)}{cf-de+if}\right)\right)}{2f}-\frac{i(\cdots}{\cdots}\right)\right)}{d}$ …
derivativedivides	$\frac{\frac{ad\ln(cf-de-f(dx+c))}{f}-bd\left(-\frac{\ln(cf-de-f(dx+c))\arctan(dx+c)}{f}+\frac{i\ln(cf-de-f(dx+c))\left(\ln\left(\frac{if+f(dx+c)}{cf-de+if}\right)-\ln\left(\frac{if-f(dx+c)}{-cf+de+if}\right)\right)}{2f}\cdots\right)}{d}$ …
default	$\frac{\frac{ad\ln(cf-de-f(dx+c))}{f}-bd\left(-\frac{\ln(cf-de-f(dx+c))\arctan(dx+c)}{f}+\frac{i\ln(cf-de-f(dx+c))\left(\ln\left(\frac{if+f(dx+c)}{cf-de+if}\right)-\ln\left(\frac{if-f(dx+c)}{-cf+de+if}\right)\right)}{2f}\cdots\right)}{d}$ …
risch	$\frac{a\ln(icf-ide+(-idx-ic+1)f-f)}{f}+\frac{ib\,\mathrm{dilog}\left(\frac{icf-ide+(-idx-ic+1)f-f}{icf-ide-f}\right)}{2f}+\frac{ib\ln(-idx-ic+1)\ln\left(\frac{icf-ide+(-idx-ic+\cdots}{icf-ide-f}\right)}{2f}$ …

input
```
int((a+b*arctan(d*x+c))/(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
a*ln(f*x+e)/f+b/d*(d*ln(f*(d*x+c)-c*f+d*e)/f*arctan(d*x+c)-d*(-1/2*I*ln(f*
(d*x+c)-c*f+d*e)*(ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))-ln((I*f+f*(d*x+c))/(c*
f-d*e+I*f)))/f-1/2*I*(dilog((I*f-f*(d*x+c))/(d*e+I*f-c*f))-dilog((I*f+f*(d
*x+c))/(c*f-d*e+I*f)))/f))
```

3.28.5 Fricas [F]

$$\int \frac{a+b\arctan(c+dx)}{e+fx}\,dx=\int \frac{b\arctan\left(dx+c\right)+a}{fx+e}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))/(f*x+e),x, algorithm="fricas")
```

output
```
integral((b*arctan(d*x + c) + a)/(f*x + e), x)
```

3.28.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b \arctan(c + dx)}{e + fx} \, dx = \text{Timed out}$$

input
```
integrate((a+b*atan(d*x+c))/(f*x+e),x)
```

output
```
Timed out
```

3.28.7 Maxima [F]

$$\int \frac{a + b \arctan(c + dx)}{e + fx} \, dx = \int \frac{b \arctan(dx + c) + a}{fx + e} \, dx$$

input
```
integrate((a+b*arctan(d*x+c))/(f*x+e),x, algorithm="maxima")
```

output
```
2*b*integrate(1/2*arctan(d*x + c)/(f*x + e), x) + a*log(f*x + e)/f
```

3.28.8 Giac [F]

$$\int \frac{a + b \arctan(c + dx)}{e + fx} \, dx = \int \frac{b \arctan(dx + c) + a}{fx + e} \, dx$$

input
```
integrate((a+b*arctan(d*x+c))/(f*x+e),x, algorithm="giac")
```

output
```
sage0*x
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(c+dx)}{e+fx}\,dx = \int \frac{a+b\,\mathrm{atan}(c+d\,x)}{e+f\,x}\,dx$$

input
```
int((a + b*atan(c + d*x))/(e + f*x),x)
```

output
```
int((a + b*atan(c + d*x))/(e + f*x), x)
```

3.29 $\int \frac{a+b\arctan(c+dx)}{(e+fx)^2}\,dx$

3.29.1 Optimal result

Integrand size = 18, antiderivative size = 151

$$\int \frac{a+b\arctan(c+dx)}{(e+fx)^2}\,dx = \frac{bd(de-cf)\arctan(c+dx)}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)} - \frac{a+b\arctan(c+dx)}{f(e+fx)} + \frac{bd\log(e+fx)}{d^2e^2-2cdef+(1+c^2)\,f^2} - \frac{bd\log\left(1+c^2+2cdx+d^2x^2\right)}{2\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}$$

output
```
b*d*(-c*f+d*e)*arctan(d*x+c)/f/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+(-a-b*arcta
n(d*x+c))/f/(f*x+e)+b*d*ln(f*x+e)/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-1/2*b*d*
ln(d^2*x^2+2*c*d*x+c^2+1)/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)
```

3.29.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.80

$$\int \frac{a+b\arctan(c+dx)}{(e+fx)^2}\,dx = \frac{-\frac{a+b\arctan(c+dx)}{e+fx} + \frac{bd(i(-de+(i+c)f)\log(i-c-dx)+i(de+if-cf)\log(i+c+dx)+2f\log(d(e+fx)))}{2(d^2e^2-2cdef+(1+c^2)f^2)}}{f}$$

input
```
Integrate[(a + b*ArcTan[c + d*x])/(e + f*x)^2,x]
```

output
```
(-((a + b*ArcTan[c + d*x])/(e + f*x)) + (b*d*(I*(-(d*e) + (I + c)*f)*Log[I
 - c - d*x] + I*(d*e + I*f - c*f)*Log[I + c + d*x] + 2*f*Log[d*(e + f*x)])
)/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)))/f
```

3.29.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5568, 2081, 1144, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(c+dx)}{(e+fx)^2}dx$$

$$\Big\downarrow \ 5568$$

$$\frac{bd\int \frac{1}{(e+fx)((c+dx)^2+1)}dx}{f}-\frac{a+b\arctan(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \ 2081$$

$$\frac{bd\int \frac{1}{(e+fx)(c^2+2dxc+d^2x^2+1)}dx}{f}-\frac{a+b\arctan(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \ 1144$$

$$\frac{bd\left(\frac{\int \frac{d(de-2cf-dfx)}{c^2+2dxc+d^2x^2+1}dx}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{f\log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2}\right)}{f}-\frac{a+b\arctan(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \ 27$$

$$\frac{bd\left(\frac{d\int \frac{de-2cf-dfx}{c^2+2dxc+d^2x^2+1}dx}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{f\log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2}\right)}{f}-\frac{a+b\arctan(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \ 1142$$

$$\frac{bd\left(\frac{d\left((de-cf)\int \frac{1}{c^2+2dxc+d^2x^2+1}dx-\frac{f\int \frac{2d(c+dx)}{c^2+2dxc+d^2x^2+1}dx}{2d}\right)}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{f\log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2}\right)}{f}-\frac{a+b\arctan(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \ 27$$

$$\frac{bd\left(\frac{d\left((de-cf)\int \frac{1}{c^2+2dxc+d^2x^2+1}dx-f\int \frac{c+dx}{c^2+2dxc+d^2x^2+1}dx\right)}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{f\log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2}\right)}{f}-\frac{a+b\arctan(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{bd\left(\frac{d\left(-f\int \frac{c+dx}{c^2+2dxc+d^2x^2+1}dx-2(de-cf)\int \frac{1}{-4d^2-(2xd^2+2cd)^2}d(2xd^2+2cd)\right)}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{f\log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2}\right)}{f}-\frac{a+b\arctan(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{bd\left(\frac{d\left(\frac{\arctan\left(\frac{2cd+2d^2x}{2d}\right)(de-cf)}{d}-f\int \frac{c+dx}{c^2+2dxc+d^2x^2+1}dx\right)}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{f\log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2}\right)}{f}-\frac{a+b\arctan(c+dx)}{f(e+fx)}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{bd\left(\frac{d\left(\frac{\arctan\left(\frac{2cd+2d^2x}{2d}\right)(de-cf)}{d}-\frac{f\log(c^2+2cdx+d^2x^2+1)}{2d}\right)}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{f\log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2}\right)}{f}-\frac{a+b\arctan(c+dx)}{f(e+fx)}$$

input
```
Int[(a + b*ArcTan[c + d*x])/(e + f*x)^2,x]
```

output
```
-((a + b*ArcTan[c + d*x])/(f*(e + f*x))) + (b*d*((f*Log[e + f*x])/(d^2*e^2
 - 2*c*d*e*f + (1 + c^2)*f^2) + (d*(((d*e - c*f)*ArcTan[(2*c*d + 2*d^2*x)/
(2*d)])/d - (f*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*d)))/(d^2*e^2 - 2*c*d*
e*f + (1 + c^2)*f^2)))/f
```

3.29. $\int \frac{a+b\arctan(c+dx)}{(e+fx)^2}\,dx$

3.29.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2081
```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

rule 5568
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcTan[
c + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[p, 0] && ILtQ[m, -1]
```

3.29.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.06

method	result
parts	$-\frac{a}{(fx+e)f}+\frac{b\left(-\frac{d^2\arctan(dx+c)}{(f(dx+c)-cf+de)f}+\frac{d^2\left(\frac{f\ln(f(dx+c)-cf+de)}{c^2f^2-2cdef+d^2e^2+f^2}+\frac{-\frac{f\ln\left(1+(dx+c)^2\right)}{2}+(-cf+de)\arctan(dx+c)}{c^2f^2-2cdef+d^2e^2+f^2}\right)}{f}\right)}{d}$
derivativedivides	$\frac{\frac{ad^2}{(cf-de-f(dx+c))f}+bd^2\left(\frac{\arctan(dx+c)}{(cf-de-f(dx+c))f}-\frac{\frac{\frac{f\ln\left(1+(dx+c)^2\right)}{2}+(cf-de)\arctan(dx+c)}{c^2f^2-2cdef+d^2e^2+f^2}-\frac{f\ln(cf-de-f(dx+c))}{c^2f^2-2cdef+d^2e^2+f^2}}{f}\right)}{d}$
default	$\frac{\frac{ad^2}{(cf-de-f(dx+c))f}+bd^2\left(\frac{\arctan(dx+c)}{(cf-de-f(dx+c))f}-\frac{\frac{\frac{f\ln\left(1+(dx+c)^2\right)}{2}+(cf-de)\arctan(dx+c)}{c^2f^2-2cdef+d^2e^2+f^2}-\frac{f\ln(cf-de-f(dx+c))}{c^2f^2-2cdef+d^2e^2+f^2}}{f}\right)}{d}$
parallelrisch	$\frac{-2x\arctan(dx+c)bcd^3f^2+2x\arctan(dx+c)bd^4ef+2\ln(fx+e)xbd^3f^2-\ln(d^2x^2+2cdx+c^2+1)xbd^3f^2-2\arctan(dx+c\cdots}{2(f\cdots}$
risch	$\frac{ib\ln(1+i(dx+c))}{2f(fx+e)}+\frac{-ibf^2\ln(1-i(dx+c))-i\ln((cdf-d^2e-3idf)x-2icf-ide+c^2f-cde+3f)bcdf^2x-i\ln((cdf-d^2e-\cdots}{\cdots}$

input
```
int((a+b*arctan(d*x+c))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/(f*x+e)/f+b/d*(-d^2/(f*(d*x+c)-c*f+d*e)/f*arctan(d*x+c)+d^2/f*(1/(c^2*f
^2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(f*(d*x+c)-c*f+d*e)+1/(c^2*f^2-2*c*d*e*f+d^2
*e^2+f^2)*(-1/2*f*ln(1+(d*x+c)^2)+(-c*f+d*e)*arctan(d*x+c))))
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.26

$$\int\frac{a+b\arctan(c+dx)}{(e+fx)^2}\,dx = -\frac{2\,ad^2e^2-4\,acdef+2\,(ac^2+a)f^2-2\,(bcdef-(bc^2+b)f^2+(bd^2ef-bcdf^2)x)\arctan(dx+c)+(bdf\cdots}{2\,(d^2e^3f-2\,cde^2f^2+(c^2+1)ef^3+(d^2e^2f^2-2\,cde\cdots}$$

input
```
integrate((a+b*arctan(d*x+c))/(f*x+e)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*d^2*e^2 - 4*a*c*d*e*f + 2*(a*c^2 + a)*f^2 - 2*(b*c*d*e*f - (b*c^
2 + b)*f^2 + (b*d^2*e*f - b*c*d*f^2)*x)*arctan(d*x + c) + (b*d*f^2*x + b*d
*e*f)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 2*(b*d*f^2*x + b*d*e*f)*log(f*x +
 e))/(d^2*e^3*f - 2*c*d*e^2*f^2 + (c^2 + 1)*e*f^3 + (d^2*e^2*f^2 - 2*c*d*e
*f^3 + (c^2 + 1)*f^4)*x)
```

3.29.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(c+dx)}{(e+fx)^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*atan(d*x+c))/(f*x+e)**2,x)
```

output
```
Timed out
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.17

$$\int \frac{a+b\arctan(c+dx)}{(e+fx)^2}\,dx = \frac{1}{2}\left(d\left(\frac{2\,(d^2e-cdf)\arctan\left(\frac{d^2x+cd}{d}\right)}{(d^2e^2f-2\,cdef^2+(c^2+1)f^3)d}-\frac{\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^2e^2-2\,cdef+(c^2+1)f^2}+\frac{2\,\log\left(fx+e\right)}{d^2e^2-2\,cdef+(c^2+1)f^2}\right)-\right. - \frac{a}{f^2x+ef}$$

input
```
integrate((a+b*arctan(d*x+c))/(f*x+e)^2,x, algorithm="maxima")
```

output
```
1/2*(d*(2*(d^2*e - c*d*f)*arctan((d^2*x + c*d)/d)/((d^2*e^2*f - 2*c*d*e*f^
2 + (c^2 + 1)*f^3)*d) - log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2 - 2*c*d*
e*f + (c^2 + 1)*f^2) + 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 + 1)*f^2
)) - 2*arctan(d*x + c)/(f^2*x + e*f))*b - a/(f^2*x + e*f)
```

3.29.8 Giac [F]

$$\int \frac{a+b\arctan(c+dx)}{(e+fx)^2}\,dx=\int \frac{b\arctan(dx+c)+a}{(fx+e)^2}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))/(f*x+e)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.29.9 Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.84

$$\int \frac{a+b\arctan(c+dx)}{(e+fx)^2}\,dx=\frac{b\,d\,\ln(e+f\,x)}{d^2\,e^2-2\,c\,d\,e\,f+(c^2+1)\,f^2}-\frac{b\,\mathrm{atan}(c+d\,x)}{f\,(e+f\,x)}-\frac{a}{x\,f^2+e\,f}-\frac{b\,d\,\ln(c+d\,x-\mathrm{i})\,1\mathrm{i}}{2\,f\,(d\,e-c\,f+f\,1\mathrm{i})}-\frac{b\,d\,\ln(c+d\,x+1\mathrm{i})}{2\,f\,(f-c\,f\,1\mathrm{i}+d\,e\,1\mathrm{i})}$$

input
```
int((a + b*atan(c + d*x))/(e + f*x)^2,x)
```

output
```
(b*d*log(e + f*x))/(f^2*(c^2 + 1) + d^2*e^2 - 2*c*d*e*f) - (b*atan(c + d*x
))/(f*(e + f*x)) - a/(e*f + f^2*x) - (b*d*log(c + d*x - 1i)*1i)/(2*f*(f*1i
 - c*f + d*e)) - (b*d*log(c + d*x + 1i))/(2*f*(f - c*f*1i + d*e*1i))
```

3.30 $\int \frac{a+b\arctan(c+dx)}{(e+fx)^3}\,dx$

3.30.1 Optimal result

Integrand size = 18, antiderivative size = 227

$$\begin{aligned}\int \frac{a+b\arctan(c+dx)}{(e+fx)^3}\,dx = &-\frac{bd}{2\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)(e+fx)}\\&+\frac{bd^2(de+f-cf)(de-(1+c)f)\arctan(c+dx)}{2f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)^2}\\&-\frac{a+b\arctan(c+dx)}{2f(e+fx)^2}+\frac{bd^2(de-cf)\log(e+fx)}{\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)^2}\\&-\frac{bd^2(de-cf)\log\left(1+c^2+2cdx+d^2x^2\right)}{2\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)^2}\end{aligned}$$

output
```
-1/2*b*d/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)/(f*x+e)+1/2*b*d^2*(-c*f+d*e+f)*(d
*e-(1+c)*f)*arctan(d*x+c)/f/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^2+1/2*(-a-b*ar
ctan(d*x+c))/f/(f*x+e)^2+b*d^2*(-c*f+d*e)*ln(f*x+e)/(d^2*e^2-2*c*d*e*f+(c^
2+1)*f^2)^2-1/2*b*d^2*(-c*f+d*e)*ln(d^2*x^2+2*c*d*x+c^2+1)/(d^2*e^2-2*c*d*
e*f+(c^2+1)*f^2)^2
```

3.30.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.77

$$\int \frac{a+b\arctan(c+dx)}{(e+fx)^3}\,dx$$
$$= \frac{-\frac{a+b\arctan(c+dx)}{(e+fx)^2}+\frac{1}{2}bd^2\left(-\frac{2f}{d(d^2e^2-2cdef+(1+c^2)f^2)(e+fx)}-\frac{i\log(i-c-dx)}{(de-(-i+c)f)^2}+\frac{i\log(i+c+dx)}{(de-(i+c)f)^2}-\frac{4f(-de+cf)\log(d(e+fx))}{(d^2e^2-2cdef+(1+c^2)f^2)^2}\right)}{2f}$$

input
```
Integrate[(a + b*ArcTan[c + d*x])/(e + f*x)^3,x]
```

output
```
(-((a + b*ArcTan[c + d*x])/(e + f*x)^2) + (b*d^2*((-2*f)/(d*(d^2*e^2 - 2*c
*d*e*f + (1 + c^2)*f^2)*(e + f*x)) - (I*Log[I - c - d*x])/(d*e - (-I + c)*
f)^2 + (I*Log[I + c + d*x])/(d*e - (I + c)*f)^2 - (4*f*(-(d*e) + c*f)*Log[
d*(e + f*x)])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2))/2)/(2*f)
```

3.30.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5568, 2081, 1145, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(c+dx)}{(e+fx)^3}\,dx$$
$$\Big\downarrow \text{ 5568}$$
$$\frac{bd\int \frac{1}{(e+fx)^2((c+dx)^2+1)}dx}{2f}-\frac{a+b\arctan(c+dx)}{2f(e+fx)^2}$$
$$\Big\downarrow \text{ 2081}$$
$$\frac{bd\int \frac{1}{(e+fx)^2(c^2+2dxc+d^2x^2+1)}dx}{2f}-\frac{a+b\arctan(c+dx)}{2f(e+fx)^2}$$
$$\Big\downarrow \text{ 1145}$$

$$\frac{bd\left(\frac{\int \frac{d(de-2cf-dfx)}{(e+fx)\left(c^2+2dxc+d^2x^2+1\right)}dx}{(c^2+1)f^2-2cdef+d^2e^2}-\frac{f}{(e+fx)((c^2+1)f^2-2cdef+d^2e^2)}\right)}{2f}-\frac{a+b\arctan(c+dx)}{2f(e+fx)^2}$$

$\big\downarrow$ 27

$$\frac{bd\left(\frac{d\int \frac{de-2cf-dfx}{(e+fx)\left(c^2+2dxc+d^2x^2+1\right)}dx}{(c^2+1)f^2-2cdef+d^2e^2}-\frac{f}{(e+fx)((c^2+1)f^2-2cdef+d^2e^2)}\right)}{2f}-\frac{a+b\arctan(c+dx)}{2f(e+fx)^2}$$

$\big\downarrow$ 1200

$$\frac{bd\left(\frac{d\int \left(\frac{2(de-cf)f^2}{\left(d^2e^2-2cdfe+\left(c^2+1\right)f^2\right)(e+fx)}+\frac{d\left(d^2e^2-4cdfe-\left(1-3c^2\right)f^2-2df(de-cf)x\right)}{\left(d^2e^2-2cdfe+\left(c^2+1\right)f^2\right)\left(c^2+2dxc+d^2x^2+1\right)}\right)dx}{(c^2+1)f^2-2cdef+d^2e^2}-\frac{f}{(e+fx)((c^2+1)f^2-2cdef+d^2e^2)}\right)}{2f}-\frac{a+b\arctan(c+dx)}{2f(e+fx)^2}$$

$\big\downarrow$ 2009

$$\frac{bd\left(\frac{d\left(\frac{\arctan(c+dx)(-cf+de+f)(de-(c+1)f)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}-\frac{f(de-cf)\log\left(c^2+2cdx+d^2x^2+1\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}+\frac{2f(de-cf)\log(e+fx)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}\right)}{(c^2+1)f^2-2cdef+d^2e^2}-\frac{f}{(e+fx)((c^2+1)f^2-2cdef+d^2e^2)}\right)}{2f}-\frac{a+b\arctan(c+dx)}{2f(e+fx)^2}$$

input
```
Int[(a + b*ArcTan[c + d*x])/(e + f*x)^3,x]
```

output
```
-1/2*(a + b*ArcTan[c + d*x])/(f*(e + f*x)^2) + (b*d*(-(f/((d^2*e^2 - 2*c*d
*e*f + (1 + c^2)*f^2)*(e + f*x))) + (d*(((d*e + f - c*f)*(d*e - (1 + c)*f)
*ArcTan[c + d*x])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (2*f*(d*e - c*f)
*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (f*(d*e - c*f)*Log[
1 + c^2 + 2*c*d*x + d^2*x^2])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)))/(d^2
*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)))/(2*f)
```

3.30. $\int \frac{a+b\arctan(c+dx)}{(e+fx)^3}\,dx$

3.30.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2081
```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

rule 5568
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcTan[
c + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[p, 0] && ILtQ[m, -1]
```

3.30.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.08

method	result
parts	$-\frac{a}{2(fx+e)^2 f}+\frac{b\left(-\frac{d^3\arctan(dx+c)}{2(f(dx+c)-cf+de)^2 f}+\frac{d^3\left(-\frac{f}{\left(c^2f^2-2cdef+d^2e^2+f^2\right)(f(dx+c)-cf+de)}-\frac{2(cf-de)f\ln(f(dx+c)-cf+}{\left(c^2f^2-2cdef+d^2e^2+f^2\right)}\right.}{2}\right.}{d}$
derivativedivides	$\frac{-\frac{a\,d^3}{2(cf-de-f(dx+c))^2 f}-b\,d^3\left(\frac{\arctan(dx+c)}{2(cf-de-f(dx+c))^2 f}-\frac{\frac{f}{\left(c^2f^2-2cdef+d^2e^2+f^2\right)(cf-de-f(dx+c))}-\frac{2f(cf-de)\ln(cf-de-f(dx}{\left(c^2f^2-2cdef+d^2e^2+f^2\right.}}{}\right.}{d}$
default	$\frac{-\frac{a\,d^3}{2(cf-de-f(dx+c))^2 f}-b\,d^3\left(\frac{\arctan(dx+c)}{2(cf-de-f(dx+c))^2 f}-\frac{\frac{f}{\left(c^2f^2-2cdef+d^2e^2+f^2\right)(cf-de-f(dx+c))}-\frac{2f(cf-de)\ln(cf-de-f(dx}{\left(c^2f^2-2cdef+d^2e^2+f^2\right.}}{}\right.}{d}$
parallelrisch	$-\frac{2x^2\arctan(dx+c)bc\,d^5e\,f^4-2x\arctan(dx+c)b\,c^2d^4e\,f^4+4x\arctan(dx+c)bc\,d^5e^2f^3+4\ln(fx+e)xbc\,d^4e\,f^4-2\ln(d^2x}{}$
risch	Expression too large to display

input
```
int((a+b*arctan(d*x+c))/(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/(f*x+e)^2/f+b/d*(-1/2*d^3/(f*(d*x+c)-c*f+d*e)^2/f*arctan(d*x+c)+1/2
*d^3/f*(-f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)/(f*(d*x+c)-c*f+d*e)-2*(c*f-d*e)
*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*ln(f*(d*x+c)-c*f+d*e)+1/(c^2*f^2-2*c*
d*e*f+d^2*e^2+f^2)^2*(1/2*(2*c*f^2-2*d*e*f)*ln(1+(d*x+c)^2)+(c^2*f^2-2*c*d
*e*f+d^2*e^2-f^2)*arctan(d*x+c))))
```

3.30.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 682 vs. 2(219) = 438.

Time = 1.22 (sec) , antiderivative size = 682, normalized size of antiderivative = 3.00

$$\int \frac{a+b\arctan(c+dx)}{(e+fx)^3}\,dx =$$
$$-\frac{ad^4e^4-(4\,ac-b)d^3e^3f+2\,(3\,ac^2-bc+a)d^2e^2f^2-(4\,ac^3-bc^2+4\,ac-b)def^3+(ac^4+2\,ac^2+a)f}{}$$

input
```
integrate((a+b*arctan(d*x+c))/(f*x+e)^3,x, algorithm="fricas")
```

output
```
-1/2*(a*d^4*e^4 - (4*a*c - b)*d^3*e^3*f + 2*(3*a*c^2 - b*c + a)*d^2*e^2*f^
2 - (4*a*c^3 - b*c^2 + 4*a*c - b)*d*e*f^3 + (a*c^4 + 2*a*c^2 + a)*f^4 + (b
*d^3*e^2*f^2 - 2*b*c*d^2*e*f^3 + (b*c^2 + b)*d*f^4)*x - (2*b*c*d^3*e^3*f -
 (5*b*c^2 + 3*b)*d^2*e^2*f^2 + 4*(b*c^3 + b*c)*d*e*f^3 - (b*c^4 + 2*b*c^2
+ b)*f^4 + (b*d^4*e^2*f^2 - 2*b*c*d^3*e*f^3 + (b*c^2 - b)*d^2*f^4)*x^2 + 2
*(b*d^4*e^3*f - 2*b*c*d^3*e^2*f^2 + (b*c^2 - b)*d^2*e*f^3)*x)*arctan(d*x +
 c) + (b*d^3*e^3*f - b*c*d^2*e^2*f^2 + (b*d^3*e*f^3 - b*c*d^2*f^4)*x^2 + 2
*(b*d^3*e^2*f^2 - b*c*d^2*e*f^3)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 2*(
b*d^3*e^3*f - b*c*d^2*e^2*f^2 + (b*d^3*e*f^3 - b*c*d^2*f^4)*x^2 + 2*(b*d^3
*e^2*f^2 - b*c*d^2*e*f^3)*x)*log(f*x + e))/(d^4*e^6*f - 4*c*d^3*e^5*f^2 +
2*(3*c^2 + 1)*d^2*e^4*f^3 - 4*(c^3 + c)*d*e^3*f^4 + (c^4 + 2*c^2 + 1)*e^2*
f^5 + (d^4*e^4*f^3 - 4*c*d^3*e^3*f^4 + 2*(3*c^2 + 1)*d^2*e^2*f^5 - 4*(c^3
+ c)*d*e*f^6 + (c^4 + 2*c^2 + 1)*f^7)*x^2 + 2*(d^4*e^5*f^2 - 4*c*d^3*e^4*f
^3 + 2*(3*c^2 + 1)*d^2*e^3*f^4 - 4*(c^3 + c)*d*e^2*f^5 + (c^4 + 2*c^2 + 1)
*e*f^6)*x)
```

3.30.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(c+dx)}{(e+fx)^3}\,dx=\text{Timed out}$$

input
```
integrate((a+b*atan(d*x+c))/(f*x+e)**3,x)
```

output
```
Timed out
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.80

$$\int \frac{a+b\arctan(c+dx)}{(e+fx)^3}\,dx=$$
$$-\frac{1}{2}\left(d\left(\frac{(d^2e-cdf)\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^4e^4-4\,cd^3e^3f+2\,(3\,c^2+1)d^2e^2f^2-4\,(c^3+c)def^3+(c^4+2\,c^2+1)\,f^4}-\frac{}{d^4e^4-4\,cd^3e^3f+2}\right.\right.$$
$$-\frac{a}{2\,(f^3x^2+2\,ef^2x+e^2f)}$$

input
```
integrate((a+b*arctan(d*x+c))/(f*x+e)^3,x, algorithm="maxima")
```

output
```
-1/2*(d*((d^2*e - c*d*f)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^4*e^4 - 4*c*d
^3*e^3*f + 2*(3*c^2 + 1)*d^2*e^2*f^2 - 4*(c^3 + c)*d*e*f^3 + (c^4 + 2*c^2
+ 1)*f^4) - 2*(d^2*e - c*d*f)*log(f*x + e)/(d^4*e^4 - 4*c*d^3*e^3*f + 2*(3
*c^2 + 1)*d^2*e^2*f^2 - 4*(c^3 + c)*d*e*f^3 + (c^4 + 2*c^2 + 1)*f^4) - (d^
4*e^2 - 2*c*d^3*e*f + (c^2 - 1)*d^2*f^2)*arctan((d^2*x + c*d)/d)/((d^4*e^4
*f - 4*c*d^3*e^3*f^2 + 2*(3*c^2 + 1)*d^2*e^2*f^3 - 4*(c^3 + c)*d*e*f^4 + (
c^4 + 2*c^2 + 1)*f^5)*d) + 1/(d^2*e^3 - 2*c*d*e^2*f + (c^2 + 1)*e*f^2 + (d
^2*e^2*f - 2*c*d*e*f^2 + (c^2 + 1)*f^3)*x)) + arctan(d*x + c)/(f^3*x^2 + 2
*e*f^2*x + e^2*f))*b - 1/2*a/(f^3*x^2 + 2*e*f^2*x + e^2*f)
```

3.30.8 Giac [F]

$$\int \frac{a+b\arctan(c+dx)}{(e+fx)^3}\,dx = \int \frac{b\arctan(dx+c)+a}{(fx+e)^3}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))/(f*x+e)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.30.9 Mupad [B] (verification not implemented)

Time = 8.02 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.76

$$\int \frac{a+b\arctan(c+dx)}{(e+fx)^3}\,dx = \frac{b\,d^3\,e\,\ln(e+f\,x)}{\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)^2} - \frac{a\,f}{2\,(e+f\,x)^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)} - \frac{b\,d\,e}{2\,(e+f\,x)^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)} - \frac{a\,c^2\,f}{2\,(e+f\,x)^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)} - \frac{b\,\mathrm{atan}(c+d\,x)}{2\,f\,(e+f\,x)^2} - \frac{b\,c\,d^2\,f\,\ln(e+f\,x)}{\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)^2} + \frac{a\,c\,d\,e}{(e+f\,x)^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)} - \frac{b\,d\,f\,x}{2\,(e+f\,x)^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)} - \frac{a\,d^2\,e^2}{2\,f\,(e+f\,x)^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)} - \frac{b\,d^2\,\ln(c+d\,x-\mathrm{i})\,1\mathrm{i}}{4\,f\,(d\,e-c\,f+f\,1\mathrm{i})^2} + \frac{b\,d^2\,\ln(c+d\,x+1\mathrm{i})\,1\mathrm{i}}{4\,f\,(c\,f-d\,e+f\,1\mathrm{i})^2}$$

input
```
int((a + b*atan(c + d*x))/(e + f*x)^3,x)
```

output
```
(b*d^2*log(c + d*x + 1i)*1i)/(4*f*(f*1i + c*f - d*e)^2) - (a*f)/(2*(e + f*
x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) - (b*d*e)/(2*(e + f*x)^2*(f^2
+ c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) - (b*d^2*log(c + d*x - 1i)*1i)/(4*f*(f*1
i - c*f + d*e)^2) - (b*atan(c + d*x))/(2*f*(e + f*x)^2) - (a*c^2*f)/(2*(e
+ f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) + (b*d^3*e*log(e + f*x))/(
f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)^2 - (b*c*d^2*f*log(e + f*x))/(f^2 + c
^2*f^2 + d^2*e^2 - 2*c*d*e*f)^2 + (a*c*d*e)/((e + f*x)^2*(f^2 + c^2*f^2 +
d^2*e^2 - 2*c*d*e*f)) - (b*d*f*x)/(2*(e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2
- 2*c*d*e*f)) - (a*d^2*e^2)/(2*f*(e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*
c*d*e*f))
```

3.31 $\int (e+fx)^2(a+b\arctan(c+dx))^2\,dx$

3.31.1 Optimal result

Integrand size = 20, antiderivative size = 382

$$\begin{aligned}
&\int (e+fx)^2(a+b\arctan(c+dx))^2\,dx \\
&= \frac{b^2f^2x}{3d^2}-\frac{2abf(de-cf)x}{d^2}-\frac{b^2f^2\arctan(c+dx)}{3d^3} \\
&\quad -\frac{2b^2f(de-cf)(c+dx)\arctan(c+dx)}{d^3}-\frac{bf^2(c+dx)^2(a+b\arctan(c+dx))}{3d^3} \\
&\quad +\frac{i(3d^2e^2-6cdef-(1-3c^2)f^2)(a+b\arctan(c+dx))^2}{3d^3} \\
&\quad -\frac{(de-cf)(d^2e^2-2cdef-(3-c^2)f^2)(a+b\arctan(c+dx))^2}{3d^3f} \\
&\quad +\frac{(e+fx)^3(a+b\arctan(c+dx))^2}{3f} \\
&\quad +\frac{2b(3d^2e^2-6cdef-(1-3c^2)f^2)(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{3d^3} \\
&\quad +\frac{b^2f(de-cf)\log(1+(c+dx)^2)}{d^3} \\
&\quad +\frac{ib^2(3d^2e^2-6cdef-(1-3c^2)f^2)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{3d^3}
\end{aligned}$$

output
```
1/3*b^2*f^2*x/d^2-2*a*b*f*(-c*f+d*e)*x/d^2-1/3*b^2*f^2*arctan(d*x+c)/d^3-2
*b^2*f*(-c*f+d*e)*(d*x+c)*arctan(d*x+c)/d^3-1/3*b*f^2*(d*x+c)^2*(a+b*arcta
n(d*x+c))/d^3+1/3*I*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*arctan(d*x+c
))^2/d^3-1/3*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f-(-c^2+3)*f^2)*(a+b*arctan(d*x+c
))^2/d^3/f+1/3*(f*x+e)^3*(a+b*arctan(d*x+c))^2/f+2/3*b*(3*d^2*e^2-6*c*d*e*
f-(-3*c^2+1)*f^2)*(a+b*arctan(d*x+c))*ln(2/(1+I*(d*x+c)))/d^3+b^2*f*(-c*f+
d*e)*ln(1+(d*x+c)^2)/d^3+1/3*I*b^2*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*po
lylog(2,1-2/(1+I*(d*x+c)))/d^3
```

3.31.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 801 vs. 2(382) = 764.

Time = 7.96 (sec) , antiderivative size = 801, normalized size of antiderivative = 2.10

$$\int (e+fx)^2(a+b\arctan(c+dx))^2\,dx = a^2e^2x+a^2efx^2+\frac{1}{3}a^2f^2x^3$$
$$+\frac{ab(-dfx(6de-4cf+dfx)+2(3def-3c^2def+c^3f^2+3c(d^2e^2-f^2)+d^3x(3e^2+3efx+f^2x^2))\,\text{arct}}{3d^3}$$
$$+\frac{b^2e^2\left(\arctan(c+dx)\left((-i+c+dx)\arctan(c+dx)+2\log\left(1+e^{2i\arctan(c+dx)}\right)\right)-i\,\text{PolyLog}\left(2,-e^{2i\,\text{arct}}\right.}{d}$$
$$+\frac{b^2ef\left((1+2ic-c^2+d^2x^2)\arctan(c+dx)^2-2\arctan(c+dx)\left(c+dx+2c\log\left(1+e^{2i\arctan(c+dx)}\right)\right)+\right.}{d^2}$$
$$+\frac{b^2f^2(1+(c+dx)^2)^{3/2}\left(\frac{c+dx}{\sqrt{1+(c+dx)^2}}+\frac{6c(c+dx)\arctan(c+dx)}{\sqrt{1+(c+dx)^2}}+\frac{3(c+dx)\arctan(c+dx)^2}{\sqrt{1+(c+dx)^2}}+\frac{3c^2(c+dx)\arctan(c+dx)^2}{\sqrt{1+(c+dx)^2}}+i\right.}{}$$

input
```
Integrate[(e + f*x)^2*(a + b*ArcTan[c + d*x])^2,x]
```

output
```
a^2*e^2*x + a^2*e*f*x^2 + (a^2*f^2*x^3)/3 + (a*b*(-(d*f*x*(6*d*e - 4*c*f +
 d*f*x)) + 2*(3*d*e*f - 3*c^2*d*e*f + c^3*f^2 + 3*c*(d^2*e^2 - f^2) + d^3*
x*(3*e^2 + 3*e*f*x + f^2*x^2))*ArcTan[c + d*x] + (-3*d^2*e^2 + 6*c*d*e*f +
 (1 - 3*c^2)*f^2)*Log[1 + (c + d*x)^2]))/(3*d^3) + (b^2*e^2*(ArcTan[c + d*
x]*((-I + c + d*x)*ArcTan[c + d*x] + 2*Log[1 + E^((2*I)*ArcTan[c + d*x])])
 - I*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]))/d + (b^2*e*f*((1 + (2*I)*c -
 c^2 + d^2*x^2)*ArcTan[c + d*x]^2 - 2*ArcTan[c + d*x]*(c + d*x + 2*c*Log[1
 + E^((2*I)*ArcTan[c + d*x])]) + Log[1 + (c + d*x)^2] + (2*I)*c*PolyLog[2,
 -E^((2*I)*ArcTan[c + d*x])]))/d^2 + (b^2*f^2*(1 + (c + d*x)^2)^(3/2)*((c
+ d*x)/Sqrt[1 + (c + d*x)^2] + (6*c*(c + d*x)*ArcTan[c + d*x])/Sqrt[1 + (c
 + d*x)^2] + (3*(c + d*x)*ArcTan[c + d*x]^2)/Sqrt[1 + (c + d*x)^2] + (3*c^
2*(c + d*x)*ArcTan[c + d*x]^2)/Sqrt[1 + (c + d*x)^2] + I*ArcTan[c + d*x]^2
*Cos[3*ArcTan[c + d*x]] - (3*I)*c^2*ArcTan[c + d*x]^2*Cos[3*ArcTan[c + d*x
]] - 2*ArcTan[c + d*x]*Cos[3*ArcTan[c + d*x]]*Log[1 + E^((2*I)*ArcTan[c +
d*x])] + 6*c^2*ArcTan[c + d*x]*Cos[3*ArcTan[c + d*x]]*Log[1 + E^((2*I)*Arc
Tan[c + d*x])] + 6*c*Cos[3*ArcTan[c + d*x]]*Log[1/Sqrt[1 + (c + d*x)^2]] +
 ((3*I - 12*c - (9*I)*c^2)*ArcTan[c + d*x]^2 + 2*ArcTan[c + d*x]*(-2 + (-3
 + 9*c^2)*Log[1 + E^((2*I)*ArcTan[c + d*x])]) + 18*c*Log[1/Sqrt[1 + (c + d
*x)^2]])/Sqrt[1 + (c + d*x)^2] - ((4*I)*(-1 + 3*c^2)*PolyLog[2, -E^((2*I)*
ArcTan[c + d*x])])/(1 + (c + d*x)^2)^(3/2) + Sin[3*ArcTan[c + d*x]] + 6...
```

3.31.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5570, 27, 5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2(a+b\arctan(c+dx))^2\,dx$$

$$\Big\downarrow \ 5570$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^2(a+b\arctan(c+dx))^2}{d^2}d(c+dx)}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{\int (de-cf+f(c+dx))^2(a+b\arctan(c+dx))^2d(c+dx)}{d^3}$$

$$\Big\downarrow \ 5389$$

$$\frac{\frac{(f(c+dx)-cf+de)^3(a+b\arctan(c+dx))^2}{3f}-\frac{2b\int\left((c+dx)(a+b\arctan(c+dx))f^3+3(de-cf)(a+b\arctan(c+dx))f^2+\frac{\left((de-cf)\left(d^2e^2-2cdfe-\left(3-c^2\right.\right.\right.}{3f}}{d^3}$$

$\downarrow$ 2009

$$\frac{\frac{(f(c+dx)-cf+de)^3(a+b\arctan(c+dx))^2}{3f}-\frac{2b\left(-\frac{if\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)(a+b\arctan(c+dx))^2}{2b}+\frac{(de-cf)\left(-\left(3-c^2\right)f^2-2cdef+d^2e^2\right)(a+b\arctan\right.}{2b}}{}}{}$$

input
```
Int[(e + f*x)^2*(a + b*ArcTan[c + d*x])^2,x]
```

output
```
(((d*e - c*f + f*(c + d*x))^3*(a + b*ArcTan[c + d*x])^2)/(3*f) - (2*b*(-1/
2*(b*f^3*(c + d*x)) + 3*a*f^2*(d*e - c*f)*(c + d*x) + (b*f^3*ArcTan[c + d*
x])/2 + 3*b*f^2*(d*e - c*f)*(c + d*x)*ArcTan[c + d*x] + (f^3*(c + d*x)^2*(
a + b*ArcTan[c + d*x]))/2 - ((I/2)*f*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*
f^2)*(a + b*ArcTan[c + d*x])^2)/b + ((d*e - c*f)*(d^2*e^2 - 2*c*d*e*f - (3
 - c^2)*f^2)*(a + b*ArcTan[c + d*x])^2)/(2*b) - f*(3*d^2*e^2 - 6*c*d*e*f -
 (1 - 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))] - (3*b*
f^2*(d*e - c*f)*Log[1 + (c + d*x)^2])/2 - (I/2)*b*f*(3*d^2*e^2 - 6*c*d*e*f
 - (1 - 3*c^2)*f^2)*PolyLog[2, 1 - 2/(1 + I*(c + d*x))]))/(3*f))/d^3
```

3.31.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 5570
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.31.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1017 vs. $2(362) = 724$.

Time = 0.67 (sec) , antiderivative size = 1018, normalized size of antiderivative = 2.66

method	result	size
parts	Expression too large to display	1018
derivativedivides	Expression too large to display	1042
default	Expression too large to display	1042
risch	Expression too large to display	2416

input
```
int((f*x+e)^2*(a+b*arctan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*(f*x+e)^3/f+b^2/d*(1/3/d^2*f^2*arctan(d*x+c)^2*(d*x+c)^3-1/d^2*f^2
*arctan(d*x+c)^2*(d*x+c)^2*c+1/d*f*arctan(d*x+c)^2*(d*x+c)^2*e+1/d^2*f^2*a
rctan(d*x+c)^2*(d*x+c)*c^2-2/d*f*arctan(d*x+c)^2*(d*x+c)*c*e+arctan(d*x+c)
^2*(d*x+c)*e^2-1/3/d^2*f^2*arctan(d*x+c)^2*c^3+1/d*f*arctan(d*x+c)^2*c^2*e
-arctan(d*x+c)^2*c*e^2+1/3*d/f*arctan(d*x+c)^2*e^3-2/3/d^2/f*(1/2*arctan(d
*x+c)*f^3*(d*x+c)^2-3*arctan(d*x+c)*c*f^3*(d*x+c)+3*arctan(d*x+c)*d*e*f^2*
(d*x+c)+3/2*arctan(d*x+c)*ln(1+(d*x+c)^2)*c^2*f^3-3*arctan(d*x+c)*ln(1+(d*
x+c)^2)*c*d*e*f^2+3/2*arctan(d*x+c)*ln(1+(d*x+c)^2)*d^2*e^2*f-1/2*arctan(d
*x+c)*ln(1+(d*x+c)^2)*f^3-arctan(d*x+c)^2*c^3*f^3+3*arctan(d*x+c)^2*c^2*d*
e*f^2-3*arctan(d*x+c)^2*c*d^2*e^2*f+arctan(d*x+c)^2*d^3*e^3+3*arctan(d*x+c
)^2*c*f^3-3*arctan(d*x+c)^2*d*e*f^2-1/2*f^2*(f*(d*x+c)+1/2*(-6*c*f+6*d*e)*
ln(1+(d*x+c)^2)-f*arctan(d*x+c))-1/2*f*(3*c^2*f^2-6*c*d*e*f+3*d^2*e^2-f^2)
*(-1/2*I*(ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog(-1/2*I*(d*x+
c+I))-ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I)))+1/2*I*(ln(d*x+c+I)*ln(1+(d*x+c)^2)
-1/2*ln(d*x+c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I*(d*x+c-I)))
)-1/4*(-2*c^3*f^3+6*c^2*d*e*f^2-6*c*d^2*e^2*f+2*d^3*e^3+6*c*f^3-6*d*e*f^2)
*arctan(d*x+c)^2))+1/3*a*b/d^3*f^2*ln(1+(d*x+c)^2)+2/d*a*b*c*e^2*arctan(d*
x+c)+2/3/d^3*f^2*b*a*c^3*arctan(d*x+c)+4/3/d^2*c*f^2*x*b*a-2/d*e*x*f*b*a-a
*b/d^3*f^2*ln(1+(d*x+c)^2)*c^2-a*b/d*ln(1+(d*x+c)^2)*e^2-2/d^3*c*f^2*b*a*a
rctan(d*x+c)-1/3/d*f^2*b*a*x^2-2*b/d^2*arctan(d*x+c)*a*c^2*e*f+2*a*b/d^...
```

3.31.5 Fricas [F]

$$\int (e+fx)^2(a+b\arctan(c+dx))^2\,dx=\int (fx+e)^2(b\arctan(dx+c)+a)^2\,dx$$

input
```
integrate((f*x+e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(a^2*f^2*x^2 + 2*a^2*e*f*x + a^2*e^2 + (b^2*f^2*x^2 + 2*b^2*e*f*x
+ b^2*e^2)*arctan(d*x + c)^2 + 2*(a*b*f^2*x^2 + 2*a*b*e*f*x + a*b*e^2)*arc
tan(d*x + c), x)
```

3.31.6 Sympy [F(-1)]

Timed out.

$$\int (e+fx)^2(a+b\arctan(c+dx))^2\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**2*(a+b*atan(d*x+c))**2,x)
```

output
```
Timed out
```

3.31.7 Maxima [F]

$$\int (e+fx)^2(a+b\arctan(c+dx))^2\,dx=\int (fx+e)^2(b\arctan(dx+c)+a)^2\,dx$$

input
```
integrate((f*x+e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

output
```
3/4*b^2*c^2*e^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 1/4*(3*arcta
n(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*b^2*
c^2*e^2 + 1/3*a^2*f^2*x^3 + 36*b^2*d^2*f^2*integrate(1/48*x^4*arctan(d*x +
 c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*d^2*f^2*integrate(1/48*x^4
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 72
*b^2*d^2*e*f*integrate(1/48*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2
 + 1), x) + 72*b^2*c*d*f^2*integrate(1/48*x^3*arctan(d*x + c)^2/(d^2*x^2 +
 2*c*d*x + c^2 + 1), x) + 4*b^2*d^2*f^2*integrate(1/48*x^4*log(d^2*x^2 + 2
*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^2*d^2*e*f*integr
ate(1/48*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 +
 1), x) + 6*b^2*c*d*f^2*integrate(1/48*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1
)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 36*b^2*d^2*e^2*integrate(1/48*x^2*
arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 144*b^2*c*d*e*f*inte
grate(1/48*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 36*b^
2*c^2*f^2*integrate(1/48*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 +
1), x) + 12*b^2*d^2*e*f*integrate(1/48*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1
)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 4*b^2*c*d*f^2*integrate(1/48*x^3*log
(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*d^
2*e^2*integrate(1/48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c
*d*x + c^2 + 1), x) + 12*b^2*c*d*e*f*integrate(1/48*x^2*log(d^2*x^2 + 2...
```

3.31.8 Giac [F]

$$\int (e+fx)^2(a+b\arctan(c+dx))^2\,dx=\int (fx+e)^2(b\arctan(dx+c)+a)^2\,dx$$

input
```
integrate((f*x+e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2(a+b\arctan(c+dx))^2\,dx = \int (e+f\,x)^2\,(a+b\,\mathrm{atan}(c+d\,x))^2\,dx$$

input
```
int((e + f*x)^2*(a + b*atan(c + d*x))^2,x)
```

output
```
int((e + f*x)^2*(a + b*atan(c + d*x))^2, x)
```

3.32 $\int (e+fx)(a+b\arctan(c+dx))^2\,dx$

3.32.1 Optimal result

Integrand size = 18, antiderivative size = 222

$$\begin{aligned}\int (e+fx)(a+b\arctan(c+dx))^2\,dx &= -\frac{abfx}{d}-\frac{b^2f(c+dx)\arctan(c+dx)}{d^2}+\frac{i(de-cf)(a+b\arctan(c+dx))^2}{d^2}\\ &\quad-\frac{(de+f-cf)(de-(1+c)f)(a+b\arctan(c+dx))^2}{2d^2f}\\ &\quad+\frac{(e+fx)^2(a+b\arctan(c+dx))^2}{2f}+\frac{2b(de-cf)(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2}\\ &\quad+\frac{b^2f\log\left(1+(c+dx)^2\right)}{2d^2}+\frac{ib^2(de-cf)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2}\end{aligned}$$

output
```
-a*b*f*x/d-b^2*f*(d*x+c)*arctan(d*x+c)/d^2+I*(-c*f+d*e)*(a+b*arctan(d*x+c)
)^2/d^2-1/2*(-c*f+d*e+f)*(d*e-(1+c)*f)*(a+b*arctan(d*x+c))^2/d^2/f+1/2*(f*
x+e)^2*(a+b*arctan(d*x+c))^2/f+2*b*(-c*f+d*e)*(a+b*arctan(d*x+c))*ln(2/(1+
I*(d*x+c)))/d^2+1/2*b^2*f*ln(1+(d*x+c)^2)/d^2+I*b^2*(-c*f+d*e)*polylog(2,1
-2/(1+I*(d*x+c)))/d^2
```

3.32.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.19

$$\int (e+fx)(a+b\arctan(c+dx))^2\,dx$$
$$=\frac{2a^2cde-2abcf-a^2c^2f+2a^2d^2ex-2abdfx+a^2d^2fx^2+b^2(-i+c+dx)(2de+if-cf+dfx)\arctan(}{}$$

input
```
Integrate[(e + f*x)*(a + b*ArcTan[c + d*x])^2,x]
```

output
```
(2*a^2*c*d*e - 2*a*b*c*f - a^2*c^2*f + 2*a^2*d^2*e*x - 2*a*b*d*f*x + a^2*d
^2*f*x^2 + b^2*(-I + c + d*x)*(2*d*e + I*f - c*f + d*f*x)*ArcTan[c + d*x]^
2 - 2*b*ArcTan[c + d*x]*(b*f*(c + d*x) + a*(-2*c*d*e + c^2*f - 2*d^2*e*x -
 f*(1 + d^2*x^2)) - 2*b*(d*e - c*f)*Log[1 + E^((2*I)*ArcTan[c + d*x])]) +
4*a*b*d*e*Log[1/Sqrt[1 + (c + d*x)^2]] - 2*b^2*f*Log[1/Sqrt[1 + (c + d*x)^
2]] - 4*a*b*c*f*Log[1/Sqrt[1 + (c + d*x)^2]] - (2*I)*b^2*(d*e - c*f)*PolyL
og[2, -E^((2*I)*ArcTan[c + d*x])])/(2*d^2)
```

3.32.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5570, 27, 5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)(a+b\arctan(c+dx))^2\,dx$$
$$\Big\downarrow \ 5570$$
$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)(a+b\arctan(c+dx))^2}{d}d(c+dx)}{d}$$
$$\Big\downarrow \ 27$$
$$\frac{\int (de-cf+f(c+dx))(a+b\arctan(c+dx))^2d(c+dx)}{d^2}$$
$$\Big\downarrow \ 5389$$

$$\frac{\frac{(f(c+dx)-cf+de)^2(a+b\arctan(c+dx))^2}{2f}-\frac{b\int\left((a+b\arctan(c+dx))f^2+\frac{((de-cf+f)(de-(c+1)f)+2f(de-cf)(c+dx))(a+b\arctan(c+dx))}{(c+dx)^2+1}\right)d(c+dx)}{f}}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(f(c+dx)-cf+de)^2(a+b\arctan(c+dx))^2}{2f}-\frac{b\left(-\frac{if(de-cf)(a+b\arctan(c+dx))^2}{b}+\frac{(-cf+de+f)(de-(c+1)f)(a+b\arctan(c+dx))^2}{2b}-2f(de-cf)\log\left(\frac{2}{1+i(c}\right.\right.}{}}{d^2}$$

input
```
Int[(e + f*x)*(a + b*ArcTan[c + d*x])^2,x]
```

output
```
(((d*e - c*f + f*(c + d*x))^2*(a + b*ArcTan[c + d*x])^2)/(2*f) - (b*(a*f^2
*(c + d*x) + b*f^2*(c + d*x)*ArcTan[c + d*x] - (I*f*(d*e - c*f)*(a + b*Arc
Tan[c + d*x])^2)/b + ((d*e + f - c*f)*(d*e - (1 + c)*f)*(a + b*ArcTan[c +
d*x])^2)/(2*b) - 2*f*(d*e - c*f)*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c +
 d*x))] - (b*f^2*Log[1 + (c + d*x)^2])/2 - I*b*f*(d*e - c*f)*PolyLog[2, 1
- 2/(1 + I*(c + d*x))]))/f)/d^2
```

3.32.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 5570
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.32.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.86

method	result
parts	$a^2\left(\frac{1}{2} f x^2 + e x\right) + \frac{b^2\left(\frac{\arctan(dx+c)^2 (dx+c)^2 f}{2d} - \frac{\arctan(dx+c)^2 c f (dx+c)}{d} + \arctan(dx+c)^2 e (dx+c) - \frac{-\ln\left(1+(dx+c)^2\right)}{}\right.}{}$
derivativedivides	$-\frac{a^2\left(fc(dx+c) - ed(dx+c) - \frac{f(dx+c)^2}{2}\right)}{d} - \frac{b^2\left(\arctan(dx+c)^2 fc(dx+c) - \arctan(dx+c)^2 ed(dx+c) - \frac{\arctan(dx+c)^2 f(dx+c)^2}{2} - \ln\left(1+\right.\right.}{}$
default	$-\frac{a^2\left(fc(dx+c) - ed(dx+c) - \frac{f(dx+c)^2}{2}\right)}{d} - \frac{b^2\left(\arctan(dx+c)^2 fc(dx+c) - \arctan(dx+c)^2 ed(dx+c) - \frac{\arctan(dx+c)^2 f(dx+c)^2}{2} - \ln\left(1+\right.\right.}{}$
risch	Expression too large to display

input
```
int((f*x+e)*(a+b*arctan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/2*f*x^2+e*x)+b^2/d*(1/2/d*arctan(d*x+c)^2*(d*x+c)^2*f-1/d*arctan(d*
x+c)^2*c*f*(d*x+c)+arctan(d*x+c)^2*e*(d*x+c)-1/d*(-ln(1+(d*x+c)^2)*arctan(
d*x+c)*c*f+ln(1+(d*x+c)^2)*arctan(d*x+c)*d*e-1/2*arctan(d*x+c)^2*f+arctan(
d*x+c)*(d*x+c)*f-1/2*f*ln(1+(d*x+c)^2)-1/2*(-2*c*f+2*d*e)*(-1/2*I*(ln(d*x+
c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog(-1/2*I*(d*x+c+I))-ln(d*x+c-I)
*ln(-1/2*I*(d*x+c+I)))+1/2*I*(ln(d*x+c+I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c+I)^
2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))))))+2*a*b/d*(1/2/
d*arctan(d*x+c)*(d*x+c)^2*f-1/d*arctan(d*x+c)*c*f*(d*x+c)+arctan(d*x+c)*e*
(d*x+c)-1/2/d*(f*(d*x+c)+1/2*(-2*c*f+2*d*e)*ln(1+(d*x+c)^2)-f*arctan(d*x+c
)))
```

3.32.5 Fricas [F]

$$\int (e+fx)(a+b\arctan(c+dx))^2\,dx=\int (fx+e)(b\arctan(dx+c)+a)^2\,dx$$

input
```
integrate((f*x+e)*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(a^2*f*x + a^2*e + (b^2*f*x + b^2*e)*arctan(d*x + c)^2 + 2*(a*b*f*
x + a*b*e)*arctan(d*x + c), x)
```

3.32.6 Sympy [F]

$$\int (e+fx)(a+b\arctan(c+dx))^2\,dx=\int (a+b\,\mathrm{atan}\,(c+dx))^2\,(e+fx)\,dx$$

input
```
integrate((f*x+e)*(a+b*atan(d*x+c))**2,x)
```

output
```
Integral((a + b*atan(c + d*x))**2*(e + f*x), x)
```

3.32.7 Maxima [F]

$$\int (e+fx)(a+b\arctan(c+dx))^2\,dx=\int (fx+e)(b\arctan(dx+c)+a)^2\,dx$$

input
```
integrate((f*x+e)*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

output
```
3/4*b^2*c^2*e*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 1/4*(3*arctan(
d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*b^2*c^
2*e + 12*b^2*d^2*f*integrate(1/16*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x
 + c^2 + 1), x) + b^2*d^2*f*integrate(1/16*x^3*log(d^2*x^2 + 2*c*d*x + c^2
 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*d^2*e*integrate(1/16*x^
2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^2*c*d*f*integ
rate(1/16*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 2*b^2*
d^2*f*integrate(1/16*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d
*x + c^2 + 1), x) + b^2*d^2*e*integrate(1/16*x^2*log(d^2*x^2 + 2*c*d*x + c
^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 2*b^2*c*d*f*integrate(1/16*x
^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) +
24*b^2*c*d*e*integrate(1/16*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 +
 1), x) + 12*b^2*c^2*f*integrate(1/16*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d
*x + c^2 + 1), x) + 4*b^2*d^2*e*integrate(1/16*x^2*log(d^2*x^2 + 2*c*d*x +
 c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 2*b^2*c*d*f*integrate(1/16*x
^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 2*
b^2*c*d*e*integrate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2
*c*d*x + c^2 + 1), x) + b^2*c^2*f*integrate(1/16*x*log(d^2*x^2 + 2*c*d*x +
 c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 4*b^2*c*d*e*integrate(1/16
*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) +...
```

3.32.8 Giac [F]

$$\int (e+fx)(a+b\arctan(c+dx))^2\,dx=\int (fx+e)(b\arctan(dx+c)+a)^2\,dx$$

input
```
integrate((f*x+e)*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)(a+b\arctan(c+dx))^2\,dx = \int (e+f\,x)\,\left(a+b\,\mathrm{atan}(c+d\,x)\right)^2\,dx$$

input
```
int((e + f*x)*(a + b*atan(c + d*x))^2,x)
```

output
```
int((e + f*x)*(a + b*atan(c + d*x))^2, x)
```

3.33 $\int (a+b\arctan(c+dx))^2\,dx$

3.33.1 Optimal result

Integrand size = 12, antiderivative size = 102

$$\int (a+b\arctan(c+dx))^2\,dx=\frac{i(a+b\arctan(c+dx))^2}{d}+\frac{(c+dx)(a+b\arctan(c+dx))^2}{d}+\frac{2b(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d}+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d}$$

output
```
I*(a+b*arctan(d*x+c))^2/d+(d*x+c)*(a+b*arctan(d*x+c))^2/d+2*b*(a+b*arctan(
d*x+c))*ln(2/(1+I*(d*x+c)))/d+I*b^2*polylog(2,1-2/(1+I*(d*x+c)))/d
```

3.33.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.07

$$\int (a+b\arctan(c+dx))^2\,dx$$
$$=\frac{b^2(-i+c+dx)\arctan(c+dx)^2+2b\arctan(c+dx)\left(ac+adx+b\log\left(1+e^{2i\arctan(c+dx)}\right)\right)+a\Big(ac+adx}{d}$$

input
```
Integrate[(a + b*ArcTan[c + d*x])^2,x]
```

output
```
(b^2*(-I + c + d*x)*ArcTan[c + d*x]^2 + 2*b*ArcTan[c + d*x]*(a*c + a*d*x +
 b*Log[1 + E^((2*I)*ArcTan[c + d*x])]) + a*(a*c + a*d*x + 2*b*Log[1/Sqrt[1
 + (c + d*x)^2]]) - I*b^2*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])])/d
```

3.33.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5562, 5345, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arctan(c+dx))^2\,dx$$

$$\Big\downarrow \text{5562}$$

$$\frac{\int (a+b\arctan(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{5345}$$

$$\frac{(c+dx)(a+b\arctan(c+dx))^2-2b\int \frac{(c+dx)(a+b\arctan(c+dx))}{(c+dx)^2+1}d(c+dx)}{d}$$

$$\Big\downarrow \text{5455}$$

$$\frac{(c+dx)(a+b\arctan(c+dx))^2-2b\left(-\int \frac{a+b\arctan(c+dx)}{-c-dx+i}d(c+dx)-\frac{i(a+b\arctan(c+dx))^2}{2b}\right)}{d}$$

$$\Big\downarrow \text{5379}$$

$$\frac{(c+dx)(a+b\arctan(c+dx))^2-2b\left(b\int \frac{\log\left(\frac{2}{i(c+dx)+1}\right)}{(c+dx)^2+1}d(c+dx)-\frac{i(a+b\arctan(c+dx))^2}{2b}-\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\arctan$$

$$\Big\downarrow \text{2849}$$

$$\frac{(c+dx)(a+b\arctan(c+dx))^2-2b\left(-ib\int \frac{\log\left(\frac{2}{i(c+dx)+1}\right)}{1-\frac{2}{i(c+dx)+1}}d\frac{1}{i(c+dx)+1}-\frac{i(a+b\arctan(c+dx))^2}{2b}-\log\left(\frac{2}{1+i(c+dx)}\right)(a+b$$

$$\Big\downarrow \text{2752}$$

$$\frac{(c+dx)(a+b\arctan(c+dx))^2-2b\left(-\frac{i(a+b\arctan(c+dx))^2}{2b}-\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\arctan(c+dx))-\frac{1}{2}ib\,\mathrm{PolyLog}\right.}{d}$$

input
```
Int[(a + b*ArcTan[c + d*x])^2,x]
```

output
```
((c + d*x)*(a + b*ArcTan[c + d*x])^2 - 2*b*(((-1/2*I)*(a + b*ArcTan[c + d*
x])^2)/b - (a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))] - (I/2)*b*Poly
Log[2, 1 - 2/(1 + I*(c + d*x))]))/d
```

3.33.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5562
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
 x] && IGtQ[p, 0]
```

3.33.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.36

method	result
parts	$a^2x+\frac{b^2\left(\arctan(dx+c)^2(dx+c+i)+2\arctan(dx+c)\ln\left(1+\frac{(1+i(dx+c))^2}{1+(dx+c)^2}\right)-2i\arctan(dx+c)^2-i\,\mathrm{polylog}\left(2,-\frac{(1+i(dx}{1+(dx}\right.\right.}{d}$
derivativedivides	$\frac{(dx+c)a^2-i\arctan(dx+c)^2b^2+\arctan(dx+c)^2b^2(dx+c)-i\,\mathrm{polylog}\left(2,-\frac{(1+i(dx+c))^2}{1+(dx+c)^2}\right)b^2+2\arctan(dx+c)\ln\left(1+\frac{(1+i(}{1+(d}\right.}{d}$
default	$\frac{(dx+c)a^2-i\arctan(dx+c)^2b^2+\arctan(dx+c)^2b^2(dx+c)-i\,\mathrm{polylog}\left(2,-\frac{(1+i(dx+c))^2}{1+(dx+c)^2}\right)b^2+2\arctan(dx+c)\ln\left(1+\frac{(1+i(}{1+(d}\right.}{d}$
risch	$-\frac{b^2(dx+c-i)\ln(1+i(dx+c))^2}{4d}-\frac{\ln(-idx-ic+1)^2b^2c}{4d}-\frac{\ln(-idx-ic+1)ab}{d}+\left(\frac{b^2x\ln(1-i(dx+c))}{2}-\frac{ib(2axd-b}{}\right.$

input
```
int((a+b*arctan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+b^2/d*(arctan(d*x+c)^2*(d*x+c+I)+2*arctan(d*x+c)*ln(1+(1+I*(d*x+c))^
2/(1+(d*x+c)^2))-2*I*arctan(d*x+c)^2-I*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+
c)^2)))+2*a*b/d*((d*x+c)*arctan(d*x+c)-1/2*ln(1+(d*x+c)^2))
```

3.33.5 Fricas [F]

$$\int (a+b\arctan(c+dx))^2\,dx=\int (b\arctan(dx+c)+a)^2\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2, x)
```

3.33.6 Sympy [F]

$$\int (a+b\arctan(c+dx))^2\,dx = \int (a+b\operatorname{atan}(c+dx))^2\,dx$$

input
```
integrate((a+b*atan(d*x+c))**2,x)
```

output
```
Integral((a + b*atan(c + d*x))**2, x)
```

3.33.7 Maxima [F]

$$\int (a+b\arctan(c+dx))^2\,dx = \int (b\arctan(dx+c)+a)^2\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/16*(12*c^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 4*(3*arctan(d*x
 + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*c^2 + 4*x
*arctan(d*x + c)^2 + 192*d^2*integrate(1/16*x^2*arctan(d*x + c)^2/(d^2*x^2
 + 2*c*d*x + c^2 + 1), x) + 16*d^2*integrate(1/16*x^2*log(d^2*x^2 + 2*c*d*
x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*c*d*integrate(1/16*
x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 64*d^2*integrate(1
/16*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
 + 32*c*d*integrate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2
*c*d*x + c^2 + 1), x) + 64*c*d*integrate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^
2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 16*c^2*integrate(1/16*log(d^2*x
^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - x*log(d^2*x^
2 + 2*c*d*x + c^2 + 1)^2 + 12*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d
- 12*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d + 4*arctan((d^2*x + c*d)/
d)^3/d - 128*d*integrate(1/16*x*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 +
 1), x) + 16*integrate(1/16*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 +
2*c*d*x + c^2 + 1), x))*b^2 + a^2*x + (2*(d*x + c)*arctan(d*x + c) - log((
d*x + c)^2 + 1))*a*b/d
```

3.33.8 Giac [F]

$$\int (a+b\arctan(c+dx))^2\,dx=\int (b\arctan(dx+c)+a)^2\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\arctan(c+dx))^2\,dx=\int (a+b\,\mathrm{atan}(c+d\,x))^2\,dx$$

input
```
int((a + b*atan(c + d*x))^2,x)
```

output
```
int((a + b*atan(c + d*x))^2, x)
```

3.34 $\int \frac{(a+b\arctan(c+dx))^2}{e+fx}\,dx$

3.34.1 Optimal result

Integrand size = 20, antiderivative size = 261

$$\begin{aligned}\int \frac{(a+b\arctan(c+dx))^2}{e+fx}\,dx &= -\frac{(a+b\arctan(c+dx))^2\log\left(\frac{2}{1-i(c+dx)}\right)}{f} \\ &+ \frac{(a+b\arctan(c+dx))^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} \\ &+ \frac{ib(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{f} \\ &- \frac{ib(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} \\ &- \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2f} + \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f}\end{aligned}$$

output
```
-(a+b*arctan(d*x+c))^2*ln(2/(1-I*(d*x+c)))/f+(a+b*arctan(d*x+c))^2*ln(2*d*
(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f+I*b*(a+b*arctan(d*x+c))*polylog(2,1
-2/(1-I*(d*x+c)))/f-I*b*(a+b*arctan(d*x+c))*polylog(2,1-2*d*(f*x+e)/(d*e+I
*f-c*f)/(1-I*(d*x+c)))/f-1/2*b^2*polylog(3,1-2/(1-I*(d*x+c)))/f+1/2*b^2*po
lylog(3,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f
```

3.34.2 Mathematica [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{e+fx}\,dx=\int \frac{(a+b\arctan(c+dx))^2}{e+fx}\,dx$$

input
```
Integrate[(a + b*ArcTan[c + d*x])^2/(e + f*x),x]
```

output
```
Integrate[(a + b*ArcTan[c + d*x])^2/(e + f*x), x]
```

3.34.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5570, 27, 5383}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(c+dx))^2}{e+fx}\,dx$$

$$\Big\downarrow \ 5570$$

$$\frac{\int \frac{d(a+b\arctan(c+dx))^2}{d\left(e-\frac{cf}{d}\right)+f(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \ 27$$

$$\int \frac{(a+b\arctan(c+dx))^2}{f(c+dx)-cf+de}d(c+dx)$$

$$\Big\downarrow \ 5383$$

$$-\frac{ib(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{f}+\frac{(a+b\arctan(c+dx))^2\log\left(\frac{2(f(c+dx)-cf+de)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))}{f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))^2}{f}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{2f}-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2f}$$

input
```
Int[(a + b*ArcTan[c + d*x])^2/(e + f*x),x]
```

output
```
-(((a + b*ArcTan[c + d*x])^2*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcTan
[c + d*x])^2*Log[(2*(d*e - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(
c + d*x)))])/f + (I*b*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 - I*(c +
 d*x))])/f - (I*b*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - (2*(d*e - c*f + f
*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f - (b^2*PolyLog[3, 1
 - 2/(1 - I*(c + d*x))])/(2*f) + (b^2*PolyLog[3, 1 - (2*(d*e - c*f + f*(c
+ d*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(2*f)
```

3.34.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5383
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

rule 5570
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.34.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.59 (sec) , antiderivative size = 1877, normalized size of antiderivative = 7.19

method	result	size
derivativedivides	Expression too large to display	1877
default	Expression too large to display	1877
parts	Expression too large to display	1998

input
```
int((a+b*arctan(d*x+c))^2/(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*d*ln(c*f-d*e-f*(d*x+c))/f-b^2*d*(-ln(c*f-d*e-f*(d*x+c))/f*arctan(
d*x+c)^2+2/f*(1/2*arctan(d*x+c)^2*ln(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f
*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d
*e)-1/4*I*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2
/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x
+c))^2/(1+(d*x+c)^2)))*(csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I
*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*
csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))-csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(
d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)
^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I/(1+(1+I*(d*x+c)
)^2/(1+(d*x+c)^2)))-csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*
x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn
(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*
e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)
^2)))+csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*
x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/
(1+(d*x+c)^2)))^2)*arctan(d*x+c)^2-1/2*I*arctan(d*x+c)*polylog(2,-(1+I*(d*
x+c))^2/(1+(d*x+c)^2))+1/4*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-1/2*f
/(c*f-d*e+I*f)*arctan(d*x+c)*polylog(2,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d
*x+c)^2)/(d*e+I*f-c*f))-1/2*I*f/(c*f-d*e+I*f)*arctan(d*x+c)^2*ln(1-(c*f...
```

3.34.5 Fricas [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{e+fx}\,dx = \int \frac{(b\arctan(dx+c)+a)^2}{fx+e}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^2/(f*x+e),x, algorithm="fricas")
```

output
```
integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(f*x + e),
x)
```

3.34.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{e+fx}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(d*x+c))**2/(f*x+e),x)
```

output
```
Timed out
```

3.34.7 Maxima [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{e+fx}\,dx = \int \frac{(b\arctan(dx+c)+a)^2}{fx+e}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^2/(f*x+e),x, algorithm="maxima")
```

output
```
a^2*log(f*x + e)/f + integrate(1/16*(12*b^2*arctan(d*x + c)^2 + b^2*log(d^
2*x^2 + 2*c*d*x + c^2 + 1)^2 + 32*a*b*arctan(d*x + c))/(f*x + e), x)
```

3.34.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{e+fx}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(d*x+c))^2/(f*x+e),x, algorithm="giac")
```

output
```
Timed out
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{e+fx}\,dx = \int \frac{(a+b\,\mathrm{atan}(c+d\,x))^2}{e+f\,x}\,dx$$

input
```
int((a + b*atan(c + d*x))^2/(e + f*x),x)
```

output
```
int((a + b*atan(c + d*x))^2/(e + f*x), x)
```

3.35 $\int \frac{(a+b\arctan(c+dx))^2}{(e+fx)^2}\,dx$

3.35.1 Optimal result

Integrand size = 20, antiderivative size = 568

$$
\begin{aligned}
\int \frac{(a+b\arctan(c+dx))^2}{(e+fx)^2}\,dx &= \frac{2abd(de-cf)\arctan(c+dx)}{f\,(f^2+(de-cf)^2)} + \frac{ib^2d\arctan(c+dx)^2}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&+ \frac{b^2d(de-cf)\arctan(c+dx)^2}{f\,(d^2e^2-2cdef+(1+c^2)\,f^2)} - \frac{(a+b\arctan(c+dx))^2}{f(e+fx)} \\
&+ \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} - \frac{2b^2d\arctan(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&+ \frac{2b^2d\arctan(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&+ \frac{2b^2d\arctan(c+dx)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&- \frac{abd\log\left(1+(c+dx)^2\right)}{f^2+(de-cf)^2} + \frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&- \frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2} \\
&+ \frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)\,f^2}
\end{aligned}
$$

output
```
2*a*b*d*(-c*f+d*e)*arctan(d*x+c)/f/(f^2+(-c*f+d*e)^2)+I*b^2*d*arctan(d*x+c
)^2/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+b^2*d*(-c*f+d*e)*arctan(d*x+c)^2/f/(d^
2*e^2-2*c*d*e*f+(c^2+1)*f^2)-(a+b*arctan(d*x+c))^2/f/(f*x+e)+2*a*b*d*ln(f*
x+e)/(f^2+(-c*f+d*e)^2)-2*b^2*d*arctan(d*x+c)*ln(2/(1-I*(d*x+c)))/(d^2*e^2
-2*c*d*e*f+(c^2+1)*f^2)+2*b^2*d*arctan(d*x+c)*ln(2*d*(f*x+e)/(d*e+I*f-c*f)
/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+2*b^2*d*arctan(d*x+c)*ln(2
/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-a*b*d*ln(1+(d*x+c)^2)/(f^2
+(-c*f+d*e)^2)+I*b^2*d*polylog(2,1-2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^
2+1)*f^2)-I*b^2*d*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^
2*e^2-2*c*d*e*f+(c^2+1)*f^2)+I*b^2*d*polylog(2,1-2/(1+I*(d*x+c)))/(d^2*e^2
-2*c*d*e*f+(c^2+1)*f^2)
```

3.35.2 Mathematica [A] (verified)

Time = 5.05 (sec) , antiderivative size = 419, normalized size of antiderivative = 0.74

$$\int \frac{(a+b\arctan(c+dx))^2}{(e+fx)^2}\,dx$$

$$= \frac{-\frac{a^2}{f}+\frac{2ab\left(-\left(\left(-cde+f+c^2f-d^2ex+cdfx\right)\arctan(c+dx)\right)+d(e+fx)\log\left(\frac{d(e+fx)}{\sqrt{1+(c+dx)^2}}\right)\right)}{d^2e^2-2cdef+(1+c^2)f^2}+\frac{b^2d(e+fx)\left(-\frac{e^{i\arctan\left(\frac{de-cf}{f}\right)}\arctan(c+dx)^2}{f\sqrt{1+\frac{(de-cf)^2}{f^2}}}\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(a + b*ArcTan[c + d*x])^2/(e + f*x)^2,x]
```

output
```
(-(a^2/f) + (2*a*b*(-((-(c*d*e) + f + c^2*f - d^2*e*x + c*d*f*x)*ArcTan[c
+ d*x]) + d*(e + f*x)*Log[(d*(e + f*x))/Sqrt[1 + (c + d*x)^2]]))/(d^2*e^2
- 2*c*d*e*f + (1 + c^2)*f^2) + (b^2*d*(e + f*x)*(-((E^(I*ArcTan[(d*e - c*f
)/f])*ArcTan[c + d*x]^2)/(f*Sqrt[1 + (d*e - c*f)^2/f^2])) + ((c + d*x)*Arc
Tan[c + d*x]^2)/(d*(e + f*x)) - ((d*e - c*f)*((-I)*(Pi - 2*ArcTan[(d*e - c
*f)/f])*ArcTan[c + d*x] - Pi*Log[1 + E^((-2*I)*ArcTan[c + d*x])] - 2*(ArcT
an[(d*e - c*f)/f] + ArcTan[c + d*x])*Log[1 - E^((2*I)*(ArcTan[(d*e - c*f)/
f] + ArcTan[c + d*x]))] + Pi*Log[1/Sqrt[1 + (c + d*x)^2]] + 2*ArcTan[(d*e
- c*f)/f]*Log[Sin[ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x]]] + I*PolyLog[2,
 E^((2*I)*(ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x]))]))/(f^2*(1 + (d*e - c
*f)^2/f^2))))/(d*e - c*f))/(e + f*x)
```

3.35.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5568, 7292, 5580, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(c+dx))^2}{(e+fx)^2}dx$$

$$\Big\downarrow \; 5568$$

$$\frac{2bd\int \frac{a+b\arctan(c+dx)}{(e+fx)((c+dx)^2+1)}dx}{f}-\frac{(a+b\arctan(c+dx))^2}{f(e+fx)}$$

$$\Big\downarrow \; 7292$$

$$\frac{2bd\int \frac{a+b\arctan(c+dx)}{(e+fx)(c^2+2dxc+d^2x^2+1)}dx}{f}-\frac{(a+b\arctan(c+dx))^2}{f(e+fx)}$$

$$\Big\downarrow \; 5580$$

$$\frac{2b\int \frac{d(a+b\arctan(c+dx))}{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)((c+dx)^2+1)}d(c+dx)}{f}-\frac{(a+b\arctan(c+dx))^2}{f(e+fx)}$$

$$\Big\downarrow \; 27$$

$$\frac{2bd\int \frac{a+b\arctan(c+dx)}{(de-cf+f(c+dx))((c+dx)^2+1)}d(c+dx)}{f}-\frac{(a+b\arctan(c+dx))^2}{f(e+fx)}$$

$$\Big\downarrow \; 7276$$

$$\frac{2bd\int \left(\frac{a}{(de-cf+f(c+dx))((c+dx)^2+1)}+\frac{b\arctan(c+dx)}{(de-cf+f(c+dx))((c+dx)^2+1)}\right)d(c+dx)}{f}-\frac{(a+b\arctan(c+dx))^2}{f(e+fx)}$$

$$\Big\downarrow \; 2009$$

$$-\frac{(a+b\arctan(c+dx))^2}{f(e+fx)}+$$

$$2bd\left(\frac{a\arctan(c+dx)(de-cf)}{(de-cf)^2+f^2}+\frac{af\log(f(c+dx)-cf+de)}{(de-cf)^2+f^2}-\frac{af\log\left((c+dx)^2+1\right)}{2((de-cf)^2+f^2)}+\frac{ibf\arctan(c+dx)^2}{2((c^2+1)f^2-2cdef+d^2e^2)}+\frac{b\arctan(c+dx)^2(de-cf)}{2((c^2+1)f^2-2cdef+d^2e^2)}\right.$$

input
```
Int[(a + b*ArcTan[c + d*x])^2/(e + f*x)^2,x]
```

output
```
-((a + b*ArcTan[c + d*x])^2/(f*(e + f*x))) + (2*b*d*((a*(d*e - c*f)*ArcTan
[c + d*x])/(f^2 + (d*e - c*f)^2) + ((I/2)*b*f*ArcTan[c + d*x]^2)/(d^2*e^2
- 2*c*d*e*f + (1 + c^2)*f^2) + (b*(d*e - c*f)*ArcTan[c + d*x]^2)/(2*(d^2*e
^2 - 2*c*d*e*f + (1 + c^2)*f^2)) - (b*f*ArcTan[c + d*x]*Log[2/(1 - I*(c +
d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b*f*ArcTan[c + d*x]*Log[2
/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (a*f*Log[d*e
- c*f + f*(c + d*x)])/(f^2 + (d*e - c*f)^2) + (b*f*ArcTan[c + d*x]*Log[(2*
(d*e - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(d^2*e^
2 - 2*c*d*e*f + (1 + c^2)*f^2) - (a*f*Log[1 + (c + d*x)^2])/(2*(f^2 + (d*e
 - c*f)^2)) + ((I/2)*b*f*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2
*c*d*e*f + (1 + c^2)*f^2) + ((I/2)*b*f*PolyLog[2, 1 - 2/(1 + I*(c + d*x))]
)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - ((I/2)*b*f*PolyLog[2, 1 - (2*(d*
e - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(d^2*e^2 -
 2*c*d*e*f + (1 + c^2)*f^2)))/f
```

3.35.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5568
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcTan[
c + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[p, 0] && ILtQ[m, -1]
```

rule 5580
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTan[x])
^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &
& EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

3.35.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 776, normalized size of antiderivative = 1.37

method	result
parts	$-\frac{a^2}{(fx+e)f}+\frac{b^2\left(-\frac{d^2\arctan(dx+c)^2}{(f(dx+c)-cf+de)f}+\frac{2d^2\left(\frac{\arctan(dx+c)f\ln(f(dx+c)-cf+de)}{c^2f^2-2cdef+d^2e^2+f^2}-\frac{\arctan(dx+c)f\ln\left(1+(dx+c)^2\right)}{2\left(c^2f^2-2cdef+d^2e^2+f^2\right)}-\frac{\text{arcta}}{c^2f^2-2c}\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a^2d^2}{(cf-de-f(dx+c))f}+b^2d^2\left(\frac{\arctan(dx+c)^2}{(cf-de-f(dx+c))f}-\frac{2\left(\frac{\arctan(dx+c)f\ln\left(1+(dx+c)^2\right)}{2c^2f^2-4cdef+2d^2e^2+2f^2}+\frac{\arctan(dx+c)^2cf}{c^2f^2-2cdef+d^2e^2+f^2}-\frac{\arctan(dx+c)^2}{c^2f^2-2cdef+d^2e}\right.}{}\right.}{}$
default	$\frac{\frac{a^2d^2}{(cf-de-f(dx+c))f}+b^2d^2\left(\frac{\arctan(dx+c)^2}{(cf-de-f(dx+c))f}-\frac{2\left(\frac{\arctan(dx+c)f\ln\left(1+(dx+c)^2\right)}{2c^2f^2-4cdef+2d^2e^2+2f^2}+\frac{\arctan(dx+c)^2cf}{c^2f^2-2cdef+d^2e^2+f^2}-\frac{\arctan(dx+c)^2}{c^2f^2-2cdef+d^2e}\right.}{}\right.}{}$

input
```
int((a+b*arctan(d*x+c))^2/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/(f*x+e)/f+b^2/d*(-d^2/(f*(d*x+c)-c*f+d*e)/f*arctan(d*x+c)^2+2*d^2/f*(
arctan(d*x+c)*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(f*(d*x+c)-c*f+d*e)-1/2*
arctan(d*x+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(1+(d*x+c)^2)-1/(c^2*f^2
-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*c*f+1/(c^2*f^2-2*c*d*e*f+d^2*e^2+f
^2)*arctan(d*x+c)^2*d*e-f^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*(-1/2*I*ln(f*(
d*x+c)-c*f+d*e)*(ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))-ln((I*f+f*(d*x+c))/(c*f
-d*e+I*f)))/f-1/2*I*(dilog((I*f-f*(d*x+c))/(d*e+I*f-c*f))-dilog((I*f+f*(d*
x+c))/(c*f-d*e+I*f)))/f)+1/2*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*(-1/2*I*(ln
(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+c-I)^2-dilog(-1/2*I*(d*x+c+I))-ln(d*x
+c-I)*ln(-1/2*I*(d*x+c+I)))+1/2*I*(ln(d*x+c+I)*ln(1+(d*x+c)^2)-1/2*ln(d*x+
c+I)^2-dilog(1/2*I*(d*x+c-I))-ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))))+1/2/(c^2*f
^2-2*c*d*e*f+d^2*e^2+f^2)*(c*f-d*e)*arctan(d*x+c)^2))+2*a*b/d*(-d^2/(f*(d*
x+c)-c*f+d*e)/f*arctan(d*x+c)+d^2/f*(1/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*l
n(f*(d*x+c)-c*f+d*e)+1/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*(-1/2*f*ln(1+(d*x+c
)^2)+(-c*f+d*e)*arctan(d*x+c))))
```

3.35.5 Fricas [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{(e+fx)^2}\,dx = \int \frac{(b\arctan(dx+c)+a)^2}{(fx+e)^2}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^2/(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(f^2*x^2 +
2*e*f*x + e^2), x)
```

3.35.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{(e+fx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(d*x+c))**2/(f*x+e)**2,x)
```

output
```
Timed out
```

3.35.7 Maxima [F]

$$\int \frac{(a+b\arctan(c+dx))^2}{(e+fx)^2}\,dx = \int \frac{(b\arctan(dx+c)+a)^2}{(fx+e)^2}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^2/(f*x+e)^2,x, algorithm="maxima")
```

output
```
(d*(2*(d^2*e - c*d*f)*arctan((d^2*x + c*d)/d)/((d^2*e^2*f - 2*c*d*e*f^2 +
(c^2 + 1)*f^3)*d) - log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2 - 2*c*d*e*f
+ (c^2 + 1)*f^2) + 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 + 1)*f^2)) -
 2*arctan(d*x + c)/(f^2*x + e*f))*a*b - 1/16*(4*arctan(d*x + c)^2 - 16*(f^
2*x + e*f)*integrate(1/16*(12*(d^2*f*x^2 + 2*c*d*f*x + (c^2 + 1)*f)*arctan
(d*x + c)^2 + (d^2*f*x^2 + 2*c*d*f*x + (c^2 + 1)*f)*log(d^2*x^2 + 2*c*d*x
+ c^2 + 1)^2 + 8*(d*f*x + d*e)*arctan(d*x + c) - 4*(d^2*f*x^2 + c*d*e + (d
^2*e + c*d*f)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^2*f^3*x^4 + (c^2 + 1
)*e^2*f + 2*(d^2*e*f^2 + c*d*f^3)*x^3 + (d^2*e^2*f + 4*c*d*e*f^2 + (c^2 +
1)*f^3)*x^2 + 2*(c*d*e^2*f + (c^2 + 1)*e*f^2)*x), x) - log(d^2*x^2 + 2*c*d
*x + c^2 + 1)^2)*b^2/(f^2*x + e*f) - a^2/(f^2*x + e*f)
```

3.35.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{(e+fx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(d*x+c))^2/(f*x+e)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^2}{(e+fx)^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c+d\,x))^2}{(e+f\,x)^2}\,dx$$

input
```
int((a + b*atan(c + d*x))^2/(e + f*x)^2,x)
```

output
```
int((a + b*atan(c + d*x))^2/(e + f*x)^2, x)
```

3.36 $\int (e+fx)^2(a+b\arctan(c+dx))^3\,dx$

3.36.1 Optimal result

Integrand size = 20, antiderivative size = 564

$$
\begin{aligned}
&\int (e+fx)^2(a+b\arctan(c+dx))^3\,dx \\
&= \frac{ab^2f^2x}{d^2}+\frac{b^3f^2(c+dx)\arctan(c+dx)}{d^3}-\frac{bf^2(a+b\arctan(c+dx))^2}{2d^3} \\
&-\frac{3ibf(de-cf)(a+b\arctan(c+dx))^2}{d^3} \\
&-\frac{3bf(de-cf)(c+dx)(a+b\arctan(c+dx))^2}{d^3}-\frac{bf^2(c+dx)^2(a+b\arctan(c+dx))^2}{2d^3} \\
&+\frac{i(3d^2e^2-6cdef-(1-3c^2)f^2)(a+b\arctan(c+dx))^3}{3d^3} \\
&-\frac{(de-cf)(d^2e^2-2cdef-(3-c^2)f^2)(a+b\arctan(c+dx))^3}{3d^3f} \\
&+\frac{(e+fx)^3(a+b\arctan(c+dx))^3}{3f} \\
&-\frac{6b^2f(de-cf)(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&+\frac{b(3d^2e^2-6cdef-(1-3c^2)f^2)(a+b\arctan(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&-\frac{b^3f^2\log(1+(c+dx)^2)}{2d^3}-\frac{3ib^3f(de-cf)\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&+\frac{ib^2(3d^2e^2-6cdef-(1-3c^2)f^2)(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^3} \\
&+\frac{b^3(3d^2e^2-6cdef-(1-3c^2)f^2)\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d^3}
\end{aligned}
$$

output
```
a*b^2*f^2*x/d^2+b^3*f^2*(d*x+c)*arctan(d*x+c)/d^3-1/2*b*f^2*(a+b*arctan(d*
x+c))^2/d^3-3*I*b*f*(-c*f+d*e)*(a+b*arctan(d*x+c))^2/d^3-3*b*f*(-c*f+d*e)*
(d*x+c)*(a+b*arctan(d*x+c))^2/d^3-1/2*b*f^2*(d*x+c)^2*(a+b*arctan(d*x+c))^
2/d^3+1/3*I*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*arctan(d*x+c))^3/d^3
-1/3*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f-(-c^2+3)*f^2)*(a+b*arctan(d*x+c))^3/d^3
/f+1/3*(f*x+e)^3*(a+b*arctan(d*x+c))^3/f-6*b^2*f*(-c*f+d*e)*(a+b*arctan(d*
x+c))*ln(2/(1+I*(d*x+c)))/d^3+b*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*
arctan(d*x+c))^2*ln(2/(1+I*(d*x+c)))/d^3-1/2*b^3*f^2*ln(1+(d*x+c)^2)/d^3-3
*I*b^3*f*(-c*f+d*e)*polylog(2,1-2/(1+I*(d*x+c)))/d^3+I*b^2*(3*d^2*e^2-6*c*
d*e*f-(-3*c^2+1)*f^2)*(a+b*arctan(d*x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d^3
+1/2*b^3*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*polylog(3,1-2/(1+I*(d*x+c)))
/d^3
```

3.36.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1844 vs. 2(564) = 1128.

Time = 13.86 (sec) , antiderivative size = 1844, normalized size of antiderivative = 3.27

$$\int (e+fx)^2(a+b\arctan(c+dx))^3\,dx = \text{Too large to display}$$

input
```
Integrate[(e + f*x)^2*(a + b*ArcTan[c + d*x])^3,x]
```

output
```
(a^2*(a*d^2*e^2 - 3*b*d*e*f + 2*b*c*f^2)*x)/d^2 - (a^2*f*(-2*a*d*e + b*f)*
x^2)/(2*d) + (a^3*f^2*x^3)/3 + ((3*a^2*b*c*d^2*e^2 + 3*a^2*b*d*e*f - 3*a^2
*b*c^2*d*e*f - 3*a^2*b*c*f^2 + a^2*b*c^3*f^2)*ArcTan[c + d*x])/d^3 + a^2*b
*x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTan[c + d*x] + ((-3*a^2*b*d^2*e^2 + 6*a^
2*b*c*d*e*f + a^2*b*f^2 - 3*a^2*b*c^2*f^2)*Log[1 + c^2 + 2*c*d*x + d^2*x^2
])/(2*d^3) + (3*a*b^2*e^2*((-I)*ArcTan[c + d*x]^2 + (c + d*x)*ArcTan[c + d
*x]^2 + 2*ArcTan[c + d*x]*Log[1 + E^((2*I)*ArcTan[c + d*x])] - I*PolyLog[2
, -E^((2*I)*ArcTan[c + d*x])]))/d + 6*a*b^2*e*f*(-(((c + d*x)*ArcTan[c + d
*x])/d^2) + (I*c*ArcTan[c + d*x]^2)/d^2 - (c*(c + d*x)*ArcTan[c + d*x]^2)/
d^2 + ((1 + (c + d*x)^2)*ArcTan[c + d*x]^2)/(2*d^2) - (2*c*ArcTan[c + d*x]
*Log[1 + E^((2*I)*ArcTan[c + d*x])])/d^2 - Log[1/Sqrt[1 + (c + d*x)^2]]/d^
2 + (I*c*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])])/d^2) + (b^3*e^2*((-I)*Arc
Tan[c + d*x]^3 + (c + d*x)*ArcTan[c + d*x]^3 + 3*ArcTan[c + d*x]^2*Log[1 +
 E^((2*I)*ArcTan[c + d*x])] - (3*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*A
rcTan[c + d*x])] + (3*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])/2))/d + (b^3
*e*f*(ArcTan[c + d*x]*((3*I)*ArcTan[c + d*x] + (2*I)*c*ArcTan[c + d*x]^2 +
 (1 + (c + d*x)^2)*ArcTan[c + d*x]^2 - (c + d*x)*ArcTan[c + d*x]*(3 + 2*c*
ArcTan[c + d*x]) - 6*Log[1 + E^((2*I)*ArcTan[c + d*x])] - 6*c*ArcTan[c + d
*x]*Log[1 + E^((2*I)*ArcTan[c + d*x])]) + (3*I)*(1 + 2*c*ArcTan[c + d*x])*
PolyLog[2, -E^((2*I)*ArcTan[c + d*x])] - 3*c*PolyLog[3, -E^((2*I)*ArcTa...
```

3.36.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 554, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5570, 27, 5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2(a+b\arctan(c+dx))^3\,dx$$

$$\Big\downarrow \text{5570}$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)^2(a+b\arctan(c+dx))^3}{d^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (de-cf+f(c+dx))^2(a+b\arctan(c+dx))^3d(c+dx)}{d^3}$$

$$\Big\downarrow \text{5389}$$

$$\frac{\frac{(f(c+dx)-cf+de)^3(a+b\arctan(c+dx))^3}{3f}-\frac{b\int\left((c+dx)(a+b\arctan(c+dx))^2f^3+3(de-cf)(a+b\arctan(c+dx))^2f^2+\frac{\left((de-cf)\left(d^2e^2-2cdfe-\left(3-c^2\cdots\right.\right.\right.}{f}\cdots\right.}{\cdots}}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(f(c+dx)-cf+de)^3(a+b\arctan(c+dx))^3}{3f}-\frac{b\left(-ibf\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)(a+b\arctan(c+dx))-\frac{if\left(-\left(1-3c^2\right)\cdots\right.}{\cdots}\cdots\right.}{\cdots}}{\cdots}$$

input
```
Int[(e + f*x)^2*(a + b*ArcTan[c + d*x])^3,x]
```

output
```
(((d*e - c*f + f*(c + d*x))^3*(a + b*ArcTan[c + d*x])^3)/(3*f) - (b*(-(a*b
*f^3*(c + d*x)) - b^2*f^3*(c + d*x)*ArcTan[c + d*x] + (f^3*(a + b*ArcTan[c
 + d*x])^2)/2 + (3*I)*f^2*(d*e - c*f)*(a + b*ArcTan[c + d*x])^2 + 3*f^2*(d
*e - c*f)*(c + d*x)*(a + b*ArcTan[c + d*x])^2 + (f^3*(c + d*x)^2*(a + b*Ar
cTan[c + d*x])^2)/2 - ((I/3)*f*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f^2)*(
a + b*ArcTan[c + d*x])^3)/b + ((d*e - c*f)*(d^2*e^2 - 2*c*d*e*f - (3 - c^2
)*f^2)*(a + b*ArcTan[c + d*x])^3)/(3*b) + 6*b*f^2*(d*e - c*f)*(a + b*ArcTa
n[c + d*x])*Log[2/(1 + I*(c + d*x))] - f*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c
^2)*f^2)*(a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))] + (b^2*f^3*Log
[1 + (c + d*x)^2])/2 + (3*I)*b^2*f^2*(d*e - c*f)*PolyLog[2, 1 - 2/(1 + I*(
c + d*x))] - I*b*f*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f^2)*(a + b*ArcTan
[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))] - (b^2*f*(3*d^2*e^2 - 6*c*d
*e*f - (1 - 3*c^2)*f^2)*PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/2))/f)/d^3
```

3.36.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 5570
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.36.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 28.49 (sec) , antiderivative size = 5843, normalized size of antiderivative = 10.36

method	result	size
derivativedivides	Expression too large to display	5843
default	Expression too large to display	5843
parts	Expression too large to display	6026

input
```
int((f*x+e)^2*(a+b*arctan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.36.5 Fricas [F]

$$\int (e+fx)^2(a+b\arctan(c+dx))^3\,dx=\int (fx+e)^2(b\arctan(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(a^3*f^2*x^2 + 2*a^3*e*f*x + a^3*e^2 + (b^3*f^2*x^2 + 2*b^3*e*f*x
+ b^3*e^2)*arctan(d*x + c)^3 + 3*(a*b^2*f^2*x^2 + 2*a*b^2*e*f*x + a*b^2*e^
2)*arctan(d*x + c)^2 + 3*(a^2*b*f^2*x^2 + 2*a^2*b*e*f*x + a^2*b*e^2)*arcta
n(d*x + c), x)
```

3.36.6 Sympy [F(-1)]

Timed out.

$$\int (e+fx)^2(a+b\arctan(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*(a+b*atan(d*x+c))**3,x)
```

output
```
Timed out
```

3.36.7 Maxima [F]

$$\int (e+fx)^2(a+b\arctan(c+dx))^3\,dx = \int (fx+e)^2(b\arctan(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

output
```
7/8*b^3*c^2*e^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 3*a*b^2*c^2*
e^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - (3*arctan(d*x + c)*arcta
n((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^2*e^2 - 7/32
*(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arct
an((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^2*e^2 + 1/3*a
^3*f^2*x^3 + 7/8*b^3*e^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 28*
b^3*d^2*f^2*integrate(1/32*x^4*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2
+ 1), x) + 3*b^3*d^2*f^2*integrate(1/32*x^4*arctan(d*x + c)*log(d^2*x^2 +
2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*d^2*f^2*
integrate(1/32*x^4*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 5
6*b^3*d^2*e*f*integrate(1/32*x^3*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^
2 + 1), x) + 56*b^3*c*d*f^2*integrate(1/32*x^3*arctan(d*x + c)^3/(d^2*x^2
+ 2*c*d*x + c^2 + 1), x) + 4*b^3*d^2*f^2*integrate(1/32*x^4*arctan(d*x + c
)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b
^3*d^2*e*f*integrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2
+ 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*c*d*f^2*integrate(1/32*x^
3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x +
c^2 + 1), x) + 192*a*b^2*d^2*e*f*integrate(1/32*x^3*arctan(d*x + c)^2/(d^2
*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*c*d*f^2*integrate(1/32*x^3*arcta
n(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 28*b^3*d^2*e^2*integra...
```

3.36.8 Giac [F]

$$\int (e+fx)^2(a+b\arctan(c+dx))^3\,dx = \int (fx+e)^2(b\arctan(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2(a+b\arctan(c+dx))^3\,dx = \int (e+f\,x)^2\,(a+b\,\mathrm{atan}(c+d\,x))^3\,dx$$

input
```
int((e + f*x)^2*(a + b*atan(c + d*x))^3,x)
```

output
```
int((e + f*x)^2*(a + b*atan(c + d*x))^3, x)
```

3.37 $\int (e+fx)(a+b\arctan(c+dx))^3\,dx$

3.37.1 Optimal result

Integrand size = 18, antiderivative size = 337

$$\begin{aligned}\int (e+fx)(a+b\arctan(c+dx))^3\,dx &= -\frac{3ibf(a+b\arctan(c+dx))^2}{2d^2}-\frac{3bf(c+dx)(a+b\arctan(c+dx))^2}{2d^2}\\ &\quad+\frac{i(de-cf)(a+b\arctan(c+dx))^3}{d^2}\\ &\quad-\frac{(de+f-cf)(de-(1+c)f)(a+b\arctan(c+dx))^3}{2d^2f}\\ &\quad+\frac{(e+fx)^2(a+b\arctan(c+dx))^3}{2f}-\frac{3b^2f(a+b\arctan(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2}\\ &\quad+\frac{3b(de-cf)(a+b\arctan(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2}-\frac{3ib^3f\,\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{2d^2}\\ &\quad+\frac{3ib^2(de-cf)(a+b\arctan(c+dx))\,\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2}\\ &\quad+\frac{3b^3(de-cf)\,\text{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d^2}\end{aligned}$$

output
```
-3/2*I*b*f*(a+b*arctan(d*x+c))^2/d^2-3/2*b*f*(d*x+c)*(a+b*arctan(d*x+c))^2
/d^2+I*(-c*f+d*e)*(a+b*arctan(d*x+c))^3/d^2-1/2*(-c*f+d*e+f)*(d*e-(1+c)*f)
*(a+b*arctan(d*x+c))^3/d^2/f+1/2*(f*x+e)^2*(a+b*arctan(d*x+c))^3/f-3*b^2*f
*(a+b*arctan(d*x+c))*ln(2/(1+I*(d*x+c)))/d^2+3*b*(-c*f+d*e)*(a+b*arctan(d*
x+c))^2*ln(2/(1+I*(d*x+c)))/d^2-3/2*I*b^3*f*polylog(2,1-2/(1+I*(d*x+c)))/d
^2+3*I*b^2*(-c*f+d*e)*(a+b*arctan(d*x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d^2
+3/2*b^3*(-c*f+d*e)*polylog(3,1-2/(1+I*(d*x+c)))/d^2
```

3.37.2 Mathematica [A] (verified)

Time = 5.07 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.76

$$\int (e+fx)(a+b\arctan(c+dx))^3\,dx = \frac{a^2(2ade-3bf-2acf)(c+dx)+a^3f(c+dx)^2+3a^2bf\arctan(c+dx)-3a^2b(c+dx)(cf-d(2e+fx))}{}$$

input
```
Integrate[(e + f*x)*(a + b*ArcTan[c + d*x])^3,x]
```

output
```
(a^2*(2*a*d*e - 3*b*f - 2*a*c*f)*(c + d*x) + a^3*f*(c + d*x)^2 + 3*a^2*b*f
*ArcTan[c + d*x] - 3*a^2*b*(c + d*x)*(c*f - d*(2*e + f*x))*ArcTan[c + d*x]
 + 6*a*b^2*f*(-((c + d*x)*ArcTan[c + d*x]) + ((1 + (c + d*x)^2)*ArcTan[c +
 d*x]^2)/2 - Log[1/Sqrt[1 + (c + d*x)^2]]) - 3*a^2*b*(d*e - c*f)*Log[1 + (
c + d*x)^2] + 6*a*b^2*d*e*(ArcTan[c + d*x]*((-I + c + d*x)*ArcTan[c + d*x]
 + 2*Log[1 + E^((2*I)*ArcTan[c + d*x])]) - I*PolyLog[2, -E^((2*I)*ArcTan[c
 + d*x])]) - 6*a*b^2*c*f*(ArcTan[c + d*x]*((-I + c + d*x)*ArcTan[c + d*x]
+ 2*Log[1 + E^((2*I)*ArcTan[c + d*x])]) - I*PolyLog[2, -E^((2*I)*ArcTan[c
+ d*x])]) + b^3*f*(ArcTan[c + d*x]*((3*I)*ArcTan[c + d*x] - 3*(c + d*x)*Ar
cTan[c + d*x] + (1 + (c + d*x)^2)*ArcTan[c + d*x]^2 - 6*Log[1 + E^((2*I)*A
rcTan[c + d*x])]) + (3*I)*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]) + 2*b^3*
d*e*(ArcTan[c + d*x]^2*((-I + c + d*x)*ArcTan[c + d*x] + 3*Log[1 + E^((2*I
)*ArcTan[c + d*x])]) - (3*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*ArcTan[c
 + d*x])] + (3*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])/2) - 2*b^3*c*f*(Arc
Tan[c + d*x]^2*((-I + c + d*x)*ArcTan[c + d*x] + 3*Log[1 + E^((2*I)*ArcTan
[c + d*x])]) - (3*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])
] + (3*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])/2))/(2*d^2)
```

3.37.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5570, 27, 5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)(a+b\arctan(c+dx))^3\,dx$$

$$\Big\downarrow \ \textbf{5570}$$

$$\frac{\int \frac{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)(a+b\arctan(c+dx))^3}{d}d(c+dx)}{d}$$

$$\Big\downarrow \ \textbf{27}$$

$$\frac{\int (de-cf+f(c+dx))(a+b\arctan(c+dx))^3d(c+dx)}{d^2}$$

$$\Big\downarrow \ \textbf{5389}$$

$$\frac{\frac{(f(c+dx)-cf+de)^2(a+b\arctan(c+dx))^3}{2f}-\frac{3b\int \left(f^2(a+b\arctan(c+dx))^2+\frac{((de-cf+f)(de-(c+1)f)+2f(de-cf)(c+dx))(a+b\arctan(c+dx))^2}{(c+dx)^2+1}\right)d(c+dx \ldots}{2f}}{d^2}$$

$$\Big\downarrow \ \textbf{2009}$$

$$\frac{(f(c+dx)-cf+de)^2(a+b\arctan(c+dx))^3}{2f}-\frac{3b\left(-2ibf(de-cf)\operatorname{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)(a+b\arctan(c+dx))-\frac{2if(de-cf)(a+b\arctan(c+dx))^3}{3b}+\frac{(\ldots}{}\right.}{}$$

input
```
Int[(e + f*x)*(a + b*ArcTan[c + d*x])^3,x]
```

output
```
(((d*e - c*f + f*(c + d*x))^2*(a + b*ArcTan[c + d*x])^3)/(2*f) - (3*b*(I*f
^2*(a + b*ArcTan[c + d*x])^2 + f^2*(c + d*x)*(a + b*ArcTan[c + d*x])^2 - (
((2*I)/3)*f*(d*e - c*f)*(a + b*ArcTan[c + d*x])^3)/b + ((d*e + f - c*f)*(d
*e - (1 + c)*f)*(a + b*ArcTan[c + d*x])^3)/(3*b) + 2*b*f^2*(a + b*ArcTan[c
 + d*x])*Log[2/(1 + I*(c + d*x))] - 2*f*(d*e - c*f)*(a + b*ArcTan[c + d*x]
)^2*Log[2/(1 + I*(c + d*x))] + I*b^2*f^2*PolyLog[2, 1 - 2/(1 + I*(c + d*x)
)] - (2*I)*b*f*(d*e - c*f)*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I
*(c + d*x))] - b^2*f*(d*e - c*f)*PolyLog[3, 1 - 2/(1 + I*(c + d*x))]))/(2*
f))/d^2
```

3.37.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 5570
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.37.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.33 (sec) , antiderivative size = 8267, normalized size of antiderivative = 24.53

method	result	size
parts	Expression too large to display	8267
derivativedivides	Expression too large to display	8269
default	Expression too large to display	8269

input
```
int((f*x+e)*(a+b*arctan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.37.5 Fricas [F]

$$\int (e+fx)(a+b\arctan(c+dx))^3\,dx=\int (fx+e)(b\arctan(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(a^3*f*x + a^3*e + (b^3*f*x + b^3*e)*arctan(d*x + c)^3 + 3*(a*b^2*
f*x + a*b^2*e)*arctan(d*x + c)^2 + 3*(a^2*b*f*x + a^2*b*e)*arctan(d*x + c)
, x)
```

3.37.6 Sympy [F]

$$\int (e+fx)(a+b\arctan(c+dx))^3\,dx=\int (a+b\operatorname{atan}(c+dx))^3\,(e+fx)\;dx$$

input
```
integrate((f*x+e)*(a+b*atan(d*x+c))**3,x)
```

output
```
Integral((a + b*atan(c + d*x))**3*(e + f*x), x)
```

3.37.7 Maxima [F]

$$\int (e+fx)(a+b\arctan(c+dx))^3\,dx=\int (fx+e)(b\arctan(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

output
```
7/8*b^3*c^2*e*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 3*a*b^2*c^2*e*
arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - (3*arctan(d*x + c)*arctan((d
^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^2*e - 7/32*(6*ar
ctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan((d^
2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^2*e + 7/8*b^3*e*arc
tan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 56*b^3*d^2*f*integrate(1/64*x^3
*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*d^2*f*integra
te(1/64*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 +
2*c*d*x + c^2 + 1), x) + 192*a*b^2*d^2*f*integrate(1/64*x^3*arctan(d*x + c
)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*d^2*e*integrate(1/64*x^2*ar
ctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 112*b^3*c*d*f*integrat
e(1/64*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*d^
2*f*integrate(1/64*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d
^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*d^2*e*integrate(1/64*x^2*arctan(d*
x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x
) + 12*b^3*c*d*f*integrate(1/64*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x
+ c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*d^2*e*integrate
(1/64*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*a*b^2*
c*d*f*integrate(1/64*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1),
x) + 112*b^3*c*d*e*integrate(1/64*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*...
```

3.37.8 Giac [F]

$$\int (e+fx)(a+b\arctan(c+dx))^3\,dx=\int (fx+e)(b\arctan(dx+c)+a)^3\,dx$$

input
```
integrate((f*x+e)*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)(a+b\arctan(c+dx))^3\,dx=\int (e+f\,x)\,\left(a+b\,\mathrm{atan}(c+d\,x)\right)^3\,dx$$

input
```
int((e + f*x)*(a + b*atan(c + d*x))^3,x)
```

output
```
int((e + f*x)*(a + b*atan(c + d*x))^3, x)
```

3.38 $\int (a+b\arctan(c+dx))^3\,dx$

3.38.1 Optimal result

Integrand size = 12, antiderivative size = 143

$$\int (a+b\arctan(c+dx))^3\,dx = \frac{i(a+b\arctan(c+dx))^3}{d}+\frac{(c+dx)(a+b\arctan(c+dx))^3}{d}+\frac{3b(a+b\arctan(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d}+\frac{3ib^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d}+\frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+i(c+dx)}\right)}{2d}$$

output
```
I*(a+b*arctan(d*x+c))^3/d+(d*x+c)*(a+b*arctan(d*x+c))^3/d+3*b*(a+b*arctan(
d*x+c))^2*ln(2/(1+I*(d*x+c)))/d+3*I*b^2*(a+b*arctan(d*x+c))*polylog(2,1-2/
(1+I*(d*x+c)))/d+3/2*b^3*polylog(3,1-2/(1+I*(d*x+c)))/d
```

3.38.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.48

$$\int (a+b\arctan(c+dx))^3\,dx$$
$$= \frac{2a^3(c+dx)+6a^2b(c+dx)\arctan(c+dx)-3a^2b\log\left(1+(c+dx)^2\right)+6ab^2\left(\arctan(c+dx)\left((-i+c+d\right.\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c + d*x])^3,x]
```

output
```
(2*a^3*(c + d*x) + 6*a^2*b*(c + d*x)*ArcTan[c + d*x] - 3*a^2*b*Log[1 + (c
+ d*x)^2] + 6*a*b^2*(ArcTan[c + d*x]*((-I + c + d*x)*ArcTan[c + d*x] + 2*L
og[1 + E^((2*I)*ArcTan[c + d*x])]) - I*PolyLog[2, -E^((2*I)*ArcTan[c + d*x
])]) + 2*b^3*(ArcTan[c + d*x]^2*((-I + c + d*x)*ArcTan[c + d*x] + 3*Log[1
+ E^((2*I)*ArcTan[c + d*x])]) - (3*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)
*ArcTan[c + d*x])] + (3*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])/2))/(2*d)
```

3.38.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5562, 5345, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arctan(c+dx))^3\,dx$$
$$\Big\downarrow \; 5562$$
$$\frac{\int (a+b\arctan(c+dx))^3d(c+dx)}{d}$$
$$\Big\downarrow \; 5345$$
$$\frac{(c+dx)(a+b\arctan(c+dx))^3-3b\int \frac{(c+dx)(a+b\arctan(c+dx))^2}{(c+dx)^2+1}d(c+dx)}{d}$$
$$\Big\downarrow \; 5455$$
$$\frac{(c+dx)(a+b\arctan(c+dx))^3-3b\left(-\int \frac{(a+b\arctan(c+dx))^2}{-c-dx+i}d(c+dx)-\frac{i(a+b\arctan(c+dx))^3}{3b}\right)}{d}$$

$\downarrow$ 5379

$$\frac{(c+dx)(a+b\arctan(c+dx))^3-3b\left(2b\int\frac{(a+b\arctan(c+dx))\log\left(\frac{2}{i(c+dx)+1}\right)}{(c+dx)^2+1}d(c+dx)-\frac{i(a+b\arctan(c+dx))^3}{3b}-\log\left(\frac{}{1+i(}\right.\right.}{d}$$

$\downarrow$ 5529

$$\frac{(c+dx)(a+b\arctan(c+dx))^3-3b\left(2b\left(\frac{1}{2}ib\int\frac{\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)}{(c+dx)^2+1}d(c+dx)-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)(a\right.\right.}{d}$$

$\downarrow$ 7164

$$\frac{(c+dx)(a+b\arctan(c+dx))^3-3b\left(2b\left(-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{i(c+dx)+1}\right)(a+b\arctan(c+dx))-\frac{1}{4}b\,\text{PolyLog}\left(3,\right.\right.\right.}{d}$$

input
```
Int[(a + b*ArcTan[c + d*x])^3,x]
```

output
```
((c + d*x)*(a + b*ArcTan[c + d*x])^3 - 3*b*(((-1/3*I)*(a + b*ArcTan[c + d*
x])^3)/b - (a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))] + 2*b*((-1/2
*I)*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))] - (b*PolyL
og[3, 1 - 2/(1 + I*(c + d*x))])/4)))/d
```

3.38.3.1 Defintions of rubi rules used

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5562
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
 x] && IGtQ[p, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.38.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(136) = 272.

Time = 0.78 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.95

method	result
derivativedivides	$\frac{(dx+c)a^3+b^3\left(\arctan(dx+c)^3(dx+c+i)-2i\arctan(dx+c)^3+3\arctan(dx+c)^2\ln\left(1+\frac{(1+i(dx+c))^2}{1+(dx+c)^2}\right)-3i\arctan(dx+c)\,\mathrm{p}\right.}{}$
default	$\frac{(dx+c)a^3+b^3\left(\arctan(dx+c)^3(dx+c+i)-2i\arctan(dx+c)^3+3\arctan(dx+c)^2\ln\left(1+\frac{(1+i(dx+c))^2}{1+(dx+c)^2}\right)-3i\arctan(dx+c)\,\mathrm{p}\right.}{}$
parts	$a^3x+\frac{b^3\left(\arctan(dx+c)^3(dx+c+i)-2i\arctan(dx+c)^3+3\arctan(dx+c)^2\ln\left(1+\frac{(1+i(dx+c))^2}{1+(dx+c)^2}\right)-3i\arctan(dx+c)\,\mathrm{pol}\right.}{d}$

input
```
int((a+b*arctan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*((d*x+c)*a^3+b^3*(arctan(d*x+c)^3*(d*x+c+I)-2*I*arctan(d*x+c)^3+3*arct
an(d*x+c)^2*ln(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))-3*I*arctan(d*x+c)*polylog(
2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+3/2*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c
)^2)))+3*a*b^2*(arctan(d*x+c)^2*(d*x+c+I)+2*arctan(d*x+c)*ln(1+(1+I*(d*x+c
))^2/(1+(d*x+c)^2))-2*I*arctan(d*x+c)^2-I*polylog(2,-(1+I*(d*x+c))^2/(1+(d
*x+c)^2)))+3*a^2*b*((d*x+c)*arctan(d*x+c)-1/2*ln(1+(d*x+c)^2)))
```

3.38.5 Fricas [F]

$$\int (a+b\arctan(c+dx))^3\,dx=\int (b\arctan(dx+c)+a)^3\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3, x)
```

3.38.6 Sympy [F]

$$\int (a+b\arctan(c+dx))^3\,dx=\int (a+b\operatorname{atan}(c+dx))^3\,dx$$

input
```
integrate((a+b*atan(d*x+c))**3,x)
```

output
```
Integral((a + b*atan(c + d*x))**3, x)
```

3.38.7 Maxima [F]

$$\int (a+b\arctan(c+dx))^3\,dx=\int (b\arctan(dx+c)+a)^3\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

output
```
7/8*b^3*c^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 1/8*b^3*x*arctan
(d*x + c)^3 + 3*a*b^2*c^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 3/
32*b^3*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 - (3*arctan(d*
x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^
2 - 7/32*(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x +
 c)*arctan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^2 + 7
/8*b^3*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 28*b^3*d^2*integrate(
1/32*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d^2*i
ntegrate(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*
x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*d^2*integrate(1/32*x^2*arctan(d*x
+ c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*c*d*integrate(1/32*x*arc
tan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*d^2*integrate(1/
32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x
 + c^2 + 1), x) + 6*b^3*c*d*integrate(1/32*x*arctan(d*x + c)*log(d^2*x^2 +
 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*c*d*in
tegrate(1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^
3*c*d*integrate(1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d
^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*c^2*integrate(1/32*arctan(d*x + c)
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*
a*b^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 12*b^3*d*integrate(...
```

3.38.8 Giac [F]

$$\int (a+b\arctan(c+dx))^3\,dx=\int (b\arctan(dx+c)+a)^3\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\arctan(c+dx))^3\,dx = \int (a+b\,\mathrm{atan}(c+d\,x))^3\,dx$$

input
```
int((a + b*atan(c + d*x))^3,x)
```

output
```
int((a + b*atan(c + d*x))^3, x)
```

3.39 $\int \frac{(a+b\arctan(c+dx))^3}{e+fx}\,dx$

3.39.1 Optimal result

Integrand size = 20, antiderivative size = 372

$$\begin{aligned}
&\int \frac{(a+b\arctan(c+dx))^3}{e+fx}\,dx \\
&= -\frac{(a+b\arctan(c+dx))^3\log\left(\frac{2}{1-i(c+dx)}\right)}{f} \\
&+ \frac{(a+b\arctan(c+dx))^3\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f} \\
&+ \frac{3ib(a+b\arctan(c+dx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{2f} \\
&- \frac{3ib(a+b\arctan(c+dx))^2\operatorname{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f} \\
&- \frac{3b^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)}{2f} \\
&+ \frac{3b^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(3,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{2f} \\
&- \frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1-i(c+dx)}\right)}{4f} + \frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{4f}
\end{aligned}$$

output
```
-(a+b*arctan(d*x+c))^3*ln(2/(1-I*(d*x+c)))/f+(a+b*arctan(d*x+c))^3*ln(2*d*
(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f+3/2*I*b*(a+b*arctan(d*x+c))^2*polyl
og(2,1-2/(1-I*(d*x+c)))/f-3/2*I*b*(a+b*arctan(d*x+c))^2*polylog(2,1-2*d*(f
*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f-3/2*b^2*(a+b*arctan(d*x+c))*polylog(3
,1-2/(1-I*(d*x+c)))/f+3/2*b^2*(a+b*arctan(d*x+c))*polylog(3,1-2*d*(f*x+e)/
(d*e+I*f-c*f)/(1-I*(d*x+c)))/f-3/4*I*b^3*polylog(4,1-2/(1-I*(d*x+c)))/f+3/
4*I*b^3*polylog(4,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f
```

3.39.2 Mathematica [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{e+fx}\,dx=\int \frac{(a+b\arctan(c+dx))^3}{e+fx}\,dx$$

input
```
Integrate[(a + b*ArcTan[c + d*x])^3/(e + f*x),x]
```

output
```
Integrate[(a + b*ArcTan[c + d*x])^3/(e + f*x), x]
```

3.39.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5570, 27, 5385}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(c+dx))^3}{e+fx}\,dx$$

$$\Big\downarrow \text{5570}$$

$$\frac{\int \frac{d(a+b\arctan(c+dx))^3}{d\left(e-\frac{cf}{d}\right)+f(c+dx)}d(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+b\arctan(c+dx))^3}{f(c+dx)-cf+de}d(c+dx)$$

$$\Big\downarrow \text{5385}$$

$$\frac{3b^2(a+b\arctan(c+dx))\operatorname{PolyLog}\left(3,1-\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{2f} - \frac{3b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))}{2f} - \frac{3ib(a+b\arctan(c+dx))^2\operatorname{PolyLog}\left(2,1-\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{2f} + \frac{(a+b\arctan(c+dx))^3\log\left(\frac{2(f(c+dx)-cf+de)}{(1-i(c+dx))(-cf+de+if)}\right)}{f} + \frac{3ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))^2}{2f} - \frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\arctan(c+dx))^3}{f} + \frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2(de-cf+f(c+dx))}{(de-cf+if)(1-i(c+dx))}\right)}{4f} - \frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1-i(c+dx)}\right)}{4f}$$

input
```
Int[(a + b*ArcTan[c + d*x])^3/(e + f*x),x]
```

output
```
-(((a + b*ArcTan[c + d*x])^3*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcTan
[c + d*x])^3*Log[(2*(d*e - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(
c + d*x)))])/f + (((3*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, 1 - 2/(
1 - I*(c + d*x))])/f - (((3*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, 1
 - (2*(d*e - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f
 - (3*b^2*(a + b*ArcTan[c + d*x])*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*
f) + (3*b^2*(a + b*ArcTan[c + d*x])*PolyLog[3, 1 - (2*(d*e - c*f + f*(c +
d*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(2*f) - (((3*I)/4)*b^3*Poly
Log[4, 1 - 2/(1 - I*(c + d*x))])/f + (((3*I)/4)*b^3*PolyLog[4, 1 - (2*(d*e
 - c*f + f*(c + d*x)))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f
```

3.39.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5385
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^3/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^3)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^3*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[3
*I*b*(a + b*ArcTan[c*x])^2*(PolyLog[2, 1 - 2/(1 - I*c*x)]/(2*e)), x] - Simp
[3*I*b*(a + b*ArcTan[c*x])^2*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1
 - I*c*x)))]/(2*e)), x] - Simp[3*b^2*(a + b*ArcTan[c*x])*(PolyLog[3, 1 - 2/
(1 - I*c*x)]/(2*e)), x] + Simp[3*b^2*(a + b*ArcTan[c*x])*(PolyLog[3, 1 - 2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x] - Simp[3*I*b^3*(PolyLog
[4, 1 - 2/(1 - I*c*x)]/(4*e)), x] + Simp[3*I*b^3*(PolyLog[4, 1 - 2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(4*e)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& NeQ[c^2*d^2 + e^2, 0]
```

rule 5570
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.39.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.95 (sec) , antiderivative size = 3817, normalized size of antiderivative = 10.26

method	result	size
derivativedivides	Expression too large to display	3817
default	Expression too large to display	3817
parts	Expression too large to display	4072

input
```
int((a+b*arctan(d*x+c))^3/(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*d*ln(c*f-d*e-f*(d*x+c))/f-b^3*d*(-ln(c*f-d*e-f*(d*x+c))/f*arctan(
d*x+c)^3+3/f*(1/3*arctan(d*x+c)^3*ln(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f
*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d
*e)-I*d*e*arctan(d*x+c)^2*polylog(2,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+
c)^2)/(d*e+I*f-c*f))/(2*I*f+2*c*f-2*d*e)-1/2*I*arctan(d*x+c)^2*polylog(2,-
(1+I*(d*x+c))^2/(1+(d*x+c)^2))+1/2*arctan(d*x+c)*polylog(3,-(1+I*(d*x+c))^
2/(1+(d*x+c)^2))+1/2*I*c*f/(c*f-d*e+I*f)*arctan(d*x+c)^2*polylog(2,(c*f-d*
e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+1/4*I*polylog(4,-(1+I*
(d*x+c))^2/(1+(d*x+c)^2))-1/2*I*f/(c*f-d*e+I*f)*arctan(d*x+c)*polylog(3,(c
*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/2*f/(c*f-d*e+I*
f)*arctan(d*x+c)^2*polylog(2,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(
d*e+I*f-c*f))+1/4*f/(c*f-d*e+I*f)*polylog(4,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/
(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/3*c*f/(c*f-d*e+I*f)*arctan(d*x+c)^3*ln(1-(c
*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/2*c*f/(c*f-d*e+
I*f)*arctan(d*x+c)*polylog(3,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(
d*e+I*f-c*f))-1/6*I*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*
(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1
+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*(csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2
)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+
c*f-d*e))*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))-csgn(I*(I*f*(1+I*(d...
```

3.39.5 Fricas [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{e+fx}\,dx = \int \frac{(b\arctan(dx+c)+a)^3}{fx+e}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^3/(f*x+e),x, algorithm="fricas")
```

output
```
integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arct
an(d*x + c) + a^3)/(f*x + e), x)
```

3.39.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{e+fx}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(d*x+c))**3/(f*x+e),x)
```

output
```
Timed out
```

3.39.7 Maxima [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{e+fx}\,dx = \int \frac{(b\arctan(dx+c)+a)^3}{fx+e}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^3/(f*x+e),x, algorithm="maxima")
```

output
```
a^3*log(f*x + e)/f + integrate(1/32*(28*b^3*arctan(d*x + c)^3 + 3*b^3*arct
an(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 96*a*b^2*arctan(d*x + c)^
2 + 96*a^2*b*arctan(d*x + c))/(f*x + e), x)
```

3.39.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{e+fx}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(d*x+c))^3/(f*x+e),x, algorithm="giac")
```

output
```
Timed out
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{e+fx}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c+d\,x)\right)^3}{e+f\,x}\,dx$$

input
```
int((a + b*atan(c + d*x))^3/(e + f*x),x)
```

output
```
int((a + b*atan(c + d*x))^3/(e + f*x), x)
```

3.40 $\int \frac{(a+b\arctan(c+dx))^3}{(e+fx)^2}\,dx$

3.40.1 Optimal result

Integrand size = 20, antiderivative size = 1233

$$\int \frac{(a+b\arctan(c+dx))^3}{(e+fx)^2}\,dx = \frac{3a^2bd(de-cf)\arctan(c+dx)}{f\left(f^2+(de-cf)^2\right)} + \frac{3iab^2d\arctan(c+dx)^2}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$+\frac{3ab^2d(de-cf)\arctan(c+dx)^2}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)}$$

$$+\frac{ib^3d\arctan(c+dx)^3}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$+\frac{b^3d(de-cf)\arctan(c+dx)^3}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)} - \frac{(a+b\arctan(c+dx))^3}{f(e+fx)}$$

$$+\frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2} - \frac{6ab^2d\arctan(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$-\frac{3b^3d\arctan(c+dx)^2\log\left(\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$+\frac{6ab^2d\arctan(c+dx)\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$+\frac{3b^3d\arctan(c+dx)^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$+\frac{6ab^2d\arctan(c+dx)\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$+\frac{3b^3d\arctan(c+dx)^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$-\frac{3a^2bd\log\left(1+(c+dx)^2\right)}{2\left(f^2+(de-cf)^2\right)}$$

$$+\frac{3iab^2d\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$+\frac{3ib^3d\arctan(c+dx)\,\text{PolyLog}\left(2,1-\frac{2}{1-i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$-\frac{3iab^2d\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$-\frac{3ib^3d\arctan(c+dx)\,\text{PolyLog}\left(2,1-\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$+\frac{3iab^2d\,\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$+\frac{3ib^3d\arctan(c+dx)\,\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{d^2e^2-2cdef+(1+c^2)f^2}$$

$$\frac{3b^3d\,\text{PolyLog}\left(3,1-\frac{2}{\cdots}\right)}{\cdots}$$

output
```
3*a^2*b*d*(-c*f+d*e)*arctan(d*x+c)/f/(f^2+(-c*f+d*e)^2)+3*I*b^3*d*arctan(d
*x+c)*polylog(2,1-2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*a*b^2
*d*(-c*f+d*e)*arctan(d*x+c)^2/f/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*I*a*b^2*
d*polylog(2,1-2/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+b^3*d*(-c*f
+d*e)*arctan(d*x+c)^3/f/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-(a+b*arctan(d*x+c)
)^3/f/(f*x+e)+3*a^2*b*d*ln(f*x+e)/(f^2+(-c*f+d*e)^2)-6*a*b^2*d*arctan(d*x+
c)*ln(2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3*b^3*d*arctan(d*x+
c)^2*ln(2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+6*a*b^2*d*arctan(
d*x+c)*ln(2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2
+1)*f^2)+3*b^3*d*arctan(d*x+c)^2*ln(2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)
))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+6*a*b^2*d*arctan(d*x+c)*ln(2/(1+I*(d*x+
c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*b^3*d*arctan(d*x+c)^2*ln(2/(1+I*(d*
x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3/2*a^2*b*d*ln(1+(d*x+c)^2)/(f^2+(-
c*f+d*e)^2)+I*b^3*d*arctan(d*x+c)^3/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3*I*b^
3*d*arctan(d*x+c)*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^
2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*I*b^3*d*arctan(d*x+c)*polylog(2,1-2/(1+I*(d
*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3*I*a*b^2*d*polylog(2,1-2*d*(f*x+e
)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*I*a*b^2*d
*polylog(2,1-2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*I*a*b^2*d*
arctan(d*x+c)^2/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3/2*b^3*d*polylog(3,1-2...
```

3.40.2 Mathematica [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(e+fx)^2}\,dx = \int \frac{(a+b\arctan(c+dx))^3}{(e+fx)^2}\,dx$$

input
```
Integrate[(a + b*ArcTan[c + d*x])^3/(e + f*x)^2,x]
```

output
```
Integrate[(a + b*ArcTan[c + d*x])^3/(e + f*x)^2, x]
```

3.40.3 Rubi [A] (verified)

Time = 2.24 (sec) , antiderivative size = 1265, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5568, 7292, 5580, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(c+dx))^3}{(e+fx)^2}dx$$

$$\Big\downarrow \text{5568}$$

$$\frac{3bd\int \frac{(a+b\arctan(c+dx))^2}{(e+fx)((c+dx)^2+1)}dx}{f}-\frac{(a+b\arctan(c+dx))^3}{f(e+fx)}$$

$$\Big\downarrow \text{7292}$$

$$\frac{3bd\int \frac{(a+b\arctan(c+dx))^2}{(e+fx)(c^2+2dxc+d^2x^2+1)}dx}{f}-\frac{(a+b\arctan(c+dx))^3}{f(e+fx)}$$

$$\Big\downarrow \text{5580}$$

$$\frac{3b\int \frac{d(a+b\arctan(c+dx))^2}{\left(d\left(e-\frac{cf}{d}\right)+f(c+dx)\right)((c+dx)^2+1)}d(c+dx)}{f}-\frac{(a+b\arctan(c+dx))^3}{f(e+fx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{3bd\int \frac{(a+b\arctan(c+dx))^2}{(de-cf+f(c+dx))((c+dx)^2+1)}d(c+dx)}{f}-\frac{(a+b\arctan(c+dx))^3}{f(e+fx)}$$

$$\Big\downarrow \text{7276}$$

$$\frac{3bd\int \left(\frac{a^2}{(de-cf+f(c+dx))((c+dx)^2+1)}+\frac{2b\arctan(c+dx)a}{(de-cf+f(c+dx))((c+dx)^2+1)}+\frac{b^2\arctan(c+dx)^2}{(de-cf+f(c+dx))((c+dx)^2+1)}\right)d(c+dx)}{f}-\frac{(a+b\arctan(c+dx))^3}{f(e+fx)}$$

$$\Big\downarrow \text{2009}$$

$$3bd\left(\frac{ib^2f\arctan(c+dx)^3}{3(d^2e^2-2cdfe+(c^2+1)f^2)}+\frac{b^2(de-cf)\arctan(c+dx)^3}{3(d^2e^2-2cdfe+(c^2+1)f^2)}-\frac{b^2f\log\left(\frac{2}{1-i(c+dx)}\right)\arctan(c+dx)^2}{d^2e^2-2cdfe+(c^2+1)f^2}+\frac{b^2f\log\left(\frac{2}{i(c+dx)+1}\right)\arctan(c+dx)^2}{d^2e^2-2cdfe+(c^2+1)f^2}+\right.$$

$$\frac{(a+b\arctan(c+dx))^3}{f(e+fx)}$$

input
```
Int[(a + b*ArcTan[c + d*x])^3/(e + f*x)^2,x]
```

output
```
-((a + b*ArcTan[c + d*x])^3/(f*(e + f*x))) + (3*b*d*((a^2*(d*e - c*f)*ArcT
an[c + d*x])/(f^2 + (d*e - c*f)^2) + (I*a*b*f*ArcTan[c + d*x]^2)/(d^2*e^2
- 2*c*d*e*f + (1 + c^2)*f^2) + (a*b*(d*e - c*f)*ArcTan[c + d*x]^2)/(d^2*e^
2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((I/3)*b^2*f*ArcTan[c + d*x]^3)/(d^2*e^2
- 2*c*d*e*f + (1 + c^2)*f^2) + (b^2*(d*e - c*f)*ArcTan[c + d*x]^3)/(3*(d^2
*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)) - (2*a*b*f*ArcTan[c + d*x]*Log[2/(1 - I
*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (b^2*f*ArcTan[c + d*
x]^2*Log[2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (2*
a*b*f*ArcTan[c + d*x]*Log[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1
+ c^2)*f^2) + (b^2*f*ArcTan[c + d*x]^2*Log[2/(1 + I*(c + d*x))])/(d^2*e^2
- 2*c*d*e*f + (1 + c^2)*f^2) + (a^2*f*Log[d*e - c*f + f*(c + d*x)])/(f^2 +
 (d*e - c*f)^2) + (2*a*b*f*ArcTan[c + d*x]*Log[(2*(d*e - c*f + f*(c + d*x)
))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2
)*f^2) + (b^2*f*ArcTan[c + d*x]^2*Log[(2*(d*e - c*f + f*(c + d*x)))/((d*e
+ I*f - c*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) -
(a^2*f*Log[1 + (c + d*x)^2])/(2*(f^2 + (d*e - c*f)^2)) + (I*a*b*f*PolyLog[
2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (I*b^
2*f*ArcTan[c + d*x]*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*
e*f + (1 + c^2)*f^2) + (I*a*b*f*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d^2*
e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (I*b^2*f*ArcTan[c + d*x]*PolyLog[2, ...
```

3.40.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5568
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcTan[
c + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[p, 0] && ILtQ[m, -1]
```

rule 5580
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subs
t[Int[(((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTan[x])
^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &
& EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

3.40.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.29 (sec) , antiderivative size = 3708, normalized size of antiderivative = 3.01

method	result	size
derivativedivides	Expression too large to display	3708
default	Expression too large to display	3708
parts	Expression too large to display	3834

input
```
int((a+b*arctan(d*x+c))^3/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*d^2/(c*f-d*e-f*(d*x+c))/f+b^3*d^2*(1/(c*f-d*e-f*(d*x+c))/f*arctan
(d*x+c)^3-3/f*(1/2*arctan(d*x+c)^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*ln(1+
(d*x+c)^2)+arctan(d*x+c)^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*c*f-arctan(d*x+
c)^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*d*e-arctan(d*x+c)^2*f/(c^2*f^2-2*c*d*
e*f+d^2*e^2+f^2)*ln(c*f-d*e-f*(d*x+c))-f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*a
rctan(d*x+c)^2*ln((1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+f/(c^2*f^2-2*c*d*e*f+
d^2*e^2+f^2)*arctan(d*x+c)^2*ln(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I
*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)-1
/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f^2/(c*f-d*e+I*f)*arctan(d*x+c)*polylog(2
,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-I/(c^2*f^2-2*c
*d*e*f+d^2*e^2+f^2)*f^2/(c*f-d*e+I*f)*arctan(d*x+c)^2*ln(1-(c*f-d*e+I*f)*(
1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+I/(c^2*f^2-2*c*d*e*f+d^2*e^2+f
^2)*f^2*c/(c*f-d*e+I*f)*arctan(d*x+c)*polylog(2,(c*f-d*e+I*f)*(1+I*(d*x+c)
)^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/2*I/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f^2
/(c*f-d*e+I*f)*polylog(3,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+
I*f-c*f))-1/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f^2*c/(c*f-d*e+I*f)*arctan(d*x
+c)^2*ln(1-(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-1/2/
(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f^2*c/(c*f-d*e+I*f)*polylog(3,(c*f-d*e+I*f
)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+1/3*I*f/(c^2*f^2-2*c*d*e*f+
d^2*e^2+f^2)*arctan(d*x+c)^3+1/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*f*d*e/(c...
```

3.40.5 Fricas [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(e+fx)^2}\,dx=\int \frac{(b\arctan(dx+c)+a)^3}{(fx+e)^2}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^3/(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arct
an(d*x + c) + a^3)/(f^2*x^2 + 2*e*f*x + e^2), x)
```

3.40.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{(e+fx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(d*x+c))**3/(f*x+e)**2,x)
```

output
```
Timed out
```

3.40.7 Maxima [F]

$$\int \frac{(a+b\arctan(c+dx))^3}{(e+fx)^2}\,dx = \int \frac{(b\arctan(dx+c)+a)^3}{(fx+e)^2}\,dx$$

input
```
integrate((a+b*arctan(d*x+c))^3/(f*x+e)^2,x, algorithm="maxima")
```

output
```
3/2*(d*(2*(d^2*e - c*d*f)*arctan((d^2*x + c*d)/d)/((d^2*e^2*f - 2*c*d*e*f^
2 + (c^2 + 1)*f^3)*d) - log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*e^2 - 2*c*d*
e*f + (c^2 + 1)*f^2) + 2*log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 + 1)*f^2
)) - 2*arctan(d*x + c)/(f^2*x + e*f))*a^2*b - a^3/(f^2*x + e*f) - 1/32*(4*
b^3*arctan(d*x + c)^3 - 3*b^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2
+ 1)^2 - 32*(f^2*x + e*f)*integrate(1/32*(28*(b^3*d^2*f*x^2 + 2*b^3*c*d*f*
x + (b^3*c^2 + b^3)*f)*arctan(d*x + c)^3 + 12*(8*a*b^2*d^2*f*x^2 + b^3*d*e
 + (16*a*b^2*c + b^3)*d*f*x + 8*(a*b^2*c^2 + a*b^2)*f)*arctan(d*x + c)^2 -
 12*(b^3*d^2*f*x^2 + b^3*c*d*e + (b^3*d^2*e + b^3*c*d*f)*x)*arctan(d*x + c
)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 3*(b^3*d*f*x + b^3*d*e - (b^3*d^2*f*x
^2 + 2*b^3*c*d*f*x + (b^3*c^2 + b^3)*f)*arctan(d*x + c))*log(d^2*x^2 + 2*c
*d*x + c^2 + 1)^2)/(d^2*f^3*x^4 + (c^2 + 1)*e^2*f + 2*(d^2*e*f^2 + c*d*f^3
)*x^3 + (d^2*e^2*f + 4*c*d*e*f^2 + (c^2 + 1)*f^3)*x^2 + 2*(c*d*e^2*f + (c^
2 + 1)*e*f^2)*x), x))/(f^2*x + e*f)
```

3.40.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{(e+fx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(d*x+c))^3/(f*x+e)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(c+dx))^3}{(e+fx)^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c+d\,x))^3}{(e+f\,x)^2}\,dx$$

input
```
int((a + b*atan(c + d*x))^3/(e + f*x)^2,x)
```

output
```
int((a + b*atan(c + d*x))^3/(e + f*x)^2, x)
```

3.41 $\int (e+fx)^m(a+b\arctan(c+dx))\,dx$

3.41.1 Optimal result

Integrand size = 18, antiderivative size = 177

$$\begin{aligned}\int (e+fx)^m(a+b\arctan(c+dx))\,dx &= \frac{(e+fx)^{1+m}(a+b\arctan(c+dx))}{f(1+m)} \\ &\quad - \frac{ibd(e+fx)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{d(e+fx)}{de+if-cf}\right)}{2f(de+(i-c)f)(1+m)(2+m)} \\ &\quad + \frac{ibd(e+fx)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{d(e+fx)}{de-(i+c)f}\right)}{2f(de-(i+c)f)(1+m)(2+m)}\end{aligned}$$

output
```
(f*x+e)^(1+m)*(a+b*arctan(d*x+c))/f/(1+m)-1/2*I*b*d*(f*x+e)^(2+m)*hypergeo
m([1, 2+m],[3+m],d*(f*x+e)/(d*e+I*f-c*f))/f/(d*e+(I-c)*f)/(1+m)/(2+m)+1/2*
I*b*d*(f*x+e)^(2+m)*hypergeom([1, 2+m],[3+m],d*(f*x+e)/(d*e-(I+c)*f))/f/(d
*e-(I+c)*f)/(1+m)/(2+m)
```

3.41.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.92

$$\int (e+fx)^m(a+b\arctan(c+dx))\,dx$$
$$=\frac{(e+fx)^{1+m}\left(2(a+b\arctan(c+dx))+\frac{bd(e+fx)\left((de-(i+c)f)\operatorname{Hypergeometric2F1}\left(1,2+m,3+m,\frac{d(e+fx)}{de-(-i+c)f}\right)+(-de+(-i+c\right.}{(ide+f-icf)(de-(-i+c)f)(2+m)}}{2f(1+m)}$$

input
```
Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x]),x]
```

output
```
((e + f*x)^(1 + m)*(2*(a + b*ArcTan[c + d*x]) + (b*d*(e + f*x)*((d*e - (I
+ c)*f)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - (-I + c)*f
)] + (-(d*e) + (-I + c)*f)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x)
)/(d*e - (I + c)*f)]))/((I*d*e + f - I*c*f)*(d*e - (-I + c)*f)*(2 + m))))/
(2*f*(1 + m))
```

3.41.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.34, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5570, 5387, 485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^m(a+b\arctan(c+dx))\,dx$$
$$\Big\downarrow \text{5570}$$
$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m(a+b\arctan(c+dx))d(c+dx)}{d}$$
$$\Big\downarrow \text{5387}$$
$$\frac{\frac{d(a+b\arctan(c+dx))\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right)^{m+1}}{f(m+1)}-\frac{bd\int \frac{\left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^{m+1}}{(c+dx)^2+1}d(c+dx)}{f(m+1)}}{d}$$
$$\Big\downarrow \text{485}$$

$$\frac{\frac{d(a+b\arctan(c+dx))\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right)^{m+1}}{f(m+1)}-\frac{bd\int\left(\frac{i\left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^{m+1}}{2(-c-dx+i)}+\frac{i\left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^{m+1}}{2(c+dx+i)}\right)d(c+dx)}{f(m+1)}}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{d(a+b\arctan(c+dx))\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right)^{m+1}}{f(m+1)}-\frac{bd\left(\frac{id\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{de-cf+f(c+dx)}{de-cf+if}\right)}{2(m+2)(de+(-c+i)f)}-\frac{id\left(\frac{f(c+dx)}{d}-\frac{cf}{d}+e\right.}{}\right.}{f(m+1)}}{d}$$

input
```
Int[(e + f*x)^m*(a + b*ArcTan[c + d*x]),x]
```

output
```
((d*(e - (c*f)/d + (f*(c + d*x))/d)^(1 + m)*(a + b*ArcTan[c + d*x]))/(f*(1
 + m)) - (b*d*(((I/2)*d*(e - (c*f)/d + (f*(c + d*x))/d)^(2 + m)*Hypergeome
tric2F1[1, 2 + m, 3 + m, (d*e - c*f + f*(c + d*x))/(d*e + I*f - c*f)])/((d
*e + (I - c)*f)*(2 + m)) - ((I/2)*d*(e - (c*f)/d + (f*(c + d*x))/d)^(2 + m
)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*e - c*f + f*(c + d*x))/(d*e - (I +
 c)*f)])/((d*e - (I + c)*f)*(2 + m))))/(f*(1 + m)))/d
```

3.41.3.1 Defintions of rubi rules used

rule 485
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n, 1/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, n}, x] &
&  !IntegerQ[2*n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

rule 5570
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.41.4 Maple [F]

$$\int (fx+e)^m (a+b\arctan(dx+c))\,dx$$

input
```
int((f*x+e)^m*(a+b*arctan(d*x+c)),x)
```

output
```
int((f*x+e)^m*(a+b*arctan(d*x+c)),x)
```

3.41.5 Fricas [F]

$$\int (e+fx)^m(a+b\arctan(c+dx))\,dx = \int (b\arctan(dx+c)+a)(fx+e)^m\,dx$$

input
```
integrate((f*x+e)^m*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

output
```
integral((b*arctan(d*x + c) + a)*(f*x + e)^m, x)
```

3.41.6 Sympy [F(-1)]

Timed out.

$$\int (e+fx)^m(a+b\arctan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**m*(a+b*atan(d*x+c)),x)
```

output
```
Timed out
```

3.41.7 Maxima [F]

$$\int (e+fx)^m(a+b\arctan(c+dx))\,dx = \int (b\arctan(dx+c)+a)(fx+e)^m\,dx$$

input
```
integrate((f*x+e)^m*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*((3*e*m^2 + 2*e*m + (3*f*m^2 + 2*f*m + f)*x + e)*(f*x + e)^m*arctan(d*
x + c) + (e*m + (f*m + f)*x + e)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^2 +
 1) + 2*(f*m^3 + f*m^2 + f*m + f)*integrate(-1/2*(2*((c^2 + 1)*f*m^3 + 2*(
c^2 + 1)*f*m^2 + (c^2 + 1)*f*m + (d^2*f*m^3 + 2*d^2*f*m^2 + d^2*f*m)*x^2 +
 2*(c*d*f*m^3 + 2*c*d*f*m^2 + c*d*f*m)*x)*(f*x + e)^m*arctan(d*x + c) - ((
c^2 + 1)*f*m^3 - (c^2 + 1)*f*m + (d^2*f*m^3 - d^2*f*m)*x^2 + 2*(c*d*f*m^3
- c*d*f*m)*x)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 2*((c - 1)*d*
e*m^2 - (c + 1)*d*e*m + (d^2*f*m^2 - d^2*f*m)*x^2 + ((d^2*e + (c - 1)*d*f)
*m^2 - (d^2*e + (c + 1)*d*f)*m)*x)*(f*x + e)^m)/((c^2 + 1)*f*m^3 + (c^2 +
1)*f*m^2 + (c^2 + 1)*f*m + (d^2*f*m^3 + d^2*f*m^2 + d^2*f*m + d^2*f)*x^2 +
 (c^2 + 1)*f + 2*(c*d*f*m^3 + c*d*f*m^2 + c*d*f*m + c*d*f)*x), x))*b/(f*m^
3 + f*m^2 + f*m + f) + (f*x + e)^(m + 1)*a/(f*(m + 1))
```

3.41.8 Giac [F]

$$\int (e+fx)^m(a+b\arctan(c+dx))\,dx = \int (b\arctan(dx+c)+a)(fx+e)^m\,dx$$

input
```
integrate((f*x+e)^m*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

output
```
sage0*x
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^m(a+b\arctan(c+dx))\,dx = \int (e+f\,x)^m\,(a+b\,\mathrm{atan}(c+d\,x))\,dx$$

input
```
int((e + f*x)^m*(a + b*atan(c + d*x)),x)
```

output
```
int((e + f*x)^m*(a + b*atan(c + d*x)), x)
```

3.42 $\int (e+fx)^m (a+b\arctan(c+dx))^2\,dx$

3.42.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (e+fx)^m (a+b\arctan(c+dx))^2\,dx = \text{Int}\left((e+fx)^m (a+b\arctan(c+dx))^2, x\right)$$

output
```
Unintegrable((f*x+e)^m*(a+b*arctan(d*x+c))^2,x)
```

3.42.2 Mathematica [N/A]

Not integrable

Time = 4.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m (a+b\arctan(c+dx))^2\,dx = \int (e+fx)^m (a+b\arctan(c+dx))^2\,dx$$

input
```
Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x])^2,x]
```

output
```
Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x])^2, x]
```

3.42.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5570, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^m(a+b\arctan(c+dx))^2\,dx$$

$$\Big\downarrow \text{5570}$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m(a+b\arctan(c+dx))^2d(c+dx)}{d}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m(a+b\arctan(c+dx))^2d(c+dx)}{d}$$

input
```
Int[(e + f*x)^m*(a + b*ArcTan[c + d*x])^2,x]
```

output
```
$Aborted
```

3.42.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

rule 5570
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.42.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (fx+e)^m (a+b\arctan(dx+c))^2 \, dx$$

input
```
int((f*x+e)^m*(a+b*arctan(d*x+c))^2,x)
```

output
```
int((f*x+e)^m*(a+b*arctan(d*x+c))^2,x)
```

3.42.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int (e+fx)^m (a+b\arctan(c+dx))^2 \, dx = \int (b\arctan(dx+c)+a)^2 (fx+e)^m \, dx$$

input
```
integrate((f*x+e)^m*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)*(f*x + e)^m
, x)
```

3.42.6 Sympy [F(-1)]

Timed out.

$$\int (e+fx)^m (a+b\arctan(c+dx))^2 \, dx = \text{Timed out}$$

input
```
integrate((f*x+e)**m*(a+b*atan(d*x+c))**2,x)
```

output
```
Timed out
```

3.42.7 Maxima [N/A]

Not integrable

Time = 8.77 (sec) , antiderivative size = 504, normalized size of antiderivative = 25.20

$$\int (e+fx)^m(a+b\arctan(c+dx))^2\,dx=\int (b\arctan(dx+c)+a)^2(fx+e)^m\,dx$$

input
```
integrate((f*x+e)^m*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

output
```
(f*x + e)^(m + 1)*a^2/(f*(m + 1)) + 1/16*(4*(b^2*f*x + b^2*e)*(f*x + e)^m*
arctan(d*x + c)^2 - (b^2*f*x + b^2*e)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x +
c^2 + 1)^2 + 16*(f*m + f)*integrate(1/16*(12*((b^2*c^2 + b^2)*f*m + (b^2*d
^2*f*m + b^2*d^2*f)*x^2 + (b^2*c^2 + b^2)*f + 2*(b^2*c*d*f*m + b^2*c*d*f)*
x)*(f*x + e)^m*arctan(d*x + c)^2 + ((b^2*c^2 + b^2)*f*m + (b^2*d^2*f*m + b
^2*d^2*f)*x^2 + (b^2*c^2 + b^2)*f + 2*(b^2*c*d*f*m + b^2*c*d*f)*x)*(f*x +
e)^m*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 - 8*(b^2*d*e - 4*(a*b*c^2 + a*b)*f
*m - 4*(a*b*d^2*f*m + a*b*d^2*f)*x^2 - 4*(a*b*c^2 + a*b)*f - (8*a*b*c*d*f*
m + (8*a*b*c - b^2)*d*f)*x)*(f*x + e)^m*arctan(d*x + c) + 4*(b^2*d^2*f*x^2
 + b^2*c*d*e + (b^2*d^2*e + b^2*c*d*f)*x)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*
x + c^2 + 1))/((c^2 + 1)*f*m + (d^2*f*m + d^2*f)*x^2 + (c^2 + 1)*f + 2*(c*
d*f*m + c*d*f)*x), x))/(f*m + f)
```

3.42.8 Giac [N/A]

Not integrable

Time = 111.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int (e+fx)^m(a+b\arctan(c+dx))^2\,dx=\int (b\arctan(dx+c)+a)^2(fx+e)^m\,dx$$

input
```
integrate((f*x+e)^m*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.42.9 Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m(a+b\arctan(c+dx))^2\,dx=\int (e+f\,x)^m\,(a+b\,\mathrm{atan}(c+d\,x))^2\,dx$$

input
```
int((e + f*x)^m*(a + b*atan(c + d*x))^2,x)
```

output
```
int((e + f*x)^m*(a + b*atan(c + d*x))^2, x)
```

3.43 $\int (e+fx)^m(a+b\arctan(c+dx))^3\,dx$

3.43.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (e+fx)^m(a+b\arctan(c+dx))^3\,dx = \operatorname{Int}\left((e+fx)^m(a+b\arctan(c+dx))^3,x\right)$$

output
```
Unintegrable((f*x+e)^m*(a+b*arctan(d*x+c))^3,x)
```

3.43.2 Mathematica [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m(a+b\arctan(c+dx))^3\,dx = \int (e+fx)^m(a+b\arctan(c+dx))^3\,dx$$

input
```
Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x])^3,x]
```

output
```
Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x])^3, x]
```

3.43.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5570, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^m(a+b\arctan(c+dx))^3\,dx$$

$$\Big\downarrow \text{5570}$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m(a+b\arctan(c+dx))^3d(c+dx)}{d}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\int \left(e-\frac{cf}{d}+\frac{f(c+dx)}{d}\right)^m(a+b\arctan(c+dx))^3d(c+dx)}{d}$$

input
```
Int[(e + f*x)^m*(a + b*ArcTan[c + d*x])^3,x]
```

output
```
$Aborted
```

3.43.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

rule 5570
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.43.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (fx+e)^m (a+b\arctan(dx+c))^3 \, dx$$

input
```
int((f*x+e)^m*(a+b*arctan(d*x+c))^3,x)
```

output
```
int((f*x+e)^m*(a+b*arctan(d*x+c))^3,x)
```

3.43.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int (e+fx)^m(a+b\arctan(c+dx))^3 \, dx = \int (b\arctan(dx+c)+a)^3(fx+e)^m \, dx$$

input
```
integrate((f*x+e)^m*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arct
an(d*x + c) + a^3)*(f*x + e)^m, x)
```

3.43.6 Sympy [F(-1)]

Timed out.

$$\int (e+fx)^m(a+b\arctan(c+dx))^3 \, dx = \text{Timed out}$$

input
```
integrate((f*x+e)**m*(a+b*atan(d*x+c))**3,x)
```

output
```
Timed out
```

3.43.7 Maxima [N/A]

Not integrable

Time = 11.47 (sec) , antiderivative size = 659, normalized size of antiderivative = 32.95

$$\int (e+fx)^m(a+b\arctan(c+dx))^3\,dx=\int (b\arctan(dx+c)+a)^3(fx+e)^m\,dx$$

input
```
integrate((f*x+e)^m*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

output
```
(f*x + e)^(m + 1)*a^3/(f*(m + 1)) + 1/32*(4*(b^3*f*x + b^3*e)*(f*x + e)^m*
arctan(d*x + c)^3 - 3*(b^3*f*x + b^3*e)*(f*x + e)^m*arctan(d*x + c)*log(d^
2*x^2 + 2*c*d*x + c^2 + 1)^2 + 32*(f*m + f)*integrate(1/32*(28*((b^3*c^2 +
 b^3)*f*m + (b^3*d^2*f*m + b^3*d^2*f)*x^2 + (b^3*c^2 + b^3)*f + 2*(b^3*c*d
*f*m + b^3*c*d*f)*x)*(f*x + e)^m*arctan(d*x + c)^3 - 12*(b^3*d*e - 8*(a*b^
2*c^2 + a*b^2)*f*m - 8*(a*b^2*d^2*f*m + a*b^2*d^2*f)*x^2 - 8*(a*b^2*c^2 +
a*b^2)*f - (16*a*b^2*c*d*f*m + (16*a*b^2*c - b^3)*d*f)*x)*(f*x + e)^m*arct
an(d*x + c)^2 + 12*(b^3*d^2*f*x^2 + b^3*c*d*e + (b^3*d^2*e + b^3*c*d*f)*x)
*(f*x + e)^m*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 96*((a^2*b
*c^2 + a^2*b)*f*m + (a^2*b*d^2*f*m + a^2*b*d^2*f)*x^2 + (a^2*b*c^2 + a^2*b
)*f + 2*(a^2*b*c*d*f*m + a^2*b*c*d*f)*x)*(f*x + e)^m*arctan(d*x + c) + 3*(
((b^3*c^2 + b^3)*f*m + (b^3*d^2*f*m + b^3*d^2*f)*x^2 + (b^3*c^2 + b^3)*f +
 2*(b^3*c*d*f*m + b^3*c*d*f)*x)*(f*x + e)^m*arctan(d*x + c) + (b^3*d*f*x +
 b^3*d*e)*(f*x + e)^m)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2)/((c^2 + 1)*f*m
+ (d^2*f*m + d^2*f)*x^2 + (c^2 + 1)*f + 2*(c*d*f*m + c*d*f)*x), x))/(f*m +
 f)
```

3.43.8 Giac [N/A]

Not integrable

Time = 112.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int (e+fx)^m(a+b\arctan(c+dx))^3\,dx=\int (b\arctan(dx+c)+a)^3(fx+e)^m\,dx$$

input
```
integrate((f*x+e)^m*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.43.9 Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (e+fx)^m(a+b\arctan(c+dx))^3\,dx=\int (e+f\,x)^m\,(a+b\,\mathrm{atan}(c+d\,x))^3\,dx$$

input
```
int((e + f*x)^m*(a + b*atan(c + d*x))^3,x)
```

output
```
int((e + f*x)^m*(a + b*atan(c + d*x))^3, x)
```

3.44 $\int x^3 \arctan(a+bx)\,dx$

3.44.1 Optimal result

Integrand size = 10, antiderivative size = 106

$$\int x^3 \arctan(a+bx)\,dx = \frac{(1-6a^2)\,x}{4b^3} + \frac{a(a+bx)^2}{2b^4} - \frac{(a+bx)^3}{12b^4} - \frac{(1-6a^2+a^4)\arctan(a+bx)}{4b^4} + \frac{1}{4}x^4\arctan(a+bx) - \frac{a(1-a^2)\log\left(1+(a+bx)^2\right)}{2b^4}$$

output
```
1/4*(-6*a^2+1)*x/b^3+1/2*a*(b*x+a)^2/b^4-1/12*(b*x+a)^3/b^4-1/4*(a^4-6*a^2
+1)*arctan(b*x+a)/b^4+1/4*x^4*arctan(b*x+a)-1/2*a*(-a^2+1)*ln(1+(b*x+a)^2)
/b^4
```

3.44.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int x^3 \arctan(a+bx)\,dx = \frac{6(1-6a^2)\,bx + 12a(a+bx)^2 - 2(a+bx)^3 + 6b^4x^4\arctan(a+bx) + 3i(-i+a)^4\log(i-a-bx) - 3i(i+}{24b^4}$$

input
```
Integrate[x^3*ArcTan[a + b*x],x]
```

output
```
(6*(1 - 6*a^2)*b*x + 12*a*(a + b*x)^2 - 2*(a + b*x)^3 + 6*b^4*x^4*ArcTan[a
 + b*x] + (3*I)*(-I + a)^4*Log[I - a - b*x] - (3*I)*(I + a)^4*Log[I + a +
b*x])/(24*b^4)
```

3.44.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5570, 25, 27, 5387, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(a+bx)\,dx$$

$$\Big\downarrow \text{5570}$$

$$\frac{\int x^3 \arctan(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -x^3 \arctan(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -b^3x^3 \arctan(a+bx)d(a+bx)}{b^4}$$

$$\Big\downarrow \text{5387}$$

$$-\frac{\frac{1}{4}\int \frac{b^4x^4}{(a+bx)^2+1}d(a+bx)-\frac{1}{4}b^4x^4\arctan(a+bx)}{b^4}$$

$$\Big\downarrow \text{478}$$

$$-\frac{\frac{1}{4}\int \left(6a^2-4(a+bx)a+(a+bx)^2+\frac{a^4-6a^2+4(1-a^2)(a+bx)a+1}{(a+bx)^2+1}-1\right)d(a+bx)-\frac{1}{4}b^4x^4\arctan(a+bx)}{b^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{4}\left(-\left(1-6a^2\right)(a+bx)+2a\left(1-a^2\right)\log\left((a+bx)^2+1\right)+\left(a^4-6a^2+1\right)\arctan(a+bx)+\frac{1}{3}(a+bx)^3-2a(a+\right.}{b^4}$$

input
```
Int[x^3*ArcTan[a + b*x],x]
```

output
```
-((-1/4*(b^4*x^4*ArcTan[a + b*x]) + (-((1 - 6*a^2)*(a + b*x)) - 2*a*(a + b
*x)^2 + (a + b*x)^3/3 + (1 - 6*a^2 + a^4)*ArcTan[a + b*x] + 2*a*(1 - a^2)*
Log[1 + (a + b*x)^2])/4)/b^4)
```

3.44.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

rule 5570
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.44.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.24

method	result
parallelrisch	$\frac{3\arctan(bx+a)x^4b^4-b^3x^3+3ab^2x^2-3\arctan(bx+a)a^4+6a^3\ln(b^2x^2+2abx+a^2+1)-9a^2bx+18\arctan(bx+a)a^2+15a^3}{12b^4}$
parts	$\frac{x^4\arctan(bx+a)}{4}-\frac{b\left(\frac{\frac{1}{3}b^2x^3-ab\,x^2+3a^2x-x}{b^4}+\frac{(-4a^3b+4ab)\ln(b^2x^2+2abx+a^2+1)}{2b^2}+\frac{\left(-3a^4-2a^2+1-\frac{(-4a^3b+4ab)a}{b}\right)\text{arc}}{b\;b^4}\right.}{4}$
derivativedivides	$\frac{\frac{\arctan(bx+a)a^4}{4}-\arctan(bx+a)a^3(bx+a)+\frac{3\arctan(bx+a)a^2(bx+a)^2}{2}-\arctan(bx+a)a(bx+a)^3+\frac{\arctan(bx+a)(bx+a)^4}{4}-\frac{3a^2}{}}{b^4}$
default	$\frac{\frac{\arctan(bx+a)a^4}{4}-\arctan(bx+a)a^3(bx+a)+\frac{3\arctan(bx+a)a^2(bx+a)^2}{2}-\arctan(bx+a)a(bx+a)^3+\frac{\arctan(bx+a)(bx+a)^4}{4}-\frac{3a^2}{}}{b^4}$
risch	$-\frac{ix^4\ln(1+i(bx+a))}{8}+\frac{ix^4\ln(1-i(bx+a))}{8}-\frac{x^3}{12b}-\frac{a^4\arctan(bx+a)}{4b^4}+\frac{a\,x^2}{4b^2}+\frac{a^3\ln(b^2x^2+2abx+a^2+1)}{2b^4}-\frac{3a}{4}$

input
```
int(x^3*arctan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/12*(3*arctan(b*x+a)*x^4*b^4-b^3*x^3+3*a*b^2*x^2-3*arctan(b*x+a)*a^4+6*a^
3*ln(b^2*x^2+2*a*b*x+a^2+1)-9*a^2*b*x+18*arctan(b*x+a)*a^2+15*a^3-6*a*ln(b
^2*x^2+2*a*b*x+a^2+1)+3*b*x-3*arctan(b*x+a)-9*a)/b^4
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.82

$$\int x^3\arctan(a+bx)\,dx = -\frac{b^3x^3-3\,ab^2x^2+3\,(3\,a^2-1)bx-3\,(b^4x^4-a^4+6\,a^2-1)\arctan(bx+a)-6\,(a^3-a)\log(b^2x^2+2\,abx}{12\,b^4}$$

input
```
integrate(x^3*arctan(b*x+a),x, algorithm="fricas")
```

output
```
-1/12*(b^3*x^3 - 3*a*b^2*x^2 + 3*(3*a^2 - 1)*b*x - 3*(b^4*x^4 - a^4 + 6*a^
2 - 1)*arctan(b*x + a) - 6*(a^3 - a)*log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^4
```

3.44.6 Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.46

$$\int x^3\arctan(a+bx)\,dx$$
$$=\begin{cases} -\frac{a^4\operatorname{atan}(a+bx)}{4b^4}+\frac{a^3\log(a^2+2abx+b^2x^2+1)}{2b^4}-\frac{3a^2x}{4b^3}+\frac{3a^2\operatorname{atan}(a+bx)}{2b^4}+\frac{ax^2}{4b^2}-\frac{a\log(a^2+2abx+b^2x^2+1)}{2b^4}+\frac{x^4\operatorname{atan}(a+bx)}{4}- \\ \frac{x^4\operatorname{atan}(a)}{4} \end{cases}$$

input
```
integrate(x**3*atan(b*x+a),x)
```

output
```
Piecewise((-a**4*atan(a + b*x)/(4*b**4) + a**3*log(a**2 + 2*a*b*x + b**2*x
**2 + 1)/(2*b**4) - 3*a**2*x/(4*b**3) + 3*a**2*atan(a + b*x)/(2*b**4) + a*
x**2/(4*b**2) - a*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*b**4) + x**4*atan
(a + b*x)/4 - x**3/(12*b) + x/(4*b**3) - atan(a + b*x)/(4*b**4), Ne(b, 0))
, (x**4*atan(a)/4, True))
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98

$$\int x^3\arctan(a+bx)\,dx=\frac{1}{4}x^4\arctan(bx+a)$$
$$-\frac{1}{12}b\left(\frac{b^2x^3-3\,abx^2+3\,(3\,a^2-1)x}{b^4}+\frac{3\,(a^4-6\,a^2+1)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^5}-\frac{6\,(a^3-a)\log(b^2x^2+2\,abx+\right.$$

input
```
integrate(x^3*arctan(b*x+a),x, algorithm="maxima")
```

output
```
1/4*x^4*arctan(b*x + a) - 1/12*b*((b^2*x^3 - 3*a*b*x^2 + 3*(3*a^2 - 1)*x)/
b^4 + 3*(a^4 - 6*a^2 + 1)*arctan((b^2*x + a*b)/b)/b^5 - 6*(a^3 - a)*log(b^
2*x^2 + 2*a*b*x + a^2 + 1)/b^5)
```

3.44.8 Giac [F]

$$\int x^3 \arctan(a+bx)\,dx = \int x^3 \arctan(bx+a)\,dx$$

input
```
integrate(x^3*arctan(b*x+a),x, algorithm="giac")
```

output
```
sage0*x
```

3.44.9 Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.25

$$\int x^3 \arctan(a+bx)\,dx = \frac{x^4\,\mathrm{atan}(a+b\,x)}{4} - \frac{\mathrm{atan}(a+b\,x)}{4\,b^4} + \frac{x}{4\,b^3} - \frac{x^3}{12\,b} + \frac{a^3\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,b^4} + \frac{3\,a^2\,\mathrm{atan}(a+b\,x)}{2\,b^4} - \frac{a^4\,\mathrm{atan}(a+b\,x)}{4\,b^4} + \frac{a\,x^2}{4\,b^2} - \frac{3\,a^2\,x}{4\,b^3} - \frac{a\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,b^4}$$

input
```
int(x^3*atan(a + b*x),x)
```

output
```
(x^4*atan(a + b*x))/4 - atan(a + b*x)/(4*b^4) + x/(4*b^3) - x^3/(12*b) + (
a^3*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^4) + (3*a^2*atan(a + b*x))/(2*b
^4) - (a^4*atan(a + b*x))/(4*b^4) + (a*x^2)/(4*b^2) - (3*a^2*x)/(4*b^3) -
(a*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^4)
```

3.45 $\int x^2 \arctan(a + bx)\, dx$

3.45.1 Optimal result

Integrand size = 10, antiderivative size = 79

$$\int x^2 \arctan(a+bx)\, dx = \frac{ax}{b^2} - \frac{(a+bx)^2}{6b^3} - \frac{a(3-a^2)\arctan(a+bx)}{3b^3} + \frac{1}{3}x^3\arctan(a+bx) + \frac{(1-3a^2)\log\left(1+(a+bx)^2\right)}{6b^3}$$

output
```
a*x/b^2-1/6*(b*x+a)^2/b^3-1/3*a*(-a^2+3)*arctan(b*x+a)/b^3+1/3*x^3*arctan(
b*x+a)+1/6*(-3*a^2+1)*ln(1+(b*x+a)^2)/b^3
```

3.45.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.44

$$\int x^2 \arctan(a+bx)\, dx = \frac{\frac{1}{3}b\left(-\frac{a}{b}+\frac{a+bx}{b}\right)^3\arctan(a+bx) - \frac{1}{3}b\left(-\frac{3ax}{b^2}+\frac{(a+bx)^2}{2b^3}-\frac{(1+ia)^3\log(i-a-bx)}{2b^3}-\frac{(1-ia)^3\log(i+a+bx)}{2b^3}\right)}{b}$$

input
```
Integrate[x^2*ArcTan[a + b*x],x]
```

output
```
((b*(-(a/b) + (a + b*x)/b)^3*ArcTan[a + b*x])/3 - (b*((-3*a*x)/b^2 + (a +
b*x)^2/(2*b^3) - ((1 + I*a)^3*Log[I - a - b*x])/(2*b^3) - ((1 - I*a)^3*Log
[I + a + b*x])/(2*b^3)))/3)/b
```

3.45.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5570, 27, 5387, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(a+bx)\,dx$$

$$\Big\downarrow \ 5570$$

$$\frac{\int x^2 \arctan(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \ 27$$

$$\frac{\int b^2x^2 \arctan(a+bx)d(a+bx)}{b^3}$$

$$\Big\downarrow \ 5387$$

$$\frac{\frac{1}{3}\int -\frac{b^3x^3}{(a+bx)^2+1}d(a+bx)+\frac{1}{3}b^3x^3\arctan(a+bx)}{b^3}$$

$$\Big\downarrow \ 478$$

$$\frac{\frac{1}{3}\int \left(2a-bx-\frac{a(3-a^2)-(1-3a^2)(a+bx)}{(a+bx)^2+1}\right)d(a+bx)+\frac{1}{3}b^3x^3\arctan(a+bx)}{b^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{1}{3}\left(-a(3-a^2)\arctan(a+bx)+\frac{1}{2}(1-3a^2)\log\left((a+bx)^2+1\right)-\frac{1}{2}(a+bx)^2+3a(a+bx)\right)+\frac{1}{3}b^3x^3\arctan(a+b}{b^3}$$

input
```
Int[x^2*ArcTan[a + b*x],x]
```

output
```
((b^3*x^3*ArcTan[a + b*x])/3 + (3*a*(a + b*x) - (a + b*x)^2/2 - a*(3 - a^2
)*ArcTan[a + b*x] + ((1 - 3*a^2)*Log[1 + (a + b*x)^2])/2)/3)/b^3
```

3.45.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

rule 5570
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.45.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.29

method	result
parallelrisch	$-\frac{-2\arctan(bx+a)x^3b^3+b^2x^2-2\arctan(bx+a)a^3+3a^2\ln(b^2x^2+2abx+a^2+1)-4abx+6a\arctan(bx+a)+7a^2-1-\ln(b^2}{6b^3}$
derivativedivides	$\frac{-\frac{\arctan(bx+a)a^3}{3}+\arctan(bx+a)a^2(bx+a)-\arctan(bx+a)a(bx+a)^2+\frac{\arctan(bx+a)(bx+a)^3}{3}+(bx+a)a-\frac{(bx+a)^2}{6}+\frac{(-3a^2+}{}}{b^3}$
default	$\frac{-\frac{\arctan(bx+a)a^3}{3}+\arctan(bx+a)a^2(bx+a)-\arctan(bx+a)a(bx+a)^2+\frac{\arctan(bx+a)(bx+a)^3}{3}+(bx+a)a-\frac{(bx+a)^2}{6}+\frac{(-3a^2+}{}}{b^3}$
parts	$\frac{x^3\arctan(bx+a)}{3}-\frac{b\left(-\frac{-\frac{1}{2}x^2b+2ax}{b^3}+\frac{\frac{(3a^2b-b)\ln(b^2x^2+2abx+a^2+1)}{2b^2}+\frac{\left(2a^3+2a-\frac{(3a^2b-b)a}{b}\right)\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}}{b^3}\right)}{3}$
risch	$-\frac{ix^3\ln(1+i(bx+a))}{6}+\frac{ix^3\ln(1-i(bx+a))}{6}+\frac{a^3\arctan(bx+a)}{3b^3}-\frac{x^2}{6b}-\frac{a^2\ln(b^2x^2+2abx+a^2+1)}{2b^3}+\frac{2ax}{3b^2}-\frac{a\,a}{}$

input
```
int(x^2*arctan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-2*arctan(b*x+a)*x^3*b^3+b^2*x^2-2*arctan(b*x+a)*a^3+3*a^2*ln(b^2*x^
2+2*a*b*x+a^2+1)-4*a*b*x+6*a*arctan(b*x+a)+7*a^2-1-ln(b^2*x^2+2*a*b*x+a^2+
1))/b^3
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int x^2\arctan(a+bx)\,dx = -\frac{b^2x^2-4\,abx-2\,(b^3x^3+a^3-3\,a)\arctan(bx+a)+(3\,a^2-1)\log(b^2x^2+2\,abx+a^2+1)}{6\,b^3}$$

input
```
integrate(x^2*arctan(b*x+a),x, algorithm="fricas")
```

output
```
-1/6*(b^2*x^2 - 4*a*b*x - 2*(b^3*x^3 + a^3 - 3*a)*arctan(b*x + a) + (3*a^2
 - 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^3
```

3.45.6 Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.48

$$\int x^2 \arctan(a+bx)\,dx$$
$$= \begin{cases} \frac{a^3 \operatorname{atan}(a+bx)}{3b^3} - \frac{a^2 \log(a^2+2abx+b^2x^2+1)}{2b^3} + \frac{2ax}{3b^2} - \frac{a \operatorname{atan}(a+bx)}{b^3} + \frac{x^3 \operatorname{atan}(a+bx)}{3} - \frac{x^2}{6b} + \frac{\log(a^2+2abx+b^2x^2+1)}{6b^3} & \text{for } b \neq \\ \frac{x^3 \operatorname{atan}(a)}{3} & \text{otherw} \end{cases}$$

input
```
integrate(x**2*atan(b*x+a),x)
```

output
```
Piecewise((a**3*atan(a + b*x)/(3*b**3) - a**2*log(a**2 + 2*a*b*x + b**2*x*
*2 + 1)/(2*b**3) + 2*a*x/(3*b**2) - a*atan(a + b*x)/b**3 + x**3*atan(a + b
*x)/3 - x**2/(6*b) + log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(6*b**3), Ne(b, 0
)), (x**3*atan(a)/3, True))
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.08

$$\int x^2 \arctan(a+bx)\,dx = \frac{1}{3}x^3 \arctan(bx+a)$$
$$-\frac{1}{6}b\left(\frac{bx^2-4ax}{b^3} - \frac{2(a^3-3a)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^4} + \frac{(3a^2-1)\log(b^2x^2+2abx+a^2+1)}{b^4}\right)$$

input
```
integrate(x^2*arctan(b*x+a),x, algorithm="maxima")
```

output
```
1/3*x^3*arctan(b*x + a) - 1/6*b*((b*x^2 - 4*a*x)/b^3 - 2*(a^3 - 3*a)*arcta
n((b^2*x + a*b)/b)/b^4 + (3*a^2 - 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^4)
```

3.45.8 Giac [F]

$$\int x^2 \arctan(a+bx)\,dx = \int x^2 \arctan(bx+a)\,dx$$

input
```
integrate(x^2*arctan(b*x+a),x, algorithm="giac")
```

output
```
sage0*x
```

3.45.9 Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.29

$$\int x^2 \arctan(a+bx)\,dx = \frac{\ln(a^2+2\,a\,b\,x+b^2\,x^2+1)}{6\,b^3} + \frac{x^3\,\mathrm{atan}(a+b\,x)}{3} - \frac{x^2}{6\,b} - \frac{a^2\,\ln(a^2+2\,a\,b\,x+b^2\,x^2+1)}{2\,b^3} + \frac{a^3\,\mathrm{atan}(a+b\,x)}{3\,b^3} - \frac{a\,\mathrm{atan}(a+b\,x)}{b^3} + \frac{2\,a\,x}{3\,b^2}$$

input
```
int(x^2*atan(a + b*x),x)
```

output
```
log(a^2 + b^2*x^2 + 2*a*b*x + 1)/(6*b^3) + (x^3*atan(a + b*x))/3 - x^2/(6*
b) - (a^2*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^3) + (a^3*atan(a + b*x))/
(3*b^3) - (a*atan(a + b*x))/b^3 + (2*a*x)/(3*b^2)
```

3.46 $\int x \arctan(a+bx)\,dx$

3.46.1 Optimal result

Integrand size = 8, antiderivative size = 60

$$\int x \arctan(a+bx)\,dx = -\frac{x}{2b}+\frac{(1-a^2)\arctan(a+bx)}{2b^2}+\frac{1}{2}x^2\arctan(a+bx)+\frac{a\log\left(1+(a+bx)^2\right)}{2b^2}$$

output
```
-1/2*x/b+1/2*(-a^2+1)*arctan(b*x+a)/b^2+1/2*x^2*arctan(b*x+a)+1/2*a*ln(1+(
b*x+a)^2)/b^2
```

3.46.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.50

$$\int x \arctan(a+bx)\,dx = \frac{-2bx+2b^2x^2\arctan(a+bx)+i(-i+a)^2\log(i-a-bx)+i\log(i+a+bx)+2a\log(i+a+bx)-ia^2\,\mathrm{l}}{4b^2}$$

input
```
Integrate[x*ArcTan[a + b*x],x]
```

output
```
(-2*b*x + 2*b^2*x^2*ArcTan[a + b*x] + I*(-I + a)^2*Log[I - a - b*x] + I*Lo
g[I + a + b*x] + 2*a*Log[I + a + b*x] - I*a^2*Log[I + a + b*x])/(4*b^2)
```

3.46.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5570, 25, 27, 5387, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(a+bx)\,dx$$

$$\Big\downarrow \text{5570}$$

$$\frac{\int x \arctan(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -x \arctan(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -bx \arctan(a+bx)d(a+bx)}{b^2}$$

$$\Big\downarrow \text{5387}$$

$$-\frac{\frac{1}{2}\int \frac{b^2x^2}{(a+bx)^2+1}d(a+bx)-\frac{1}{2}b^2x^2\arctan(a+bx)}{b^2}$$

$$\Big\downarrow \text{478}$$

$$-\frac{\frac{1}{2}\int \left(1-\frac{-a^2+2(a+bx)a+1}{(a+bx)^2+1}\right)d(a+bx)-\frac{1}{2}b^2x^2\arctan(a+bx)}{b^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{2}\left(-\left(1-a^2\right)\arctan(a+bx)+a\left(-\log\left((a+bx)^2+1\right)\right)+a+bx\right)-\frac{1}{2}b^2x^2\arctan(a+bx)}{b^2}$$

input
```
Int[x*ArcTan[a + b*x],x]
```

output
```
-((-1/2*(b^2*x^2*ArcTan[a + b*x]) + (a + b*x - (1 - a^2)*ArcTan[a + b*x] -
 a*Log[1 + (a + b*x)^2])/2)/b^2)
```

3.46.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

rule 5570
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.46.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{\arctan(bx+a)(bx+a)^2}{2}-\arctan(bx+a)a(bx+a)-\frac{bx}{2}-\frac{a}{2}+\frac{a\ln\left(1+(bx+a)^2\right)}{2}+\frac{\arctan(bx+a)}{2}}{b^2}$
default	$\frac{\frac{\arctan(bx+a)(bx+a)^2}{2}-\arctan(bx+a)a(bx+a)-\frac{bx}{2}-\frac{a}{2}+\frac{a\ln\left(1+(bx+a)^2\right)}{2}+\frac{\arctan(bx+a)}{2}}{b^2}$
parallelrisch	$\frac{\arctan(bx+a)b^2x^2-\arctan(bx+a)a^2+a\ln(b^2x^2+2abx+a^2+1)-bx+\arctan(bx+a)+2a}{2b^2}$
parts	$\frac{x^2\arctan(bx+a)}{2}-\frac{b\left(\frac{x}{b^2}+\frac{-\frac{a\ln\left(b^2x^2+2abx+a^2+1\right)}{b}+\frac{\left(a^2-1\right)\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}}{b^2}\right)}{2}$
risch	$-\frac{ix^2\ln(1+i(bx+a))}{4}+\frac{ix^2\ln(1-i(bx+a))}{4}-\frac{a^2\arctan(bx+a)}{2b^2}+\frac{a\ln\left(b^2x^2+2abx+a^2+1\right)}{2b^2}-\frac{x}{2b}+\frac{\arctan(bx+a}{2b^2}$

input
```
int(x*arctan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/2*arctan(b*x+a)*(b*x+a)^2-arctan(b*x+a)*a*(b*x+a)-1/2*b*x-1/2*a+1
/2*a*ln(1+(b*x+a)^2)+1/2*arctan(b*x+a))
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int x\arctan(a+bx)\,dx = -\frac{bx-\left(b^2x^2-a^2+1\right)\arctan\left(bx+a\right)-a\log\left(b^2x^2+2\,abx+a^2+1\right)}{2\,b^2}$$

input
```
integrate(x*arctan(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(b*x - (b^2*x^2 - a^2 + 1)*arctan(b*x + a) - a*log(b^2*x^2 + 2*a*b*x
+ a^2 + 1))/b^2
```

3.46.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

$$\int x\arctan(a+bx)\,dx$$
$$=\begin{cases}-\frac{a^2\operatorname{atan}(a+bx)}{2b^2}+\frac{a\log(a^2+2abx+b^2x^2+1)}{2b^2}+\frac{x^2\operatorname{atan}(a+bx)}{2}-\frac{x}{2b}+\frac{\operatorname{atan}(a+bx)}{2b^2} & \text{for } b\neq 0\\ \frac{x^2\operatorname{atan}(a)}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*atan(b*x+a),x)
```

output
```
Piecewise((-a**2*atan(a + b*x)/(2*b**2) + a*log(a**2 + 2*a*b*x + b**2*x**2
 + 1)/(2*b**2) + x**2*atan(a + b*x)/2 - x/(2*b) + atan(a + b*x)/(2*b**2),
Ne(b, 0)), (x**2*atan(a)/2, True))
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

$$\int x\arctan(a+bx)\,dx$$
$$=\frac{1}{2}x^2\arctan(bx+a)-\frac{1}{2}b\left(\frac{x}{b^2}+\frac{(a^2-1)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^3}-\frac{a\log(b^2x^2+2abx+a^2+1)}{b^3}\right)$$

input
```
integrate(x*arctan(b*x+a),x, algorithm="maxima")
```

output
```
1/2*x^2*arctan(b*x + a) - 1/2*b*(x/b^2 + (a^2 - 1)*arctan((b^2*x + a*b)/b)
/b^3 - a*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^3)
```

3.46.8 Giac [F]

$$\int x\arctan(a+bx)\,dx=\int x\arctan(bx+a)\,dx$$

input
```
integrate(x*arctan(b*x+a),x, algorithm="giac")
```

output
```
sage0*x
```

3.46.9 Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int x\arctan(a+bx)\,dx=\frac{x^2\,\mathrm{atan}(a+b\,x)}{2}+\frac{\frac{\mathrm{atan}(a+b\,x)}{2}-\frac{b\,x}{2}-\frac{a^2\,\mathrm{atan}(a+b\,x)}{2}+\frac{a\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2}}{b^2}$$

input
```
int(x*atan(a + b*x),x)
```

output
```
(x^2*atan(a + b*x))/2 + (atan(a + b*x)/2 - (b*x)/2 - (a^2*atan(a + b*x))/2
 + (a*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/2)/b^2
```

3.47 $\int \arctan(a+bx)\,dx$

3.47.1 Optimal result

Integrand size = 6, antiderivative size = 33

$$\int \arctan(a+bx)\,dx=\frac{(a+bx)\arctan(a+bx)}{b}-\frac{\log\left(1+(a+bx)^2\right)}{2b}$$

output
```
(b*x+a)*arctan(b*x+a)/b-1/2*ln(1+(b*x+a)^2)/b
```

3.47.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \arctan(a+bx)\,dx=-\frac{-2(a+bx)\arctan(a+bx)+\log\left(1+a^2+2abx+b^2x^2\right)}{2b}$$

input
```
Integrate[ArcTan[a + b*x],x]
```

output
```
-1/2*(-2*(a + b*x)*ArcTan[a + b*x] + Log[1 + a^2 + 2*a*b*x + b^2*x^2])/b
```

3.47.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5562, 5345, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(a+bx)\,dx$$

$$\Big\downarrow \ 5562$$

$$\frac{\int \arctan(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \ 5345$$

$$\frac{(a+bx)\arctan(a+bx)-\int \frac{a+bx}{(a+bx)^2+1}d(a+bx)}{b}$$

$$\Big\downarrow \ 240$$

$$\frac{(a+bx)\arctan(a+bx)-\frac{1}{2}\log\left((a+bx)^2+1\right)}{b}$$

input
```
Int[ArcTan[a + b*x],x]
```

output
```
((a + b*x)*ArcTan[a + b*x] - Log[1 + (a + b*x)^2]/2)/b
```

3.47.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5562
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
 x] && IGtQ[p, 0]
```

3.47.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{(bx+a)\arctan(bx+a)-\frac{\ln\left(1+(bx+a)^2\right)}{2}}{b}$	30
default	$\frac{(bx+a)\arctan(bx+a)-\frac{\ln\left(1+(bx+a)^2\right)}{2}}{b}$	30
parallelrisch	$-\frac{-2x\arctan(bx+a)b^2-2\arctan(bx+a)ab+\ln(b^2x^2+2abx+a^2+1)b}{2b^2}$	49
parts	$x\arctan(bx+a)-b\left(\frac{\ln(b^2x^2+2abx+a^2+1)}{2b^2}-\frac{a\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b^2}\right)$	60
risch	$-\frac{ix\ln(1+i(bx+a))}{2}+\frac{ix\ln(1-i(bx+a))}{2}+\frac{a\arctan(bx+a)}{b}-\frac{\ln(b^2x^2+2abx+a^2+1)}{2b}$	66

input
```
int(arctan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*((b*x+a)*arctan(b*x+a)-1/2*ln(1+(b*x+a)^2))
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \arctan(a+bx)\,dx = \frac{2\,(bx+a)\arctan(bx+a)-\log(b^2x^2+2\,abx+a^2+1)}{2\,b}$$

input
```
integrate(arctan(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*(b*x + a)*arctan(b*x + a) - log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b
```

3.47.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \arctan(a+bx)\,dx = \begin{cases} \frac{a\operatorname{atan}(a+bx)}{b} + x\operatorname{atan}(a+bx) - \frac{\log(a^2+2abx+b^2x^2+1)}{2b} & \text{for } b \neq 0 \\ x\operatorname{atan}(a) & \text{otherwise} \end{cases}$$

input
```
integrate(atan(b*x+a),x)
```

output
```
Piecewise((a*atan(a + b*x)/b + x*atan(a + b*x) - log(a**2 + 2*a*b*x + b**2
*x**2 + 1)/(2*b), Ne(b, 0)), (x*atan(a), True))
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \arctan(a+bx)\,dx = \frac{2\,(bx+a)\arctan(bx+a) - \log\left((bx+a)^2+1\right)}{2\,b}$$

input
```
integrate(arctan(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(2*(b*x + a)*arctan(b*x + a) - log((b*x + a)^2 + 1))/b
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \arctan(a+bx)\,dx = \frac{2\,(bx+a)\arctan(bx+a) - \log\left((bx+a)^2+1\right)}{2\,b}$$

input
```
integrate(arctan(b*x+a),x, algorithm="giac")
```

output
```
1/2*(2*(b*x + a)*arctan(b*x + a) - log((b*x + a)^2 + 1))/b
```

3.47.9 Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \arctan(a+bx)\,dx = x\,\mathrm{atan}(a+b\,x) - \frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)-2\,a\,\mathrm{atan}(a+b\,x)}{2\,b}$$

input
```
int(atan(a + b*x),x)
```

output
```
x*atan(a + b*x) - (log(a^2 + b^2*x^2 + 2*a*b*x + 1) - 2*a*atan(a + b*x))/(
2*b)
```

3.48 $\int \frac{\arctan(a+bx)}{x} \, dx$

3.48.1 Optimal result

Integrand size = 10, antiderivative size = 120

$$\begin{aligned}\int \frac{\arctan(a+bx)}{x} \, dx = & -\arctan(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right) \\ & +\arctan(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right) \\ & +\frac{1}{2}i\operatorname{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right) \\ & -\frac{1}{2}i\operatorname{PolyLog}\left(2,1-\frac{2bx}{(i-a)(1-i(a+bx))}\right)\end{aligned}$$

output
```
-arctan(b*x+a)*ln(2/(1-I*(b*x+a)))+arctan(b*x+a)*ln(2*b*x/(I-a)/(1-I*(b*x+
a)))+1/2*I*polylog(2,1-2/(1-I*(b*x+a)))-1/2*I*polylog(2,1-2*b*x/(I-a)/(1-I
*(b*x+a)))
```

3.48.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.42

$$\int \frac{\arctan(a+bx)}{x}\,dx = -\frac{1}{2}i\log(1+i(a+bx))\log\left(\frac{i\left(-\frac{a}{b}+\frac{a+bx}{b}\right)}{-\frac{1}{b}-\frac{ia}{b}}\right) + \frac{1}{2}i\log(1-i(a+bx))\log\left(-\frac{i\left(-\frac{a}{b}+\frac{a+bx}{b}\right)}{-\frac{1}{b}+\frac{ia}{b}}\right) + \frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i(1-i(a+bx))}{i+a}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i(1+i(a+bx))}{-i+a}\right)$$

input
```
Integrate[ArcTan[a + b*x]/x,x]
```

output
```
(-1/2*I)*Log[1 + I*(a + b*x)]*Log[(I*(-(a/b) + (a + b*x)/b))/(-b^(-1) - (I
*a)/b)] + (I/2)*Log[1 - I*(a + b*x)]*Log[((-I)*(-(a/b) + (a + b*x)/b))/(-b
^(-1) + (I*a)/b)] + (I/2)*PolyLog[2, (I*(1 - I*(a + b*x)))/(I + a)] - (I/2
)*PolyLog[2, ((-I)*(1 + I*(a + b*x)))/(-I + a)]
```

3.48.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5570, 25, 27, 5381, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(a+bx)}{x}\,dx$$

$$\Big\downarrow \text{5570}$$

$$\frac{\int \frac{\arctan(a+bx)}{x}d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{\arctan(a+bx)}{x}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\int -\frac{\arctan(a+bx)}{bx}d(a+bx)$$

$$\Big\downarrow \text{5381}$$

$$\int \frac{\log\left(\frac{2}{1-i(a+bx)}\right)}{(a+bx)^2+1}d(a+bx)-\int \frac{\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)}{(a+bx)^2+1}d(a+bx)-\arctan(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)+\arctan(a+bx)\log\left(\frac{2bx}{(-a+i)(1-i(a+bx))}\right)$$

$$\Big\downarrow \text{2849}$$

$$i\int \frac{\log\left(\frac{2}{1-i(a+bx)}\right)}{1-\frac{2}{1-i(a+bx)}}d\frac{1}{1-i(a+bx)}-\int \frac{\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)}{(a+bx)^2+1}d(a+bx)-\arctan(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)+\arctan(a+bx)\log\left(\frac{2bx}{(-a+i)(1-i(a+bx))}\right)$$

$$\Big\downarrow \text{2752}$$

$$-\int \frac{\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)}{(a+bx)^2+1}d(a+bx)-\arctan(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)+\arctan(a+bx)\log\left(\frac{2bx}{(-a+i)(1-i(a+bx))}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)$$

$$\Big\downarrow \text{2897}$$

$$-\arctan(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)+\arctan(a+bx)\log\left(\frac{2bx}{(-a+i)(1-i(a+bx))}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2bx}{(i-a)(1-i(a+bx))}\right)$$

input
```
Int[ArcTan[a + b*x]/x,x]
```

output
```
-(ArcTan[a + b*x]*Log[2/(1 - I*(a + b*x))]) + ArcTan[a + b*x]*Log[(2*b*x)/
((I - a)*(1 - I*(a + b*x)))] + (I/2)*PolyLog[2, 1 - 2/(1 - I*(a + b*x))] -
 (I/2)*PolyLog[2, 1 - (2*b*x)/((I - a)*(1 - I*(a + b*x)))]
```

3.48. $\int \frac{\arctan(a+bx)}{x}\,dx$

3.48.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5381
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[b*(c/e)
   Int[Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Simp[b*(c/e)   Int[Log[2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x]) /; FreeQ[{a
, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

rule 5570
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.48.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.78

method	result
risch	$\frac{i\,\mathrm{dilog}\left(-\frac{ixb}{ia-1}\right)}{2}+\frac{i\ln(-ibx-ia+1)\ln\left(-\frac{ixb}{ia-1}\right)}{2}-\frac{i\,\mathrm{dilog}\left(\frac{ixb}{-ia-1}\right)}{2}-\frac{i\ln(ibx+ia+1)\ln\left(\frac{ixb}{-ia-1}\right)}{2}$
parts	$\ln(x)\arctan(bx+a)-b\left(-\frac{i\ln(x)\left(\ln\left(\frac{-bx-a+i}{i-a}\right)-\ln\left(\frac{bx+a+i}{i+a}\right)\right)}{2b}-\frac{i\left(\mathrm{dilog}\left(\frac{-bx-a+i}{i-a}\right)-\mathrm{dilog}\left(\frac{bx+a+i}{i+a}\right)\right.}{2b}\right.$
derivativedivides	$\ln(-bx)\arctan(bx+a)-\frac{i\ln(-bx)\ln\left(\frac{bx+a+i}{i+a}\right)}{2}+\frac{i\ln(-bx)\ln\left(\frac{-bx-a+i}{i-a}\right)}{2}-\frac{i\,\mathrm{dilog}\left(\frac{bx+a+i}{i+a}\right)}{2}+$
default	$\ln(-bx)\arctan(bx+a)-\frac{i\ln(-bx)\ln\left(\frac{bx+a+i}{i+a}\right)}{2}+\frac{i\ln(-bx)\ln\left(\frac{-bx-a+i}{i-a}\right)}{2}-\frac{i\,\mathrm{dilog}\left(\frac{bx+a+i}{i+a}\right)}{2}+$

input
```
int(arctan(b*x+a)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*dilog(-I*x*b/(I*a-1))+1/2*I*ln(1-I*a-I*b*x)*ln(-I*x*b/(I*a-1))-1/2*I
*dilog(I*x*b/(-I*a-1))-1/2*I*ln(1+I*a+I*b*x)*ln(I*x*b/(-I*a-1))
```

3.48.5 Fricas [F]

$$\int \frac{\arctan(a+bx)}{x}\,dx=\int \frac{\arctan(bx+a)}{x}\,dx$$

input
```
integrate(arctan(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(arctan(b*x + a)/x, x)
```

3.48.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{x}\,dx=\text{Timed out}$$

input
```
integrate(atan(b*x+a)/x,x)
```

output
```
Timed out
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.12

$$\int \frac{\arctan(a+bx)}{x}\,dx = -\frac{1}{2}\arctan\left(\frac{bx}{a^2+1}, -\frac{abx}{a^2+1}\right)\log\left(b^2x^2+2\,abx+a^2+1\right) + \frac{1}{2}\arctan\left(bx+a\right)\log\left(\frac{b^2x^2}{a^2+1}\right) + \arctan\left(bx+a\right)\log\left(x\right) - \arctan\left(\frac{b^2x+ab}{b}\right)\log\left(x\right) - \frac{1}{2}i\,\mathrm{Li}_2\left(\frac{i\,bx+i\,a+1}{i\,a+1}\right) + \frac{1}{2}i\,\mathrm{Li}_2\left(\frac{i\,bx+i\,a-1}{i\,a-1}\right)$$

input
```
integrate(arctan(b*x+a)/x,x, algorithm="maxima")
```

output
```
-1/2*arctan2(b*x/(a^2 + 1), -a*b*x/(a^2 + 1))*log(b^2*x^2 + 2*a*b*x + a^2
+ 1) + 1/2*arctan(b*x + a)*log(b^2*x^2/(a^2 + 1)) + arctan(b*x + a)*log(x)
 - arctan((b^2*x + a*b)/b)*log(x) - 1/2*I*dilog((I*b*x + I*a + 1)/(I*a + 1
)) + 1/2*I*dilog((I*b*x + I*a - 1)/(I*a - 1))
```

3.48.8 Giac [F]

$$\int \frac{\arctan(a+bx)}{x}\,dx = \int \frac{\arctan\left(bx+a\right)}{x}\,dx$$

input
```
integrate(arctan(b*x+a)/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{x}\,dx = \int \frac{\operatorname{atan}(a+b\,x)}{x}\,dx$$

input
```
int(atan(a + b*x)/x,x)
```

output
```
int(atan(a + b*x)/x, x)
```

3.49 $\int \frac{\arctan(a+bx)}{x^2} \, dx$

3.49.1 Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \frac{\arctan(a+bx)}{x^2} \, dx = -\frac{ab\arctan(a+bx)}{1+a^2} - \frac{\arctan(a+bx)}{x} + \frac{b\log(x)}{1+a^2} - \frac{b\log\left(1+(a+bx)^2\right)}{2(1+a^2)}$$

output
```
-a*b*arctan(b*x+a)/(a^2+1)-arctan(b*x+a)/x+b*ln(x)/(a^2+1)-1/2*b*ln(1+(b*x
+a)^2)/(a^2+1)
```

3.49.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(a+bx)}{x^2} \, dx = -\frac{\arctan(a+bx)}{x} + \frac{b(2\log(x) + i(i+a)\log(i-a-bx) + (-1-ia)\log(i+a+bx))}{2(1+a^2)}$$

input
```
Integrate[ArcTan[a + b*x]/x^2,x]
```

output
```
-(ArcTan[a + b*x]/x) + (b*(2*Log[x] + I*(I + a)*Log[I - a - b*x] + (-1 - I
*a)*Log[I + a + b*x]))/(2*(1 + a^2))
```

3.49.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5568, 896, 25, 479, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(a+bx)}{x^2} \, dx$$

$$\Big\downarrow \text{5568}$$

$$b \int \frac{1}{x\left((a+bx)^2+1\right)}dx - \frac{\arctan(a+bx)}{x}$$

$$\Big\downarrow \text{896}$$

$$b \int \frac{1}{bx\left((a+bx)^2+1\right)}d(a+bx) - \frac{\arctan(a+bx)}{x}$$

$$\Big\downarrow \text{25}$$

$$-b \int -\frac{1}{bx\left((a+bx)^2+1\right)}d(a+bx) - \frac{\arctan(a+bx)}{x}$$

$$\Big\downarrow \text{479}$$

$$b\left(\frac{\log(-bx)}{a^2+1} - \frac{\int \frac{2a+bx}{(a+bx)^2+1}d(a+bx)}{a^2+1}\right) - \frac{\arctan(a+bx)}{x}$$

$$\Big\downarrow \text{452}$$

$$b\left(\frac{\log(-bx)}{a^2+1} - \frac{a\int \frac{1}{(a+bx)^2+1}d(a+bx) + \int \frac{a+bx}{(a+bx)^2+1}d(a+bx)}{a^2+1}\right) - \frac{\arctan(a+bx)}{x}$$

$$\Big\downarrow \text{216}$$

$$b\left(\frac{\log(-bx)}{a^2+1} - \frac{\int \frac{a+bx}{(a+bx)^2+1}d(a+bx) + a\arctan(a+bx)}{a^2+1}\right) - \frac{\arctan(a+bx)}{x}$$

$$\Big\downarrow \text{240}$$

$$b\left(\frac{\log(-bx)}{a^2+1} - \frac{a\arctan(a+bx) + \frac{1}{2}\log\left((a+bx)^2+1\right)}{a^2+1}\right) - \frac{\arctan(a+bx)}{x}$$

input
```
Int[ArcTan[a + b*x]/x^2,x]
```

output
```
-(ArcTan[a + b*x]/x) + b*(Log[-(b*x)]/(1 + a^2) - (a*ArcTan[a + b*x] + Log
[1 + (a + b*x)^2]/2)/(1 + a^2))
```

3.49.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 479
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[d*(Log
[RemoveContent[c + d*x, x]]/(b*c^2 + a*d^2)), x] + Simp[b/(b*c^2 + a*d^2)
 Int[(c - d*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 5568
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcTan[
c + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[p, 0] && ILtQ[m, -1]
```

3.49.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result
derivativedivides	$b\left(-\frac{\arctan(bx+a)}{bx}-\frac{\frac{\ln\left(1+(bx+a)^2\right)}{2}+a\arctan(bx+a)}{a^2+1}+\frac{\ln(-bx)}{a^2+1}\right)$
default	$b\left(-\frac{\arctan(bx+a)}{bx}-\frac{\frac{\ln\left(1+(bx+a)^2\right)}{2}+a\arctan(bx+a)}{a^2+1}+\frac{\ln(-bx)}{a^2+1}\right)$
parts	$-\frac{\arctan(bx+a)}{x}+b\left(\frac{\ln(x)}{a^2+1}-\frac{b\left(\frac{\ln\left(b^2x^2+2abx+a^2+1\right)}{2b}+\frac{a\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}\right)}{a^2+1}\right)$
parallelrisch	$\frac{-2x\arctan(bx+a)a^2b^2+2b^2\ln(x)ax-b^2\ln\left(b^2x^2+2abx+a^2+1\right)ax-2\arctan(bx+a)a^3b-2\arctan(bx+a)ab}{2xab(a^2+1)}$
risch	$\frac{i\ln(1+i(bx+a))}{2x}-\frac{i\left(a^2\ln(1-i(bx+a))+\ln(1-i(bx+a))-i\ln\left(\left(-a^2b+3iab\right)x-3a+2ia^2-a^3\right)bx+\ln\left(\left(-a^2b+3iab\right)x-3\right.\right.}{2x}$

input
```
int(arctan(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
b*(-1/b/x*arctan(b*x+a)-1/(a^2+1)*(1/2*ln(1+(b*x+a)^2)+a*arctan(b*x+a))+1/
(a^2+1)*ln(-b*x))
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(a+bx)}{x^2}\,dx = -\frac{bx\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,bx\log\left(x\right)+2\left(abx+a^2+1\right)\arctan\left(bx+a\right)}{2\left(a^2+1\right)x}$$

input
```
integrate(arctan(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(b*x*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*b*x*log(x) + 2*(a*b*x + a^2
 + 1)*arctan(b*x + a))/((a^2 + 1)*x)
```

3.49.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.71

$$\int \frac{\arctan(a+bx)}{x^2}\,dx = \begin{cases} -\frac{ib\,\mathrm{atan}\,(bx-i)}{2} - \frac{\mathrm{atan}\,(bx-i)}{x} - \frac{i}{2x} & \text{for } a=-i \\ \frac{ib\,\mathrm{atan}\,(bx+i)}{2} - \frac{\mathrm{atan}\,(bx+i)}{x} + \frac{i}{2x} & \text{for } a=i \\ -\frac{2a^2\,\mathrm{atan}\,(a+bx)}{2a^2x+2x} - \frac{2abx\,\mathrm{atan}\,(a+bx)}{2a^2x+2x} + \frac{2bx\log(x)}{2a^2x+2x} - \frac{bx\log(a^2+2abx+b^2x^2+1)}{2a^2x+2x} - \frac{2\,\mathrm{atan}\,(a+bx)}{2a^2x+2x} & \text{otherwise} \end{cases}$$

input
```
integrate(atan(b*x+a)/x**2,x)
```

output
```
Piecewise(((-I*b*atan(b*x - I)/2 - atan(b*x - I)/x - I/(2*x), Eq(a, -I)), (
I*b*atan(b*x + I)/2 - atan(b*x + I)/x + I/(2*x), Eq(a, I)), (-2*a**2*atan(
a + b*x)/(2*a**2*x + 2*x) - 2*a*b*x*atan(a + b*x)/(2*a**2*x + 2*x) + 2*b*x
*log(x)/(2*a**2*x + 2*x) - b*x*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*a**2
*x + 2*x) - 2*atan(a + b*x)/(2*a**2*x + 2*x), True))
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.24

$$\int \frac{\arctan(a+bx)}{x^2}\,dx = -\frac{1}{2}\,b\left(\frac{2\,a\arctan\left(\frac{b^2x+ab}{b}\right)}{a^2+1} + \frac{\log(b^2x^2+2\,abx+a^2+1)}{a^2+1} - \frac{2\,\log(x)}{a^2+1}\right) - \frac{\arctan(bx+a)}{x}$$

input
```
integrate(arctan(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
-1/2*b*(2*a*arctan((b^2*x + a*b)/b)/(a^2 + 1) + log(b^2*x^2 + 2*a*b*x + a^
2 + 1)/(a^2 + 1) - 2*log(x)/(a^2 + 1)) - arctan(b*x + a)/x
```

3.49.8 Giac [F]

$$\int \frac{\arctan(a+bx)}{x^2}\,dx = \int \frac{\arctan(bx+a)}{x^2}\,dx$$

input
```
integrate(arctan(b*x+a)/x^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.49.9 Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int \frac{\arctan(a+bx)}{x^2}\,dx = -\frac{\operatorname{atan}(a+b\,x)}{x} - \frac{\frac{b\,x\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2} - b\,x\,\ln\left(x\right) + a\,b\,x\,\operatorname{atan}(a+b\,x)}{x\,\left(a^2+1\right)}$$

input
```
int(atan(a + b*x)/x^2,x)
```

output
```
- atan(a + b*x)/x - ((b*x*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/2 - b*x*log(x)
 + a*b*x*atan(a + b*x))/(x*(a^2 + 1))
```

3.50 $\int \frac{\arctan(a+bx)}{x^3} \, dx$

3.50.1 Optimal result

Integrand size = 10, antiderivative size = 96

$$\int \frac{\arctan(a+bx)}{x^3} \, dx = -\frac{b}{2(1+a^2)x} - \frac{(1-a^2)b^2\arctan(a+bx)}{2(1+a^2)^2} - \frac{\arctan(a+bx)}{2x^2} - \frac{ab^2\log(x)}{(1+a^2)^2} + \frac{ab^2\log(1+(a+bx)^2)}{2(1+a^2)^2}$$

output
```
-1/2*b/(a^2+1)/x-1/2*(-a^2+1)*b^2*arctan(b*x+a)/(a^2+1)^2-1/2*arctan(b*x+a
)/x^2-a*b^2*ln(x)/(a^2+1)^2+1/2*a*b^2*ln(1+(b*x+a)^2)/(a^2+1)^2
```

3.50.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$\int \frac{\arctan(a+bx)}{x^3} \, dx = \frac{-2\arctan(a+bx) + \frac{bx(-4abx\log(x)-i(i+a)^2bx\log(i-a-bx)+(-i+a)(-2(i+a)+(1+ia)bx\log(i+a+bx)))}{(1+a^2)^2}}{4x^2}$$

input
```
Integrate[ArcTan[a + b*x]/x^3,x]
```

output
```
(-2*ArcTan[a + b*x] + (b*x*(-4*a*b*x*Log[x] - I*(I + a)^2*b*x*Log[I - a -
b*x] + (-I + a)*(-2*(I + a) + (1 + I*a)*b*x*Log[I + a + b*x])))/(1 + a^2)^
2)/(4*x^2)
```

3.50.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5568, 896, 480, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(a+bx)}{x^3} \, dx$$

$$\Big\downarrow \text{5568}$$

$$\frac{1}{2} b \int \frac{1}{x^2 \left((a+bx)^2+1\right)} dx - \frac{\arctan(a+bx)}{2x^2}$$

$$\Big\downarrow \text{896}$$

$$\frac{1}{2} b^2 \int \frac{1}{b^2 x^2 \left((a+bx)^2+1\right)} d(a+bx) - \frac{\arctan(a+bx)}{2x^2}$$

$$\Big\downarrow \text{480}$$

$$\frac{1}{2} b^2 \left(\frac{\int -\frac{2a+bx}{bx\left((a+bx)^2+1\right)} d(a+bx)}{a^2+1} - \frac{1}{\left(a^2+1\right) bx}\right) - \frac{\arctan(a+bx)}{2x^2}$$

$$\Big\downarrow \text{657}$$

$$\frac{1}{2} b^2 \left(\frac{\int \left(\frac{a^2+2(a+bx)a-1}{\left(a^2+1\right)\left((a+bx)^2+1\right)} - \frac{2a}{\left(a^2+1\right) bx}\right) d(a+bx)}{a^2+1} - \frac{1}{\left(a^2+1\right) bx}\right) - \frac{\arctan(a+bx)}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} b^2 \left(\frac{-\frac{\left(1-a^2\right)\arctan(a+bx)}{a^2+1} - \frac{2a\log(-bx)}{a^2+1} + \frac{a\log\left((a+bx)^2+1\right)}{a^2+1}}{a^2+1} - \frac{1}{\left(a^2+1\right) bx}\right) - \frac{\arctan(a+bx)}{2x^2}$$

input
```
Int[ArcTan[a + b*x]/x^3,x]
```

output
```
-1/2*ArcTan[a + b*x]/x^2 + (b^2*(-(1/((1 + a^2)*b*x)) + (-(((1 - a^2)*ArcT
an[a + b*x])/(1 + a^2)) - (2*a*Log[-(b*x)])/(1 + a^2) + (a*Log[1 + (a + b*
x)^2])/(1 + a^2))/(1 + a^2)))/2
```

3.50.3.1 Defintions of rubi rules used

rule 480
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[n, -1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5568
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcTan[
c + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[p, 0] && ILtQ[m, -1]
```

3.50.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

method	result
derivativedivides	$b^{2}\left(-\frac{\arctan(bx+a)}{2b^{2}x^{2}}-\frac{1}{2(a^{2}+1)bx}-\frac{a\ln(-bx)}{(a^{2}+1)^{2}}+\frac{a\ln\left(1+(bx+a)^{2}\right)+(a^{2}-1)\arctan(bx+a)}{2(a^{2}+1)^{2}}\right)$
default	$b^{2}\left(-\frac{\arctan(bx+a)}{2b^{2}x^{2}}-\frac{1}{2(a^{2}+1)bx}-\frac{a\ln(-bx)}{(a^{2}+1)^{2}}+\frac{a\ln\left(1+(bx+a)^{2}\right)+(a^{2}-1)\arctan(bx+a)}{2(a^{2}+1)^{2}}\right)$
parts	$-\frac{\arctan(bx+a)}{2x^{2}}+\frac{b\left(-\frac{1}{(a^{2}+1)x}-\frac{2ab\ln(x)}{(a^{2}+1)^{2}}+\frac{b^{2}\left(\frac{a\ln\left(b^{2}x^{2}+2abx+a^{2}+1\right)}{b}+\frac{(a^{2}-1)\arctan\left(\frac{2b^{2}x+2ab}{2b}\right)}{b}\right)}{(a^{2}+1)^{2}}\right)}{2}$
parallelrisch	$-\frac{-x^{2}\arctan(bx+a)a^{2}b^{2}+2a\,b^{2}\ln(x)x^{2}-a\,b^{2}\ln(b^{2}x^{2}+2abx+a^{2}+1)x^{2}+\arctan(bx+a)b^{2}x^{2}-2a\,b^{2}x^{2}+\arctan(bx+a)a^{4}}{2x^{2}(a^{4}+2a^{2}+1)}$
risch	$\frac{i\ln(1+i(bx+a))}{4x^{2}}-\frac{i(a^{4}\ln(1-i(bx+a))+2a^{2}\ln(1-i(bx+a))+\ln(1-i(bx+a))-\ln((a^{6}b-4ia^{5}b+9a^{4}b+8ia^{3}b-9a^{2}b-4ia}{}$

input
```
int(arctan(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
b^2*(-1/2/b^2/x^2*arctan(b*x+a)-1/2/(a^2+1)/b/x-1/(a^2+1)^2*a*ln(-b*x)+1/2
/(a^2+1)^2*(a*ln(1+(b*x+a)^2)+(a^2-1)*arctan(b*x+a)))
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.99

$$\int \frac{\arctan(a+bx)}{x^{3}}\,dx = \frac{ab^{2}x^{2}\log\left(b^{2}x^{2}+2\,abx+a^{2}+1\right)-2\,ab^{2}x^{2}\log\left(x\right)-(a^{2}+1)bx+((a^{2}-1)b^{2}x^{2}-a^{4}-2\,a^{2}-1)\arctan\left(b\right.}{2\left(a^{4}+2\,a^{2}+1\right)x^{2}}$$

input
```
integrate(arctan(b*x+a)/x^3,x, algorithm="fricas")
```

output
```
1/2*(a*b^2*x^2*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*a*b^2*x^2*log(x) - (a^
2 + 1)*b*x + ((a^2 - 1)*b^2*x^2 - a^4 - 2*a^2 - 1)*arctan(b*x + a))/((a^4
+ 2*a^2 + 1)*x^2)
```

3.50.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 382, normalized size of antiderivative = 3.98

$$\int \frac{\arctan(a+bx)}{x^3}\,dx$$
$$= \begin{cases} -\frac{b^2\operatorname{atan}(bx-i)}{8} - \frac{b}{8x} - \frac{\operatorname{atan}(bx-i)}{2x^2} - \frac{i}{8x^2} \\ -\frac{b^2\operatorname{atan}(bx+i)}{8} - \frac{b}{8x} - \frac{\operatorname{atan}(bx+i)}{2x^2} + \frac{i}{8x^2} \\ -\frac{a^4\operatorname{atan}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} + \frac{a^2b^2x^2\operatorname{atan}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} - \frac{a^2bx}{2a^4x^2+4a^2x^2+2x^2} - \frac{2a^2\operatorname{atan}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} - \frac{2ab^2x^2\log(x)}{2a^4x^2+4a^2x^2+2x^2} + \frac{ab^2x^2\log(a^2+2ab}{2a^4x^2+4a^2x} \end{cases}$$

input
```
integrate(atan(b*x+a)/x**3,x)
```

output
```
Piecewise(((-b**2*atan(b*x - I)/8 - b/(8*x) - atan(b*x - I)/(2*x**2) - I/(8
*x**2), Eq(a, -I)), (-b**2*atan(b*x + I)/8 - b/(8*x) - atan(b*x + I)/(2*x*
*2) + I/(8*x**2), Eq(a, I)), (-a**4*atan(a + b*x)/(2*a**4*x**2 + 4*a**2*x*
*2 + 2*x**2) + a**2*b**2*x**2*atan(a + b*x)/(2*a**4*x**2 + 4*a**2*x**2 + 2
*x**2) - a**2*b*x/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - 2*a**2*atan(a + b
*x)/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - 2*a*b**2*x**2*log(x)/(2*a**4*x*
*2 + 4*a**2*x**2 + 2*x**2) + a*b**2*x**2*log(a**2 + 2*a*b*x + b**2*x**2 +
1)/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - b**2*x**2*atan(a + b*x)/(2*a**4*
x**2 + 4*a**2*x**2 + 2*x**2) - b*x/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) -
atan(a + b*x)/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2), True))
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

$$\int \frac{\arctan(a+bx)}{x^3}\,dx$$
$$= \frac{1}{2}\left(\frac{(a^2-1)b\arctan\left(\frac{b^2x+ab}{b}\right)}{a^4+2a^2+1} + \frac{ab\log\left(b^2x^2+2abx+a^2+1\right)}{a^4+2a^2+1} - \frac{2ab\log(x)}{a^4+2a^2+1} - \frac{1}{(a^2+1)x}\right)b - \frac{\arctan(bx+a)}{2x^2}$$

input
```
integrate(arctan(b*x+a)/x^3,x, algorithm="maxima")
```

output
```
1/2*((a^2 - 1)*b*arctan((b^2*x + a*b)/b)/(a^4 + 2*a^2 + 1) + a*b*log(b^2*x
^2 + 2*a*b*x + a^2 + 1)/(a^4 + 2*a^2 + 1) - 2*a*b*log(x)/(a^4 + 2*a^2 + 1)
 - 1/((a^2 + 1)*x))*b - 1/2*arctan(b*x + a)/x^2
```

3.50.8 Giac [F]

$$\int \frac{\arctan(a+bx)}{x^3}\,dx = \int \frac{\arctan\left(bx+a\right)}{x^3}\,dx$$

input
```
integrate(arctan(b*x+a)/x^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.50.9 Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.42

$$\int \frac{\arctan(a+bx)}{x^3}\,dx = \frac{a\,b^2\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,{\left(a^2+1\right)}^2} - \frac{\frac{b\,x}{2}+\mathrm{atan}(a+b\,x)\,\left(\frac{a^2}{2}+\frac{1}{2}\right)+\frac{b^2\,x^2\,\mathrm{atan}(a+b\,x)}{2}+\frac{x^3\,\left(b^3-3\,a^2\,b^3\right)}{2\,\left(a^4+2\,a^2+1\right)}-\frac{a\,b^4\,x^4}{{\left(a^2+1\right)}^2}+a\,b\,x\,\mathrm{atan}(a+b\,x)}{a^2\,x^2+2\,a\,b\,x^3+b^2\,x^4+x^2} - \frac{\mathrm{atan}\left(\frac{2\,x\,b^2+2\,a\,b}{2\,\sqrt{b^2\,\left(a^2+1\right)-a^2\,b^2}}\right)\,\left(b^3-a^2\,b^3\right)}{\sqrt{b^2}\,\left(2\,a^4+4\,a^2+2\right)} - \frac{a\,b^2\,\ln\left(x\right)}{{\left(a^2+1\right)}^2}$$

input
```
int(atan(a + b*x)/x^3,x)
```

output
```
(a*b^2*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*(a^2 + 1)^2) - ((b*x)/2 + atan
(a + b*x)*(a^2/2 + 1/2) + (b^2*x^2*atan(a + b*x))/2 + (x^3*(b^3 - 3*a^2*b^
3))/(2*(2*a^2 + a^4 + 1)) - (a*b^4*x^4)/(a^2 + 1)^2 + a*b*x*atan(a + b*x))
/(x^2 + a^2*x^2 + b^2*x^4 + 2*a*b*x^3) - (atan((2*a*b + 2*b^2*x)/(2*(b^2*(
a^2 + 1) - a^2*b^2)^(1/2)))*(b^3 - a^2*b^3))/((b^2)^(1/2)*(4*a^2 + 2*a^4 +
 2)) - (a*b^2*log(x))/(a^2 + 1)^2
```

3.51 $\int \frac{\arctan(a+bx)}{x^4}\,dx$

3.51.1 Optimal result

Integrand size = 10, antiderivative size = 129

$$\int \frac{\arctan(a+bx)}{x^4}\,dx = -\frac{b}{6(1+a^2)\,x^2}+\frac{2ab^2}{3(1+a^2)^2\,x}+\frac{a(3-a^2)\,b^3\arctan(a+bx)}{3(1+a^2)^3} - \frac{\arctan(a+bx)}{3x^3}-\frac{(1-3a^2)\,b^3\log(x)}{3(1+a^2)^3} + \frac{(1-3a^2)\,b^3\log\left(1+(a+bx)^2\right)}{6(1+a^2)^3}$$

output
```
-1/6*b/(a^2+1)/x^2+2/3*a*b^2/(a^2+1)^2/x+1/3*a*(-a^2+3)*b^3*arctan(b*x+a)/
(a^2+1)^3-1/3*arctan(b*x+a)/x^3-1/3*(-3*a^2+1)*b^3*ln(x)/(a^2+1)^3+1/6*(-3
*a^2+1)*b^3*ln(1+(b*x+a)^2)/(a^2+1)^3
```

3.51.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.99

$$\int \frac{\arctan(a+bx)}{x^4}\,dx = \frac{-2(1+a^2)^3\arctan(a+bx)+2(-1+3a^2)\,b^3x^3\log(x)+i(i+a)^3b^3x^3\log(i-a-bx)-(-i+a)bx((i+\cdots}{6(1+a^2)^3\,x^3}$$

input
```
Integrate[ArcTan[a + b*x]/x^4,x]
```

output
```
(-2*(1 + a^2)^3*ArcTan[a + b*x] + 2*(-1 + 3*a^2)*b^3*x^3*Log[x] + I*(I + a
)^3*b^3*x^3*Log[I - a - b*x] - (-I + a)*b*x*((I + a)*(1 + a^2 - 4*a*b*x) +
 I*(-I + a)^2*b^2*x^2*Log[I + a + b*x]))/(6*(1 + a^2)^3*x^3)
```

3.51.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5568, 896, 25, 480, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(a+bx)}{x^4}dx$$

$$\Big\downarrow \text{5568}$$

$$\frac{1}{3}b\int \frac{1}{x^3\left((a+bx)^2+1\right)}dx-\frac{\arctan(a+bx)}{3x^3}$$

$$\Big\downarrow \text{896}$$

$$\frac{1}{3}b^3\int \frac{1}{b^3x^3\left((a+bx)^2+1\right)}d(a+bx)-\frac{\arctan(a+bx)}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{3}b^3\int -\frac{1}{b^3x^3\left((a+bx)^2+1\right)}d(a+bx)-\frac{\arctan(a+bx)}{3x^3}$$

$$\Big\downarrow \text{480}$$

$$\frac{1}{3}b^3\left(-\frac{\int \frac{2a+bx}{b^2x^2((a+bx)^2+1)}d(a+bx)}{a^2+1}-\frac{1}{2\left(a^2+1\right)b^2x^2}\right)-\frac{\arctan(a+bx)}{3x^3}$$

$$\Big\downarrow \text{657}$$

$$\frac{1}{3}b^3\left(-\frac{\int \left(\frac{2a}{(a^2+1)b^2x^2}-\frac{3a^2-1}{(a^2+1)^2bx}+\frac{-a(3-a^2)-(1-3a^2)(a+bx)}{(a^2+1)^2((a+bx)^2+1)}\right)d(a+bx)}{a^2+1}-\frac{1}{2\left(a^2+1\right)b^2x^2}\right)-\frac{\arctan(a+bx)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}b^3\left(-\frac{-\frac{(3-a^2)a\arctan(a+bx)}{(a^2+1)^2}-\frac{2a}{(a^2+1)bx}+\frac{(1-3a^2)\log(-bx)}{(a^2+1)^2}-\frac{(1-3a^2)\log((a+bx)^2+1)}{2(a^2+1)^2}}{a^2+1}-\frac{1}{2(a^2+1)b^2x^2}\right)-\frac{\arctan(a+bx)}{3x^3}$$

input
```
Int[ArcTan[a + b*x]/x^4,x]
```

output
```
-1/3*ArcTan[a + b*x]/x^3 + (b^3*(-1/2*1/((1 + a^2)*b^2*x^2) - ((-2*a)/((1
+ a^2)*b*x) - (a*(3 - a^2)*ArcTan[a + b*x])/(1 + a^2)^2 + ((1 - 3*a^2)*Log
[-(b*x)])/(1 + a^2)^2 - ((1 - 3*a^2)*Log[1 + (a + b*x)^2])/(2*(1 + a^2)^2)
)/(1 + a^2)))/3
```

3.51.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 480
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[n, -1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5568
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcTan[
c + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[p, 0] && ILtQ[m, -1]
```

3.51.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.89

method	result
derivativedivides	$b^3\left(-\frac{\arctan(bx+a)}{3b^3x^3}-\frac{(-3a^2+1)\ln(-bx)}{3(a^2+1)^3}-\frac{1}{6(a^2+1)b^2x^2}+\frac{2a}{3(a^2+1)^2bx}-\frac{\frac{(3a^2-1)\ln\left(1+(bx+a)^2\right)}{2}+(a^3-3a)}{3(a^2+1)^3}\right.$
default	$b^3\left(-\frac{\arctan(bx+a)}{3b^3x^3}-\frac{(-3a^2+1)\ln(-bx)}{3(a^2+1)^3}-\frac{1}{6(a^2+1)b^2x^2}+\frac{2a}{3(a^2+1)^2bx}-\frac{\frac{(3a^2-1)\ln\left(1+(bx+a)^2\right)}{2}+(a^3-3a)}{3(a^2+1)^3}\right.$
parts	$-\frac{\arctan(bx+a)}{3x^3}+\frac{b\left(-\frac{1}{2\left(a^2+1\right)x^2}+\frac{b^2\left(3a^2-1\right)\ln(x)}{\left(a^2+1\right)^3}+\frac{2ab}{\left(a^2+1\right)^2x}-\frac{b^3\left(\frac{\left(3a^2b-b\right)\ln\left(b^2x^2+2abx+a^2+1\right)}{2b^2}+\frac{\left(4a^3-4a-\frac{(3a}{}\right.}{}\right.}{\left(a^2+1\right)^3}\right.}{3}$
parallelrisch	$\frac{-2x^3\arctan(bx+a)a^3b^3+6\ln(x)x^3a^2b^3-3\ln\left(b^2x^2+2abx+a^2+1\right)x^3a^2b^3+6x^3\arctan(bx+a)a\,b^3-7x^3a^2b^3-2b^3\ln(x)x^3}{}$
risch	Expression too large to display

input
```
int(arctan(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
b^3*(-1/3/b^3/x^3*arctan(b*x+a)-1/3*(-3*a^2+1)/(a^2+1)^3*ln(-b*x)-1/6/(a^2
+1)/b^2/x^2+2/3/(a^2+1)^2*a/b/x-1/3/(a^2+1)^3*(1/2*(3*a^2-1)*ln(1+(b*x+a)^
2)+(a^3-3*a)*arctan(b*x+a)))
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.05

$$\int \frac{\arctan(a+bx)}{x^4}\,dx = -\frac{(3\,a^2-1)b^3x^3\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,(3\,a^2-1)b^3x^3\log\left(x\right)-4\,(a^3+a)b^2x^2+(a^4+2\,a^2+1)b}{6\,(a^6+3\,a^4+3\,a^2+1)x^3}$$

input
```
integrate(arctan(b*x+a)/x^4,x, algorithm="fricas")
```

output
```
-1/6*((3*a^2 - 1)*b^3*x^3*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*(3*a^2 - 1)
*b^3*x^3*log(x) - 4*(a^3 + a)*b^2*x^2 + (a^4 + 2*a^2 + 1)*b*x + 2*((a^3 -
3*a)*b^3*x^3 + a^6 + 3*a^4 + 3*a^2 + 1)*arctan(b*x + a))/((a^6 + 3*a^4 + 3
*a^2 + 1)*x^3)
```

3.51.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 760, normalized size of antiderivative = 5.89

$$\int \frac{\arctan(a+bx)}{x^4}\,dx = \begin{cases} \frac{ib^3\operatorname{atan}(bx-i)}{24}+\frac{ib^2}{24x}-\frac{b}{24x^2}-\frac{\operatorname{atan}(bx-i)}{3x^3}-\frac{i}{18x^3} \\ -\frac{ib^3\operatorname{atan}(bx+i)}{24}-\frac{ib^2}{24x}-\frac{b}{24x^2}-\frac{\operatorname{atan}(bx+i)}{3x^3}+\frac{i}{18x^3} \\ -\frac{2a^6\operatorname{atan}(a+bx)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{a^4bx}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{6a^4\operatorname{atan}(a+bx)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{2a^3b^3x^3\operatorname{atan}(a+bx)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}+ \end{cases}$$

input
```
integrate(atan(b*x+a)/x**4,x)
```

output
```
Piecewise((I*b**3*atan(b*x - I)/24 + I*b**2/(24*x) - b/(24*x**2) - atan(b*
x - I)/(3*x**3) - I/(18*x**3), Eq(a, -I)), (-I*b**3*atan(b*x + I)/24 - I*b
**2/(24*x) - b/(24*x**2) - atan(b*x + I)/(3*x**3) + I/(18*x**3), Eq(a, I))
, (-2*a**6*atan(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x*
*3) - a**4*b*x/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 6*a*
*4*atan(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 2*
a**3*b**3*x**3*atan(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 +
6*x**3) + 4*a**3*b**2*x**2/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*
x**3) + 6*a**2*b**3*x**3*log(x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3
 + 6*x**3) - 3*a**2*b**3*x**3*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(6*a**6*
x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 2*a**2*b*x/(6*a**6*x**3 + 1
8*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 6*a**2*atan(a + b*x)/(6*a**6*x**3 +
 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) + 6*a*b**3*x**3*atan(a + b*x)/(6*a*
*6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) + 4*a*b**2*x**2/(6*a**6*x*
*3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 2*b**3*x**3*log(x)/(6*a**6*x*
*3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) + b**3*x**3*log(a**2 + 2*a*b*x
+ b**2*x**2 + 1)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - b*
x/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 2*atan(a + b*x)/(
6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3), True))
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.28

$$\int \frac{\arctan(a+bx)}{x^4}\,dx =$$

$$-\frac{1}{6}\left(\frac{2\,(a^3-3\,a)b^2\arctan\left(\frac{b^2x+ab}{b}\right)}{a^6+3\,a^4+3\,a^2+1}+\frac{(3\,a^2-1)b^2\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^6+3\,a^4+3\,a^2+1}-\frac{2\,(3\,a^2-1)b^2\log\left(x\right)}{a^6+3\,a^4+3\,a^2+1}-\right.$$

$$-\frac{\arctan\left(bx+a\right)}{3\,x^3}$$

input
```
integrate(arctan(b*x+a)/x^4,x, algorithm="maxima")
```

output
```
-1/6*(2*(a^3 - 3*a)*b^2*arctan((b^2*x + a*b)/b)/(a^6 + 3*a^4 + 3*a^2 + 1)
+ (3*a^2 - 1)*b^2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/(a^6 + 3*a^4 + 3*a^2 +
1) - 2*(3*a^2 - 1)*b^2*log(x)/(a^6 + 3*a^4 + 3*a^2 + 1) - (4*a*b*x - a^2 -
 1)/((a^4 + 2*a^2 + 1)*x^2))*b - 1/3*arctan(b*x + a)/x^3
```

3.51.8 Giac [F]

$$\int \frac{\arctan(a+bx)}{x^4}\,dx = \int \frac{\arctan(bx+a)}{x^4}\,dx$$

input
```
integrate(arctan(b*x+a)/x^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.51.9 Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.23

$$\int \frac{\arctan(a+bx)}{x^4}\,dx = -\frac{\frac{b\,x}{6}+\mathrm{atan}(a+b\,x)\,\left(\frac{a^2}{3}+\frac{1}{3}\right)+\frac{b^2\,x^2\,\mathrm{atan}(a+b\,x)}{3}+\frac{x^3\,\left(b^3-7\,a^2\,b^3\right)}{6\,\left(a^4+2\,a^2+1\right)}-\frac{a\,b^2\,x^2}{3\,\left(a^2+1\right)}-\frac{2\,a\,b^4\,x^4}{3\,{\left(a^2+1\right)}^2}+\frac{2\,a\,b\,x\,\mathrm{atan}(a+b\,x)}{3}}{a^2\,x^3+2\,a\,b\,x^4+b^2\,x^5+x^3} - \frac{\ln\left(x\right)\,\left(\frac{b^3}{3}-a^2\,b^3\right)}{a^6+3\,a^4+3\,a^2+1} - \frac{b^3\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)\,\left(3\,a^2-1\right)}{6\,\left(a^6+3\,a^4+3\,a^2+1\right)} - \frac{a\,\mathrm{atan}\left(\frac{2\,x\,b^2+2\,a\,b}{2\,\sqrt{b^2\,\left(a^2+1\right)-a^2\,b^2}}\right)\,\left(a^2-3\right)\,{\left(b^2\right)}^{3/2}}{3\,\left(a^6+3\,a^4+3\,a^2+1\right)}$$

input
```
int(atan(a + b*x)/x^4,x)
```

output
```
- ((b*x)/6 + atan(a + b*x)*(a^2/3 + 1/3) + (b^2*x^2*atan(a + b*x))/3 + (x^
3*(b^3 - 7*a^2*b^3))/(6*(2*a^2 + a^4 + 1)) - (a*b^2*x^2)/(3*(a^2 + 1)) - (
2*a*b^4*x^4)/(3*(a^2 + 1)^2) + (2*a*b*x*atan(a + b*x))/3)/(x^3 + a^2*x^3 +
 b^2*x^5 + 2*a*b*x^4) - (log(x)*(b^3/3 - a^2*b^3))/(3*a^2 + 3*a^4 + a^6 +
1) - (b^3*log(a^2 + b^2*x^2 + 2*a*b*x + 1)*(3*a^2 - 1))/(6*(3*a^2 + 3*a^4
+ a^6 + 1)) - (a*atan((2*a*b + 2*b^2*x)/(2*(b^2*(a^2 + 1) - a^2*b^2)^(1/2)
))*(a^2 - 3)*(b^2)^(3/2))/(3*(3*a^2 + 3*a^4 + a^6 + 1))
```

3.52 $\int \frac{\arctan(a+bx)}{c+dx^3} \, dx$

3.52.1 Optimal result

Integrand size = 16, antiderivative size = 863

$$\int \frac{\arctan(a+bx)}{c+dx^3}\,dx = -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\sqrt[6]{-1}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}-\sqrt[3]{-1}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-\sqrt[3]{-1}(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} - \frac{\sqrt[6]{-1}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}-\sqrt[3]{-1}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+\sqrt[3]{-1}(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{(-1)^{5/6}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}+(-1)^{2/3}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+(-1)^{2/3}(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} - \frac{(-1)^{5/6}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}+(-1)^{2/3}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}+\sqrt[6]{-1}(1-ia)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} - \frac{i\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{d}(i-a-bx)}{b\sqrt[3]{c}+(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{(-1)^{5/6}\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[6]{-1}\sqrt[3]{d}(i-a-bx)}{ib\sqrt[3]{c}-\sqrt[6]{-1}(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{\sqrt[6]{-1}\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[3]{-1}\sqrt[3]{d}(i-a-bx)}{b\sqrt[3]{c}-\sqrt[3]{-1}(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{i\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[3]{d}(i+a+bx)}{b\sqrt[3]{c}-(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} - \frac{\sqrt[6]{-1}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{d}(i+a+bx)}{b\sqrt[3]{c}+\sqrt[3]{-1}(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} - \frac{(-1)^{5/6}\,\mathrm{PolyLog}\left(2,-\frac{(-1)^{2/3}\sqrt[3]{d}(i+a+bx)}{b\sqrt[3]{c}-(-1)^{2/3}(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}}$$

output
```
-1/6*I*ln(1+I*a+I*b*x)*ln(b*(c^(1/3)+d^(1/3)*x)/(b*c^(1/3)+(I-a)*d^(1/3)))
/c^(2/3)/d^(1/3)+1/6*I*ln(1-I*a-I*b*x)*ln(b*(c^(1/3)+d^(1/3)*x)/(b*c^(1/3)
-(I+a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*(-1)^(1/6)*ln(1+I*a+I*b*x)*ln(b*(c^(1
/3)-(-1)^(1/3)*d^(1/3)*x)/(b*c^(1/3)-(-1)^(1/3)*(I-a)*d^(1/3)))/c^(2/3)/d^
(1/3)-1/6*(-1)^(1/6)*ln(1-I*a-I*b*x)*ln(b*(c^(1/3)-(-1)^(1/3)*d^(1/3)*x)/(
b*c^(1/3)+(-1)^(1/3)*(I+a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*(-1)^(5/6)*ln(1+I
*a+I*b*x)*ln(b*(c^(1/3)+(-1)^(2/3)*d^(1/3)*x)/(b*c^(1/3)+(-1)^(2/3)*(I-a)*
d^(1/3)))/c^(2/3)/d^(1/3)-1/6*(-1)^(5/6)*ln(1-I*a-I*b*x)*ln(b*(c^(1/3)+(-1
)^(2/3)*d^(1/3)*x)/(b*c^(1/3)+(-1)^(1/6)*(1-I*a)*d^(1/3)))/c^(2/3)/d^(1/3)
-1/6*I*polylog(2,d^(1/3)*(I-a-b*x)/(b*c^(1/3)+(I-a)*d^(1/3)))/c^(2/3)/d^(1
/3)+1/6*(-1)^(5/6)*polylog(2,-(-1)^(1/6)*d^(1/3)*(I-a-b*x)/(I*b*c^(1/3)-(-
1)^(1/6)*(I-a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*(-1)^(1/6)*polylog(2,-(-1)^(1
/3)*d^(1/3)*(I-a-b*x)/(b*c^(1/3)-(-1)^(1/3)*(I-a)*d^(1/3)))/c^(2/3)/d^(1/3
)+1/6*I*polylog(2,-d^(1/3)*(I+a+b*x)/(b*c^(1/3)-(I+a)*d^(1/3)))/c^(2/3)/d^
(1/3)-1/6*(-1)^(1/6)*polylog(2,(-1)^(1/3)*d^(1/3)*(I+a+b*x)/(b*c^(1/3)+(-1
)^(1/3)*(I+a)*d^(1/3)))/c^(2/3)/d^(1/3)-1/6*(-1)^(5/6)*polylog(2,-(-1)^(2/
3)*d^(1/3)*(I+a+b*x)/(b*c^(1/3)-(-1)^(2/3)*(I+a)*d^(1/3)))/c^(2/3)/d^(1/3)
```

3.52.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 701, normalized size of antiderivative = 0.81

$$\int \frac{\arctan(a+bx)}{c+dx^3}\,dx = \frac{-i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-(-i+a)\sqrt[3]{d}}\right)+i\log(-i(i+a+bx))\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-(i+a)\sqrt[3]{d}}\right)+\sqrt[6]{-1}\log(1+}{}$$

input
```
Integrate[ArcTan[a + b*x]/(c + d*x^3),x]
```

output
```
((-I)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*c^(1/3) - (-I
+ a)*d^(1/3))] + I*Log[(-I)*(I + a + b*x)]*Log[(b*(c^(1/3) + d^(1/3)*x))/(
b*c^(1/3) - (I + a)*d^(1/3))] + (-1)^(1/6)*Log[1 + I*a + I*b*x]*Log[(b*(c^
(1/3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/3)*(-I + a)*d^(1/3))]
- (-1)^(1/6)*Log[(-I)*(I + a + b*x)]*Log[(b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*
x))/(b*c^(1/3) + (-1)^(1/3)*(I + a)*d^(1/3))] - (-1)^(5/6)*Log[(-I)*(I + a
 + b*x)]*Log[(b*(c^(1/3) + (-1)^(2/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/6)*
(1 - I*a)*d^(1/3))] + (-1)^(5/6)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/3) + (-
1)^(2/3)*d^(1/3)*x))/(b*c^(1/3) - (-1)^(2/3)*(-I + a)*d^(1/3))] - I*PolyLo
g[2, (d^(1/3)*(-I + a + b*x))/(-(b*c^(1/3)) + (-I + a)*d^(1/3))] + (-1)^(5
/6)*PolyLog[2, ((-1)^(1/6)*d^(1/3)*(-I + a + b*x))/(I*b*c^(1/3) + (-1)^(1/
6)*(-I + a)*d^(1/3))] + (-1)^(1/6)*PolyLog[2, ((-1)^(1/3)*d^(1/3)*(-I + a
+ b*x))/(b*c^(1/3) + (-1)^(1/3)*(-I + a)*d^(1/3))] + I*PolyLog[2, (d^(1/3)
*(I + a + b*x))/(-(b*c^(1/3)) + (I + a)*d^(1/3))] - (-1)^(1/6)*PolyLog[2,
((-1)^(1/3)*d^(1/3)*(I + a + b*x))/(b*c^(1/3) + (-1)^(1/3)*(I + a)*d^(1/3)
)] - (-1)^(5/6)*PolyLog[2, ((-1)^(2/3)*d^(1/3)*(I + a + b*x))/(-(b*c^(1/3)
) + (-1)^(2/3)*(I + a)*d^(1/3))])/(6*c^(2/3)*d^(1/3))
```

3.52.3 Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 869, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5574, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(a+bx)}{c+dx^3}\,dx$$

$$\Big\downarrow \text{5574}$$

$$\frac{1}{2}i\int \frac{\log(-ia-ibx+1)}{dx^3+c}dx-\frac{1}{2}i\int \frac{\log(ia+ibx+1)}{dx^3+c}dx$$

$$\Big\downarrow \text{2856}$$

$$\frac{1}{2}i\int \left(-\frac{\log(-ia-ibx+1)}{3c^{2/3}\left(-\sqrt[3]{d}x-\sqrt[3]{c}\right)}-\frac{\log(-ia-ibx+1)}{3c^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{d}x-\sqrt[3]{c}\right)}-\frac{\log(-ia-ibx+1)}{3c^{2/3}\left(-(-1)^{2/3}\sqrt[3]{d}x-\sqrt[3]{c}\right)}\right)dx-\frac{1}{2}i\int \left(-\frac{\log(ia+ibx+1)}{3c^{2/3}\left(-\sqrt[3]{d}x-\sqrt[3]{c}\right)}-\frac{\log(ia+ibx+1)}{3c^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{d}x-\sqrt[3]{c}\right)}-\frac{\log(ia+ibx+1)}{3c^{2/3}\left(-(-1)^{2/3}\sqrt[3]{d}x-\sqrt[3]{c}\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}i\left(\frac{\log(-ia-ibx+1)\log\left(\frac{b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{b\sqrt[3]{c}-(a+i)\sqrt[3]{d}}\right)}{3c^{2/3}\sqrt[3]{d}}+\frac{(-1)^{2/3}\log(-ia-ibx+1)\log\left(\frac{b\left(\sqrt[3]{c}-\sqrt[3]{-1}\sqrt[3]{d}x\right)}{\sqrt[3]{-1}\sqrt[3]{d}(a+i)+b\sqrt[3]{c}}\right)}{3c^{2/3}\sqrt[3]{d}}-\frac{\sqrt[3]{-1}\log\right.$$

$$\frac{1}{2}i\left(\frac{\log(ia+ibx+1)\log\left(\frac{b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\sqrt[3]{d}(i-a)+b\sqrt[3]{c}}\right)}{3c^{2/3}\sqrt[3]{d}}+\frac{(-1)^{2/3}\log(ia+ibx+1)\log\left(\frac{b\left(\sqrt[3]{c}-\sqrt[3]{-1}\sqrt[3]{d}x\right)}{b\sqrt[3]{c}-\sqrt[3]{-1}(i-a)\sqrt[3]{d}}\right)}{3c^{2/3}\sqrt[3]{d}}-\frac{\sqrt[3]{-1}\log(ia\right.$$

input
```
Int[ArcTan[a + b*x]/(c + d*x^3),x]
```

output
```
(-1/2*I)*((Log[1 + I*a + I*b*x]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*c^(1/3) +
 (I - a)*d^(1/3))])/(3*c^(2/3)*d^(1/3)) + ((-1)^(2/3)*Log[1 + I*a + I*b*x]
*Log[(b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^(1/3) - (-1)^(1/3)*(I - a)*
d^(1/3))])/(3*c^(2/3)*d^(1/3)) - ((-1)^(1/3)*Log[1 + I*a + I*b*x]*Log[(b*(
c^(1/3) + (-1)^(2/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(2/3)*(I - a)*d^(1/3))]
)/(3*c^(2/3)*d^(1/3)) + PolyLog[2, (d^(1/3)*(I - a - b*x))/(b*c^(1/3) + (I
 - a)*d^(1/3))]/(3*c^(2/3)*d^(1/3)) - ((-1)^(1/3)*PolyLog[2, -(((-1)^(1/6)
*d^(1/3)*(I - a - b*x))/(I*b*c^(1/3) - (-1)^(1/6)*(I - a)*d^(1/3)))])/(3*c
^(2/3)*d^(1/3)) + ((-1)^(2/3)*PolyLog[2, -(((-1)^(1/3)*d^(1/3)*(I - a - b*
x))/(b*c^(1/3) - (-1)^(1/3)*(I - a)*d^(1/3)))])/(3*c^(2/3)*d^(1/3))) + (I/
2)*((Log[1 - I*a - I*b*x]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*c^(1/3) - (I +
a)*d^(1/3))])/(3*c^(2/3)*d^(1/3)) + ((-1)^(2/3)*Log[1 - I*a - I*b*x]*Log[(
b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/3)*(I + a)*d^(1/3
))])/(3*c^(2/3)*d^(1/3)) - ((-1)^(1/3)*Log[1 - I*a - I*b*x]*Log[(b*(c^(1/3
) + (-1)^(2/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/6)*(1 - I*a)*d^(1/3))])/(3
*c^(2/3)*d^(1/3)) + PolyLog[2, -((d^(1/3)*(I + a + b*x))/(b*c^(1/3) - (I +
 a)*d^(1/3)))]/(3*c^(2/3)*d^(1/3)) + ((-1)^(2/3)*PolyLog[2, ((-1)^(1/3)*d^
(1/3)*(I + a + b*x))/(b*c^(1/3) + (-1)^(1/3)*(I + a)*d^(1/3))])/(3*c^(2/3)
*d^(1/3)) - ((-1)^(1/3)*PolyLog[2, -(((-1)^(2/3)*d^(1/3)*(I + a + b*x))/(b
*c^(1/3) - (-1)^(2/3)*(I + a)*d^(1/3)))])/(3*c^(2/3)*d^(1/3)))
```

3.52. $\int \frac{\arctan(a+bx)}{c+dx^3}\,dx$

3.52.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 5574
```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[
I/2   Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Simp[I/2   Int[Log[1 +
 I*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

3.52.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.88 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.44

method	result
risch	$-\frac{ib^2\left(\sum_{\_R1=\mathrm{RootOf}\left(d\_Z^3+\left(3\,\mathrm{RootOf}\left(\_Z^2+1,\mathrm{index}=1\right)ad-3d\right)\_Z^2+\left(-6\,\mathrm{RootOf}\left(\_Z^2+1,\mathrm{index}=1\right)ad-3a^2d+3d\right)\_Z-\mathrm{RootO}}\right.}{}$
derivativedivides	$-\frac{b^3\left(\sum_{\_R=\mathrm{RootOf}\left(d\_Z^3-3ad\_Z^2+3a^2d\_Z-d\,a^3+b^3c\right)}\frac{\ln\left(bx-\_R+a\right)}{-\_R^2+2\_Ra-a^2}\right)\arctan(bx+a)}{3d}+\frac{b^3\left(\arctan(bx+a)\left(\sum_{\_R=\mathrm{RootO}}\right.\right.}{}$
default	$-\frac{b^3\left(\sum_{\_R=\mathrm{RootOf}\left(d\_Z^3-3ad\_Z^2+3a^2d\_Z-d\,a^3+b^3c\right)}\frac{\ln\left(bx-\_R+a\right)}{-\_R^2+2\_Ra-a^2}\right)\arctan(bx+a)}{3d}+\frac{b^3\left(\arctan(bx+a)\left(\sum_{\_R=\mathrm{RootO}}\right.\right.}{}$

input
```
int(arctan(b*x+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6*I*b^2/d*sum(1/(1+2*I*a*_R1-2*I*a+_R1^2-a^2-2*_R1)*(ln(1-I*a-I*b*x)*ln
((_R1+I*b*x+I*a-1)/_R1)+dilog((_R1+I*b*x+I*a-1)/_R1)),_R1=RootOf(d*_Z^3+(3
*RootOf(_Z^2+1,index=1)*a*d-3*d)*_Z^2+(-6*RootOf(_Z^2+1,index=1)*a*d-3*a^2
*d+3*d)*_Z-RootOf(_Z^2+1,index=1)*a^3*d+RootOf(_Z^2+1,index=1)*b^3*c+3*Roo
tOf(_Z^2+1,index=1)*a*d+3*a^2*d-d))+1/6*I*b^2/d*sum(1/(1-2*I*a*_R1+2*I*a+_
R1^2-a^2-2*_R1)*(ln(1+I*a+I*b*x)*ln((_R1-I*b*x-I*a-1)/_R1)+dilog((_R1-I*b*
x-I*a-1)/_R1)),_R1=RootOf(d*_Z^3+(-3*RootOf(_Z^2+1,index=1)*a*d-3*d)*_Z^2+
(6*RootOf(_Z^2+1,index=1)*a*d-3*a^2*d+3*d)*_Z+RootOf(_Z^2+1,index=1)*a^3*d
-RootOf(_Z^2+1,index=1)*b^3*c-3*RootOf(_Z^2+1,index=1)*a*d+3*a^2*d-d))
```

3.52. $\int \frac{\arctan(a+bx)}{c+dx^3}\,dx$

3.52.5 Fricas [F]

$$\int \frac{\arctan(a+bx)}{c+dx^3}\,dx = \int \frac{\arctan\left(bx+a\right)}{dx^3+c}\,dx$$

input
```
integrate(arctan(b*x+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
integral(arctan(b*x + a)/(d*x^3 + c), x)
```

3.52.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+dx^3}\,dx = \text{Timed out}$$

input
```
integrate(atan(b*x+a)/(d*x**3+c),x)
```

output
```
Timed out
```

3.52.7 Maxima [F]

$$\int \frac{\arctan(a+bx)}{c+dx^3}\,dx = \int \frac{\arctan\left(bx+a\right)}{dx^3+c}\,dx$$

input
```
integrate(arctan(b*x+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(arctan(b*x + a)/(d*x^3 + c), x)
```

3.52.8 Giac [F]

$$\int \frac{\arctan(a+bx)}{c+dx^3}\,dx = \int \frac{\arctan(bx+a)}{dx^3+c}\,dx$$

input
```
integrate(arctan(b*x+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+dx^3}\,dx = \int \frac{\operatorname{atan}(a+b\,x)}{d\,x^3+c}\,dx$$

input
```
int(atan(a + b*x)/(c + d*x^3),x)
```

output
```
int(atan(a + b*x)/(c + d*x^3), x)
```

3.53 $\int \frac{\arctan(a+bx)}{c+dx^2}\,dx$

3.53.1 Optimal result

Integrand size = 16, antiderivative size = 543

$$\begin{aligned}\int \frac{\arctan(a+bx)}{c+dx^2}\,dx = &-\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}\\ &+\frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}\\ &+\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}+\sqrt{d}x\right)}{b\sqrt{-c}+(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}\\ &-\frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{-c}+\sqrt{d}x\right)}{b\sqrt{-c}-(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}\\ &-\frac{i\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{d}(i-a-bx)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{i\,\mathrm{PolyLog}\left(2,\frac{\sqrt{d}(i-a-bx)}{b\sqrt{-c}+(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}\\ &-\frac{i\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{d}(i+a+bx)}{b\sqrt{-c}-(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{i\,\mathrm{PolyLog}\left(2,\frac{\sqrt{d}(i+a+bx)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}\end{aligned}$$

output
```
-1/4*I*ln(1+I*a+I*b*x)*ln(b*((-c)^(1/2)-x*d^(1/2))/(b*(-c)^(1/2)-(I-a)*d^(
1/2)))/(-c)^(1/2)/d^(1/2)+1/4*I*ln(1-I*a-I*b*x)*ln(b*((-c)^(1/2)-x*d^(1/2)
)/(b*(-c)^(1/2)+(I+a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*I*ln(1+I*a+I*b*x)*l
n(b*((-c)^(1/2)+x*d^(1/2))/(b*(-c)^(1/2)+(I-a)*d^(1/2)))/(-c)^(1/2)/d^(1/2
)-1/4*I*ln(1-I*a-I*b*x)*ln(b*((-c)^(1/2)+x*d^(1/2))/(b*(-c)^(1/2)-(I+a)*d^
(1/2)))/(-c)^(1/2)/d^(1/2)-1/4*I*polylog(2,-(I-a-b*x)*d^(1/2)/(b*(-c)^(1/2
)-(I-a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*I*polylog(2,(I-a-b*x)*d^(1/2)/(b*
(-c)^(1/2)+(I-a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)-1/4*I*polylog(2,-(I+a+b*x)*d
^(1/2)/(b*(-c)^(1/2)-(I+a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*I*polylog(2,(I
+a+b*x)*d^(1/2)/(b*(-c)^(1/2)+(I+a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)
```

3.53.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.75

$$\int \frac{\arctan(a+bx)}{c+dx^2}\,dx =$$
$$-\frac{i\left(\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}+(-i+a)\sqrt{d}}\right)-\log(-i(i+a+bx))\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)-\log(1+ia+ibx}{}$$

input
```
Integrate[ArcTan[a + b*x]/(c + d*x^2),x]
```

output
```
((-1/4*I)*(Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c]
 + (-I + a)*Sqrt[d])] - Log[(-I)*(I + a + b*x)]*Log[(b*(Sqrt[-c] - Sqrt[d]
*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])] - Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-
c] + Sqrt[d]*x))/(b*Sqrt[-c] - (-I + a)*Sqrt[d])] + Log[(-I)*(I + a + b*x)
]*Log[(b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d])] - PolyLog
[2, (Sqrt[d]*(-I + a + b*x))/(-(b*Sqrt[-c]) + (-I + a)*Sqrt[d])] + PolyLog
[2, (Sqrt[d]*(-I + a + b*x))/(b*Sqrt[-c] + (-I + a)*Sqrt[d])] + PolyLog[2,
 (Sqrt[d]*(I + a + b*x))/(-(b*Sqrt[-c]) + (I + a)*Sqrt[d])] - PolyLog[2, (
Sqrt[d]*(I + a + b*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])]))/(Sqrt[-c]*Sqrt[d]
)
```

3.53.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5574, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(a+bx)}{c+dx^2} \, dx$$

$$\Big\downarrow \; 5574$$

$$\frac{1}{2}i\int \frac{\log(-ia-ibx+1)}{dx^2+c}dx-\frac{1}{2}i\int \frac{\log(ia+ibx+1)}{dx^2+c}dx$$

$$\Big\downarrow \; 2856$$

$$\frac{1}{2}i\int \left(\frac{\sqrt{-c}\log(-ia-ibx+1)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)}+\frac{\sqrt{-c}\log(-ia-ibx+1)}{2c\left(\sqrt{d}x+\sqrt{-c}\right)}\right)dx-\frac{1}{2}i\int \left(\frac{\sqrt{-c}\log(ia+ibx+1)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)}+\frac{\sqrt{-c}\log(ia+ibx+1)}{2c\left(\sqrt{d}x+\sqrt{-c}\right)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}i\left(-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}(a+bx+i)}{b\sqrt{-c}-(a+i)\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}(a+bx+i)}{\sqrt{d}(a+i)+b\sqrt{-c}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\log(-ia-ibx+1)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}+(a+i)\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}-\ldots\right.$$
$$\frac{1}{2}i\left(\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}(-a-bx+i)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}(-a-bx+i)}{\sqrt{d}(i-a)+b\sqrt{-c}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\log(ia+ibx+1)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}x\right)}{b\sqrt{-c}-(-a+i)\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}-\ldots\right.$$

input
```
Int[ArcTan[a + b*x]/(c + d*x^2),x]
```

output
```
(-1/2*I)*((Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c]
 - (I - a)*Sqrt[d])])/(2*Sqrt[-c]*Sqrt[d]) - (Log[1 + I*a + I*b*x]*Log[(b*
(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] + (I - a)*Sqrt[d])])/(2*Sqrt[-c]*Sqrt[
d]) + PolyLog[2, -((Sqrt[d]*(I - a - b*x))/(b*Sqrt[-c] - (I - a)*Sqrt[d]))
]/(2*Sqrt[-c]*Sqrt[d]) - PolyLog[2, (Sqrt[d]*(I - a - b*x))/(b*Sqrt[-c] +
(I - a)*Sqrt[d])]/(2*Sqrt[-c]*Sqrt[d])) + (I/2)*((Log[1 - I*a - I*b*x]*Log
[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])])/(2*Sqrt[-c]*S
qrt[d]) - (Log[1 - I*a - I*b*x]*Log[(b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c]
 - (I + a)*Sqrt[d])])/(2*Sqrt[-c]*Sqrt[d]) - PolyLog[2, -((Sqrt[d]*(I + a
+ b*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d]))]/(2*Sqrt[-c]*Sqrt[d]) + PolyLog[2,
 (Sqrt[d]*(I + a + b*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])]/(2*Sqrt[-c]*Sqrt[
d]))
```

3.53.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 5574
```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[
I/2   Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Simp[I/2   Int[Log[1 +
 I*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

3.53.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.00

method	result
risch	$\frac{\ln(-bxi-ia+1)\ln\left(\frac{iad-b\sqrt{cd}+(-bxi-ia+1)d-d}{iad-b\sqrt{cd}-d}\right)\sqrt{cd}}{4cd}-\frac{\ln(-bxi-ia+1)\ln\left(\frac{iad+b\sqrt{cd}+(-bxi-ia+1)d-d}{iad+b\sqrt{cd}-d}\right)\sqrt{cd}}{4cd}+\frac{\text{dilog}}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(arctan(b*x+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(1-I*a-I*b*x)/c/d*ln((I*a*d-b*(c*d)^(1/2)+(1-I*a-I*b*x)*d-d)/(I*a*d-
b*(c*d)^(1/2)-d))*(c*d)^(1/2)-1/4*ln(1-I*a-I*b*x)/c/d*ln((I*a*d+b*(c*d)^(1
/2)+(1-I*a-I*b*x)*d-d)/(I*a*d+b*(c*d)^(1/2)-d))*(c*d)^(1/2)+1/4/c/d*dilog(
(I*a*d-b*(c*d)^(1/2)+(1-I*a-I*b*x)*d-d)/(I*a*d-b*(c*d)^(1/2)-d))*(c*d)^(1/
2)-1/4/c/d*dilog((I*a*d+b*(c*d)^(1/2)+(1-I*a-I*b*x)*d-d)/(I*a*d+b*(c*d)^(1
/2)-d))*(c*d)^(1/2)+1/4*ln(1+I*a+I*b*x)/c/d*ln((I*a*d+b*(c*d)^(1/2)-(1+I*a
+I*b*x)*d+d)/(I*a*d+b*(c*d)^(1/2)+d))*(c*d)^(1/2)-1/4*ln(1+I*a+I*b*x)/c/d*
ln((I*a*d-b*(c*d)^(1/2)-(1+I*a+I*b*x)*d+d)/(I*a*d-b*(c*d)^(1/2)+d))*(c*d)^
(1/2)+1/4/c/d*(c*d)^(1/2)*dilog((I*a*d+b*(c*d)^(1/2)-(1+I*a+I*b*x)*d+d)/(I
*a*d+b*(c*d)^(1/2)+d))-1/4/c/d*(c*d)^(1/2)*dilog((I*a*d-b*(c*d)^(1/2)-(1+I
*a+I*b*x)*d+d)/(I*a*d-b*(c*d)^(1/2)+d))
```

3.53.5 Fricas [F]

$$\int \frac{\arctan(a+bx)}{c+dx^2}\,dx = \int \frac{\arctan\left(bx+a\right)}{dx^2+c}\,dx$$

input
```
integrate(arctan(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctan(b*x + a)/(d*x^2 + c), x)
```

3.53.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+dx^2}\,dx = \text{Timed out}$$

input
```
integrate(atan(b*x+a)/(d*x**2+c),x)
```

output
```
Timed out
```

3.53.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8520 vs. 2(369) = 738.

Time = 3.85 (sec) , antiderivative size = 8520, normalized size of antiderivative = 15.69

$$\int \frac{\arctan(a+bx)}{c+dx^2}\,dx = \text{Too large to display}$$

input
```
integrate(arctan(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/8*b*(8*arctan(d*x/sqrt(c*d))*arctan((b^2*x + a*b)/b)/b - (4*arctan(sqrt(
d)*x/sqrt(c))*arctan2((2*a*b^2*c*d + (a*b^3*c + (a^3 + a)*b*d + (b^4*c + (
a^2 + 3)*b^2*d)*x)*sqrt(c)*sqrt(d) + (3*b^3*c*d + (a^2 + 1)*b*d^2)*x)/(b^4
*c^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + 4*(b^3*c + (a^2 + 1)*
b*d)*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + (2*a*b
^2*d*x + b^3*c + 3*(a^2 + 1)*b*d)*sqrt(c)*sqrt(d) + (a*b^3*c*d + (a^3 + a)
*b*d^2)*x)/(b^4*c^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + 4*(b^3
*c + (a^2 + 1)*b*d)*sqrt(c)*sqrt(d))) + 4*arctan(sqrt(d)*x/sqrt(c))*arctan
2((2*a*b^2*c*d - (a*b^3*c + (a^3 + a)*b*d + (b^4*c + (a^2 + 3)*b^2*d)*x)*s
qrt(c)*sqrt(d) + (3*b^3*c*d + (a^2 + 1)*b*d^2)*x)/(b^4*c^2 + 2*(a^2 + 3)*b
^2*c*d + (a^4 + 2*a^2 + 1)*d^2 - 4*(b^3*c + (a^2 + 1)*b*d)*sqrt(c)*sqrt(d)
), ((a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 - (2*a*b^2*d*x + b^3*c + 3*(
a^2 + 1)*b*d)*sqrt(c)*sqrt(d) + (a*b^3*c*d + (a^3 + a)*b*d^2)*x)/(b^4*c^2
+ 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 - 4*(b^3*c + (a^2 + 1)*b*d)*
sqrt(c)*sqrt(d))) + log(d*x^2 + c)*log(((a^2 + 1)*b^22*c^11*d + 11*(a^4 +
22*a^2 + 21)*b^20*c^10*d^2 + 55*(a^6 + 39*a^4 + 171*a^2 + 133)*b^18*c^9*d^
3 + 33*(5*a^8 + 260*a^6 + 1870*a^4 + 3876*a^2 + 2261)*b^16*c^8*d^4 + 330*(
a^10 + 61*a^8 + 570*a^6 + 1802*a^4 + 2261*a^2 + 969)*b^14*c^7*d^5 + 22*(21
*a^12 + 1386*a^10 + 15015*a^8 + 60060*a^6 + 109395*a^4 + 92378*a^2 + 29393
)*b^12*c^6*d^6 + 22*(21*a^14 + 1407*a^12 + 16401*a^10 + 75075*a^8 + 169...
```

3.53.8 Giac [F]

$$\int \frac{\arctan(a+bx)}{c+dx^2}\,dx = \int \frac{\arctan(bx+a)}{dx^2+c}\,dx$$

input
```
integrate(arctan(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+dx^2}\,dx = \int \frac{\operatorname{atan}(a+b\,x)}{d\,x^2+c}\,dx$$

input
```
int(atan(a + b*x)/(c + d*x^2),x)
```

output
```
int(atan(a + b*x)/(c + d*x^2), x)
```

3.54 $\int \frac{\arctan(a+bx)}{c+dx}\,dx$

3.54.1 Optimal result

Integrand size = 14, antiderivative size = 152

$$\int \frac{\arctan(a+bx)}{c+dx}\,dx = -\frac{\arctan(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d} + \frac{\arctan(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d} + \frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{2d} - \frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{2d}$$

output
```
-arctan(b*x+a)*ln(2/(1-I*(b*x+a)))/d+arctan(b*x+a)*ln(2*b*(d*x+c)/(b*c+I*d
-a*d)/(1-I*(b*x+a)))/d+1/2*I*polylog(2,1-2/(1-I*(b*x+a)))/d-1/2*I*polylog(
2,1-2*b*(d*x+c)/(b*c+I*d-a*d)/(1-I*(b*x+a)))/d
```

3.54.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.52

$$\int \frac{\arctan(a+bx)}{c+dx}\,dx = \frac{i\log(1-i(a+bx))\log\left(-\frac{i\left(\frac{bc-ad}{b}+\frac{d(a+bx)}{b}\right)}{-\frac{d}{b}-\frac{i(bc-ad)}{b}}\right)}{2d} - \frac{i\log(1+i(a+bx))\log\left(\frac{i\left(\frac{bc-ad}{b}+\frac{d(a+bx)}{b}\right)}{-\frac{d}{b}+\frac{i(bc-ad)}{b}}\right)}{2d} + \frac{i\,\mathrm{PolyLog}\left(2,-\frac{id(1-i(a+bx))}{bc-id-ad}\right)}{2d} - \frac{i\,\mathrm{PolyLog}\left(2,\frac{id(1+i(a+bx))}{bc+id-ad}\right)}{2d}$$

input
```
Integrate[ArcTan[a + b*x]/(c + d*x),x]
```

output
```
((I/2)*Log[1 - I*(a + b*x)]*Log[((-I)*((b*c - a*d)/b + (d*(a + b*x))/b))/(
-(d/b) - (I*(b*c - a*d))/b)])/d - ((I/2)*Log[1 + I*(a + b*x)]*Log[(I*((b*c
 - a*d)/b + (d*(a + b*x))/b))/(-(d/b) + (I*(b*c - a*d))/b)])/d + ((I/2)*Po
lyLog[2, ((-I)*d*(1 - I*(a + b*x)))/(b*c - I*d - a*d)])/d - ((I/2)*PolyLog
[2, (I*d*(1 + I*(a + b*x)))/(b*c + I*d - a*d)])/d
```

3.54.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5570, 27, 5381, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{5570}$$

$$\frac{\int \frac{b\arctan(a+bx)}{b\left(c-\frac{ad}{b}\right)+d(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\arctan(a+bx)}{d(a+bx)-ad+bc}d(a+bx)$$

$$\Big\downarrow \text{5381}$$

$$-\frac{\int \frac{\log\left(\frac{2(bc-ad+d(a+bx))}{(bc-ad+id)(1-i(a+bx))}\right)}{(a+bx)^2+1}d(a+bx)}{d}+\frac{\int \frac{\log\left(\frac{2}{1-i(a+bx)}\right)}{(a+bx)^2+1}d(a+bx)}{d}+\frac{\arctan(a+bx)\log\left(\frac{2(d(a+bx)-ad+bc)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d}-\frac{\arctan(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d}$$

$$\Big\downarrow \text{2849}$$

$$-\frac{\int \frac{\log\left(\frac{2(bc-ad+d(a+bx))}{(bc-ad+id)(1-i(a+bx))}\right)}{(a+bx)^2+1}d(a+bx)}{d}+\frac{i\int \frac{\log\left(\frac{2}{1-i(a+bx)}\right)}{1-\frac{2}{1-i(a+bx)}}d\frac{1}{1-i(a+bx)}}{d}+\frac{\arctan(a+bx)\log\left(\frac{2(d(a+bx)-ad+bc)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d}-\frac{\arctan(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{\int \frac{\log\left(\frac{2(bc-ad+d(a+bx))}{(bc-ad+id)(1-i(a+bx))}\right)}{(a+bx)^2+1}d(a+bx)}{d}+\frac{\arctan(a+bx)\log\left(\frac{2(d(a+bx)-ad+bc)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d}-\frac{\arctan(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{2d}$$

$$\Big\downarrow \text{2897}$$

$$\frac{\arctan(a+bx)\log\left(\frac{2(d(a+bx)-ad+bc)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d}-\frac{\arctan(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2(bc-ad+d(a+bx))}{(bc-ad+id)(1-i(a+bx))}\right)}{2d}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{2d}$$

input
```
Int[ArcTan[a + b*x]/(c + d*x),x]
```

output
```
-((ArcTan[a + b*x]*Log[2/(1 - I*(a + b*x))])/d) + (ArcTan[a + b*x]*Log[(2*
(b*c - a*d + d*(a + b*x)))/((b*c + I*d - a*d)*(1 - I*(a + b*x)))])/d + ((I
/2)*PolyLog[2, 1 - 2/(1 - I*(a + b*x))])/d - ((I/2)*PolyLog[2, 1 - (2*(b*c
 - a*d + d*(a + b*x)))/((b*c + I*d - a*d)*(1 - I*(a + b*x)))])/d
```

3.54. $\int \frac{\arctan(a+bx)}{c+dx}\,dx$

3.54.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5381
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[b*(c/e)
   Int[Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Simp[b*(c/e)   Int[Log[2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x]) /; FreeQ[{a
, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

rule 5570
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.54.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{\frac{b\ln(ad-bc-d(bx+a))\arctan(bx+a)}{d}+b\Big(-\frac{i\ln(ad-bc-d(bx+a))\left(\ln\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\ln\left(\frac{id-d(bx+a)}{-ad+bc+id}\right)\right)}{2d}-\frac{i\big(\operatorname{dilog}\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-}{2d}}{b}$
default	$\frac{\frac{b\ln(ad-bc-d(bx+a))\arctan(bx+a)}{d}+b\Big(-\frac{i\ln(ad-bc-d(bx+a))\left(\ln\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\ln\left(\frac{id-d(bx+a)}{-ad+bc+id}\right)\right)}{2d}-\frac{i\big(\operatorname{dilog}\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-}{2d}}{b}$
parts	$\frac{\ln(dx+c)\arctan(bx+a)}{d}-b\Big(-\frac{i\ln(dx+c)\left(\ln\left(\frac{id-ad+bc-b(dx+c)}{-ad+bc+id}\right)-\ln\left(\frac{id+ad-bc+b(dx+c)}{ad-bc+id}\right)\right)}{2db}-\frac{i\big(\operatorname{dilog}\big(\frac{id-ad+bc}{-ad+}}{}$
risch	$\frac{i\operatorname{dilog}\left(\frac{iad-ibc+(-bxi-ia+1)d-d}{iad-ibc-d}\right)}{2d}+\frac{i\ln(-bxi-ia+1)\ln\left(\frac{iad-ibc+(-bxi-ia+1)d-d}{iad-ibc-d}\right)}{2d}-\frac{i\operatorname{dilog}\big(\frac{-iad+ibc+(bxi+ia+1)}{-iad+ibc-d}}{2d}$

input
```
int(arctan(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(b*ln(a*d-b*c-d*(b*x+a))/d*arctan(b*x+a)+b*(-1/2*I*ln(a*d-b*c-d*(b*x+a
))*(ln((I*d+d*(b*x+a))/(a*d-b*c+I*d))-ln((I*d-d*(b*x+a))/(b*c+I*d-a*d)))/d
-1/2*I*(dilog((I*d+d*(b*x+a))/(a*d-b*c+I*d))-dilog((I*d-d*(b*x+a))/(b*c+I*
d-a*d)))/d))
```

3.54.5 Fricas [F]

$$\int \frac{\arctan(a+bx)}{c+dx}\,dx = \int \frac{\arctan(bx+a)}{dx+c}\,dx$$

input
```
integrate(arctan(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(arctan(b*x + a)/(d*x + c), x)
```

3.54.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+dx}\,dx = \text{Timed out}$$

input
```
integrate(atan(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

3.54.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(130) = 260.

Time = 0.36 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.87

$$\int \frac{\arctan(a+bx)}{c+dx}\,dx = \frac{\arctan(bx+a)\log(dx+c)}{d} - \frac{\arctan\left(\frac{b^2x+ab}{b}\right)\log(dx+c)}{d} - \frac{\arctan\left(\frac{bd^2x+bcd}{b^2c^2-2\,abcd+(a^2+1)d^2}, \frac{b^2c^2-abcd+(b^2cd-abd^2)x}{b^2c^2-2\,abcd+(a^2+1)d^2}\right)\log(b^2x^2+2\,abx+a^2+1) - \arctan(bx+a)\log\left(\frac{b^2d^2}{b^2c^2}\right.}{2\,d}$$

input
```
integrate(arctan(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
arctan(b*x + a)*log(d*x + c)/d - arctan((b^2*x + a*b)/b)*log(d*x + c)/d -
1/2*(arctan2((b*d^2*x + b*c*d)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2), (b^2
*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d
^2))*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - arctan(b*x + a)*log((b^2*d^2*x^2 +
 2*b^2*c*d*x + b^2*c^2)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)) + I*dilog((
I*b*d*x + (I*a + 1)*d)/(-I*b*c + (I*a + 1)*d)) - I*dilog((I*b*d*x + (I*a -
 1)*d)/(-I*b*c + (I*a - 1)*d)))/d
```

3.54.8 Giac [F]

$$\int \frac{\arctan(a+bx)}{c+dx}\,dx = \int \frac{\arctan(bx+a)}{dx+c}\,dx$$

input
```
integrate(arctan(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{atan}(a+b\,x)}{c+d\,x}\,dx$$

input
```
int(atan(a + b*x)/(c + d*x),x)
```

output
```
int(atan(a + b*x)/(c + d*x), x)
```

3.55 $\int \frac{\arctan(a+bx)}{c+\frac{d}{x}} dx$

3.55.1 Optimal result

Integrand size = 16, antiderivative size = 244

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x}} dx = -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} - \frac{id\log(1-ia-ibx)\log\left(-\frac{b(d+cx)}{(i+a)c-bd}\right)}{2c^2} + \frac{id\log(1+ia+ibx)\log\left(\frac{b(d+cx)}{(i-a)c+bd}\right)}{2c^2} + \frac{id\,\text{PolyLog}\left(2,\frac{c(i-a-bx)}{ic-ac+bd}\right)}{2c^2} - \frac{id\,\text{PolyLog}\left(2,\frac{c(i+a+bx)}{(i+a)c-bd}\right)}{2c^2}$$

output
```
-1/2*(1+I*a+I*b*x)*ln(1+I*a+I*b*x)/b/c-1/2*(1-I*a-I*b*x)*ln(-I*(I+a+b*x))/
b/c-1/2*I*d*ln(1-I*a-I*b*x)*ln(-b*(c*x+d)/((I+a)*c-b*d))/c^2+1/2*I*d*ln(1+
I*a+I*b*x)*ln(b*(c*x+d)/((I-a)*c+b*d))/c^2+1/2*I*d*polylog(2,c*(I-a-b*x)/(
I*c-a*c+b*d))/c^2-1/2*I*d*polylog(2,c*(I+a+b*x)/((I+a)*c-b*d))/c^2
```

3.55.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 771 vs. 2(244) = 488.

Time = 8.24 (sec) , antiderivative size = 771, normalized size of antiderivative = 3.16

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x}}\,dx$$
$$= \frac{-2a^2c^2\arctan(a+bx)+2abcd\arctan(a+bx)+iabcd\pi\arctan(a+bx)-ib^2d^2\pi\arctan(a+bx)-2abc^2x}{}$$

input
```
Integrate[ArcTan[a + b*x]/(c + d/x),x]
```

output
```
(-2*a^2*c^2*ArcTan[a + b*x] + 2*a*b*c*d*ArcTan[a + b*x] + I*a*b*c*d*Pi*Arc
Tan[a + b*x] - I*b^2*d^2*Pi*ArcTan[a + b*x] - 2*a*b*c^2*x*ArcTan[a + b*x]
+ 2*b^2*c*d*x*ArcTan[a + b*x] + (2*I)*a*b*c*d*ArcTan[a - (b*d)/c]*ArcTan[a
 + b*x] - (2*I)*b^2*d^2*ArcTan[a - (b*d)/c]*ArcTan[a + b*x] - b*c*d*ArcTan
[a + b*x]^2 + I*a*b*c*d*ArcTan[a + b*x]^2 - I*b^2*d^2*ArcTan[a + b*x]^2 +
(b*c*d*Sqrt[1 + a^2 - (2*a*b*d)/c + (b^2*d^2)/c^2]*ArcTan[a + b*x]^2)/E^(I
*ArcTan[a - (b*d)/c]) + a*b*c*d*Pi*Log[1 + E^((-2*I)*ArcTan[a + b*x])] - b
^2*d^2*Pi*Log[1 + E^((-2*I)*ArcTan[a + b*x])] - 2*a*b*c*d*ArcTan[a + b*x]*
Log[1 + E^((2*I)*ArcTan[a + b*x])] + 2*b^2*d^2*ArcTan[a + b*x]*Log[1 + E^(
(2*I)*ArcTan[a + b*x])] - 2*a*b*c*d*ArcTan[a - (b*d)/c]*Log[1 - E^((2*I)*(
-ArcTan[a - (b*d)/c] + ArcTan[a + b*x]))] + 2*b^2*d^2*ArcTan[a - (b*d)/c]*
Log[1 - E^((2*I)*(-ArcTan[a - (b*d)/c] + ArcTan[a + b*x]))] + 2*a*b*c*d*Ar
cTan[a + b*x]*Log[1 - E^((2*I)*(-ArcTan[a - (b*d)/c] + ArcTan[a + b*x]))]
- 2*b^2*d^2*ArcTan[a + b*x]*Log[1 - E^((2*I)*(-ArcTan[a - (b*d)/c] + ArcTa
n[a + b*x]))] - 2*a*c^2*Log[1/Sqrt[1 + (a + b*x)^2]] + 2*b*c*d*Log[1/Sqrt[
1 + (a + b*x)^2]] - a*b*c*d*Pi*Log[1/Sqrt[1 + (a + b*x)^2]] + b^2*d^2*Pi*L
og[1/Sqrt[1 + (a + b*x)^2]] + 2*a*b*c*d*ArcTan[a - (b*d)/c]*Log[Sin[ArcTan
[(-(a*c) + b*d)/c] + ArcTan[a + b*x]]] - 2*b^2*d^2*ArcTan[a - (b*d)/c]*Log
[Sin[ArcTan[(-(a*c) + b*d)/c] + ArcTan[a + b*x]]] + I*b*d*(a*c - b*d)*Poly
Log[2, -E^((2*I)*ArcTan[a + b*x])] + I*b*d*(-(a*c) + b*d)*PolyLog[2, E^...
```

3.55.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5574, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x}}dx$$

$$\Big\downarrow \text{5574}$$

$$\frac{1}{2}i\int \frac{\log(-ia-ibx+1)}{c+\frac{d}{x}}dx-\frac{1}{2}i\int \frac{\log(ia+ibx+1)}{c+\frac{d}{x}}dx$$

$$\Big\downarrow \text{2856}$$

$$\frac{1}{2}i\int \left(\frac{\log(-ia-ibx+1)}{c}-\frac{d\log(-ia-ibx+1)}{c(d+cx)}\right)dx-\frac{1}{2}i\int \left(\frac{\log(ia+ibx+1)}{c}-\frac{d\log(ia+ibx+1)}{c(d+cx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}i\left(-\frac{d\,\text{PolyLog}\left(2,\frac{c(a+bx+i)}{(a+i)c-bd}\right)}{c^2}-\frac{d\log(-ia-ibx+1)\log\left(-\frac{b(cx+d)}{-bd+(a+i)c}\right)}{c^2}+\frac{i(-ia-ibx+1)\log(-i(a+bx+i))}{bc}\right.$$
$$\frac{1}{2}i\left(-\frac{d\,\text{PolyLog}\left(2,\frac{c(-a-bx+i)}{-ac+ic+bd}\right)}{c^2}-\frac{d\log(ia+ibx+1)\log\left(\frac{b(cx+d)}{bd+(-a+i)c}\right)}{c^2}-\frac{i(ia+ibx+1)\log(ia+ibx+1)}{bc}-\frac{x}{c}\right)$$

input
```
Int[ArcTan[a + b*x]/(c + d/x),x]
```

output
```
(-1/2*I)*(-(x/c) - (I*(1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(b*c) - (d*L
og[1 + I*a + I*b*x]*Log[(b*(d + c*x))/((I - a)*c + b*d)])/c^2 - (d*PolyLog
[2, (c*(I - a - b*x))/(I*c - a*c + b*d)])/c^2) + (I/2)*(-(x/c) + (I*(1 - I
*a - I*b*x)*Log[(-I)*(I + a + b*x)])/(b*c) - (d*Log[1 - I*a - I*b*x]*Log[-
((b*(d + c*x))/((I + a)*c - b*d))])/c^2 - (d*PolyLog[2, (c*(I + a + b*x))/
((I + a)*c - b*d)])/c^2)
```

3.55.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 5574
```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[
I/2   Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Simp[I/2   Int[Log[1 +
 I*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

3.55.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{\frac{\arctan(bx+a)(bx+a)}{c}-\frac{\arctan(bx+a)db\ln(ac-bd-c(bx+a))}{c^2}+\frac{-\frac{\ln\left(a^2c^2-2abcd+b^2d^2-2ac(ac-bd-c(bx+a))+2bd(ac-bd-c(bx+a))}{2}}{}}{}$
default	$\frac{\frac{\arctan(bx+a)(bx+a)}{c}-\frac{\arctan(bx+a)db\ln(ac-bd-c(bx+a))}{c^2}+\frac{-\frac{\ln\left(a^2c^2-2abcd+b^2d^2-2ac(ac-bd-c(bx+a))+2bd(ac-bd-c(bx+a))}{2}}{}}{}$
parts	$\frac{\arctan(bx+a)x}{c}-\frac{\arctan(bx+a)d\ln(cx+d)}{c^2}-\frac{b\left(\frac{\ln\left(a^2c^2-2abcd+2abc(cx+d)+b^2d^2-2b^2d(cx+d)+b^2(cx+d)^2+c^2\right)}{2b^2}-\frac{a\,\text{arc}}{}\right.}{}$
risch	$\frac{i\ln(-bxi-ia+1)a}{2bc}+\frac{id\ln(bxi+ia+1)\ln\left(\frac{-iac+ibd+(bxi+ia+1)c-c}{-iac+ibd-c}\right)}{2c^2}-\frac{i\ln(bxi+ia+1)a}{2bc}+\frac{i\ln(-bxi-ia+1)x}{2c}-\ln$

input
```
int(arctan(b*x+a)/(c+d/x),x,method=_RETURNVERBOSE)
```

output
```
1/b*(arctan(b*x+a)/c*(b*x+a)-arctan(b*x+a)*d*b/c^2*ln(a*c-b*d-c*(b*x+a))+1
/c*(-1/2*ln(a^2*c^2-2*a*b*c*d+b^2*d^2-2*a*c*(a*c-b*d-c*(b*x+a))+2*b*d*(a*c
-b*d-c*(b*x+a))+c^2+(a*c-b*d-c*(b*x+a))^2)-b*d*(-1/2*I*ln(a*c-b*d-c*(b*x+a
))*(ln((I*c+c*(b*x+a))/(a*c-b*d+I*c))-ln((I*c-c*(b*x+a))/(I*c-a*c+b*d)))/c
-1/2*I*(dilog((I*c+c*(b*x+a))/(a*c-b*d+I*c))-dilog((I*c-c*(b*x+a))/(I*c-a*
c+b*d)))/c)))
```

3.55.5 Fricas [F]

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x}}\,dx=\int \frac{\arctan\left(bx+a\right)}{c+\frac{d}{x}}\,dx$$

input
```
integrate(arctan(b*x+a)/(c+d/x),x, algorithm="fricas")
```

output
```
integral(x*arctan(b*x + a)/(c*x + d), x)
```

3.55.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x}}\,dx=\text{Timed out}$$

input
```
integrate(atan(b*x+a)/(c+d/x),x)
```

output
```
Timed out
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.16

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x}}\,dx=$$

$$-\frac{bd\arctan\left(bx+a\right)\log\left(-\frac{b^2c^2x^2+2\,b^2cdx+b^2d^2}{2\,abcd-b^2d^2-(a^2+1)c^2}\right)+i\,bd\mathrm{Li}_2\left(-\frac{i\,bcx+(i\,a-1)c}{(-i\,a+1)c+i\,bd}\right)-i\,bd\mathrm{Li}_2\left(-\frac{i\,bcx+(i\,a+1)c}{(-i\,a-1)c+i\,bd}\right)-2\,(b\cdots}{\cdots}$$

input
```
integrate(arctan(b*x+a)/(c+d/x),x, algorithm="maxima")
```

output
```
-1/2*(b*d*arctan(b*x + a)*log(-(b^2*c^2*x^2 + 2*b^2*c*d*x + b^2*d^2)/(2*a*
b*c*d - b^2*d^2 - (a^2 + 1)*c^2)) + I*b*d*dilog(-(I*b*c*x + (I*a - 1)*c)/(
(-I*a + 1)*c + I*b*d)) - I*b*d*dilog(-(I*b*c*x + (I*a + 1)*c)/((-I*a - 1)*
c + I*b*d)) - 2*(b*c*x + a*c)*arctan(b*x + a) - (b*d*arctan2(-(b*c^2*x + b
*c*d)/(2*a*b*c*d - b^2*d^2 - (a^2 + 1)*c^2), (a*b*c*d - b^2*d^2 + (a*b*c^2
 - b^2*c*d)*x)/(2*a*b*c*d - b^2*d^2 - (a^2 + 1)*c^2)) - c)*log(b^2*x^2 + 2
*a*b*x + a^2 + 1))/(b*c^2)
```

3.55.8 Giac [F]

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x}}\,dx=\int \frac{\arctan(bx+a)}{c+\frac{d}{x}}\,dx$$

input
```
integrate(arctan(b*x+a)/(c+d/x),x, algorithm="giac")
```

output
```
sage0*x
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x}}\,dx=\int \frac{\mathrm{atan}(a+b\,x)}{c+\frac{d}{x}}\,dx$$

input
```
int(atan(a + b*x)/(c + d/x),x)
```

output
```
int(atan(a + b*x)/(c + d/x), x)
```

3.56 $\int \frac{\arctan(a+bx)}{c+\frac{d}{x^2}} \, dx$

3.56.1 Optimal result

Integrand size = 16, antiderivative size = 668

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^2}}\,dx = -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} + \frac{i\sqrt{d}\log(1+ia+ibx)\log\left(-\frac{b(\sqrt{d}-\sqrt{-c}x)}{i\sqrt{-c}-a\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} - \frac{i\sqrt{d}\log(1-ia-ibx)\log\left(\frac{b(\sqrt{d}-\sqrt{-c}x)}{i\sqrt{-c}+a\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}} + \frac{i\sqrt{d}\log(1-ia-ibx)\log\left(-\frac{b(\sqrt{d}+\sqrt{-c}x)}{(i+a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} - \frac{i\sqrt{d}\log(1+ia+ibx)\log\left(\frac{b(\sqrt{d}+\sqrt{-c}x)}{i\sqrt{-c}-a\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}} + \frac{i\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c}(i-a-bx)}{i\sqrt{-c}-a\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} - \frac{i\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c}(1+ia+ibx)}{(1+ia)\sqrt{-c}-ib\sqrt{d}}\right)}{4(-c)^{3/2}} + \frac{i\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c}(i+a+bx)}{i\sqrt{-c}+a\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} - \frac{i\sqrt{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c}(i+a+bx)}{i\sqrt{-c}+a\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}}$$

output
```
-1/2*(1+I*a+I*b*x)*ln(1+I*a+I*b*x)/b/c-1/2*(1-I*a-I*b*x)*ln(-I*(I+a+b*x))/
b/c+1/4*I*ln(1+I*a+I*b*x)*ln(-b*(-x*(-c)^(1/2)+d^(1/2))/(I*(-c)^(1/2)-a*(-
c)^(1/2)-b*d^(1/2)))*d^(1/2)/(-c)^(3/2)+1/4*I*ln(1-I*a-I*b*x)*ln(-b*(x*(-c
)^(1/2)+d^(1/2))/((I+a)*(-c)^(1/2)-b*d^(1/2)))*d^(1/2)/(-c)^(3/2)-1/4*I*ln
(1+I*a+I*b*x)*ln(b*(x*(-c)^(1/2)+d^(1/2))/(I*(-c)^(1/2)-a*(-c)^(1/2)+b*d^(
1/2)))*d^(1/2)/(-c)^(3/2)-1/4*I*ln(1-I*a-I*b*x)*ln(b*(-x*(-c)^(1/2)+d^(1/2
))/(I*(-c)^(1/2)+a*(-c)^(1/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)+1/4*I*polylog
(2,(I-a-b*x)*(-c)^(1/2)/(I*(-c)^(1/2)-a*(-c)^(1/2)-b*d^(1/2)))*d^(1/2)/(-c
)^(3/2)+1/4*I*polylog(2,(I+a+b*x)*(-c)^(1/2)/(I*(-c)^(1/2)+a*(-c)^(1/2)-b*
d^(1/2)))*d^(1/2)/(-c)^(3/2)-1/4*I*polylog(2,(1+I*a+I*b*x)*(-c)^(1/2)/((1+
I*a)*(-c)^(1/2)-I*b*d^(1/2)))*d^(1/2)/(-c)^(3/2)-1/4*I*polylog(2,(I+a+b*x)
*(-c)^(1/2)/(I*(-c)^(1/2)+a*(-c)^(1/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)
```

3.56.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 660, normalized size of antiderivative = 0.99

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^2}}\,dx =$$

$$-\frac{i\Big(2i\sqrt{-c}\log(1+ia+ibx)-2a\sqrt{-c}\log(1+ia+ibx)-2b\sqrt{-c}x\log(1+ia+ibx)+2i\sqrt{-c}\log(-i(i}{}$$

input
```
Integrate[ArcTan[a + b*x]/(c + d/x^2),x]
```

output
```
((-1/4*I)*((2*I)*Sqrt[-c]*Log[1 + I*a + I*b*x] - 2*a*Sqrt[-c]*Log[1 + I*a
+ I*b*x] - 2*b*Sqrt[-c]*x*Log[1 + I*a + I*b*x] + (2*I)*Sqrt[-c]*Log[(-I)*(
I + a + b*x)] + 2*a*Sqrt[-c]*Log[(-I)*(I + a + b*x)] + 2*b*Sqrt[-c]*x*Log[
(-I)*(I + a + b*x)] - b*Sqrt[d]*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[d] - Sqr
t[-c]*x))/((-I)*Sqrt[-c] + a*Sqrt[-c] + b*Sqrt[d])] + b*Sqrt[d]*Log[(-I)*(
I + a + b*x)]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/(I*Sqrt[-c] + a*Sqrt[-c] + b*
Sqrt[d])] - b*Sqrt[d]*Log[(-I)*(I + a + b*x)]*Log[-((b*(Sqrt[d] + Sqrt[-c]
*x))/((I + a)*Sqrt[-c] - b*Sqrt[d]))] + b*Sqrt[d]*Log[1 + I*a + I*b*x]*Log
[(b*(Sqrt[d] + Sqrt[-c]*x))/(I*Sqrt[-c] - a*Sqrt[-c] + b*Sqrt[d])] + b*Sqr
t[d]*PolyLog[2, (Sqrt[-c]*(-I + a + b*x))/((-I)*Sqrt[-c] + a*Sqrt[-c] - b*
Sqrt[d])] - b*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(-I + a + b*x))/((-I)*Sqrt[-c]
+ a*Sqrt[-c] + b*Sqrt[d])] - b*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(I + a + b*x))
/(I*Sqrt[-c] + a*Sqrt[-c] - b*Sqrt[d])] + b*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(
I + a + b*x))/(I*Sqrt[-c] + a*Sqrt[-c] + b*Sqrt[d])]))/(b*(-c)^(3/2))
```

3.56.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5574, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^2}}dx$$

$$\Big\downarrow \text{5574}$$

$$\frac{1}{2}i\int \frac{\log(-ia-ibx+1)}{c+\frac{d}{x^2}}dx-\frac{1}{2}i\int \frac{\log(ia+ibx+1)}{c+\frac{d}{x^2}}dx$$

$$\Big\downarrow \text{2856}$$

$$\frac{1}{2}i\int \left(\frac{\log(-ia-ibx+1)}{c}-\frac{d\log(-ia-ibx+1)}{c\left(cx^2+d\right)}\right)dx-\frac{1}{2}i\int \left(\frac{\log(ia+ibx+1)}{c}-\frac{d\log(ia+ibx+1)}{c\left(cx^2+d\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}i\left(\frac{\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(a+bx+i)}{\sqrt{-c}a+i\sqrt{-c}-b\sqrt{d}}\right)}{2(-c)^{3/2}}-\frac{\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(a+bx+i)}{\sqrt{-c}a+i\sqrt{-c}+b\sqrt{d}}\right)}{2(-c)^{3/2}}-\frac{\sqrt{d}\log(-ia-ibx+1)\log\left(\frac{b\left(\sqrt{d}-\right.}{a\sqrt{-c}+b}\right.}{2(-c)^{3/2}}\right.$$

$$\frac{1}{2}i\left(-\frac{\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(-a-bx+i)}{-\sqrt{-c}a+i\sqrt{-c}-b\sqrt{d}}\right)}{2(-c)^{3/2}}+\frac{\sqrt{d}\,\text{PolyLog}\left(2,\frac{\sqrt{-c}(ia+ibx+1)}{(ia+1)\sqrt{-c}-ib\sqrt{d}}\right)}{2(-c)^{3/2}}-\frac{\sqrt{d}\log(ia+ibx+1)\log\left(-\frac{b(}{a(-\sqrt{}}\right.}{2(-c)^{3/2}}\right.$$

input
```
Int[ArcTan[a + b*x]/(c + d/x^2),x]
```

output
```
(-1/2*I)*(-(x/c) - (I*(1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(b*c) - (Sqr
t[d]*Log[1 + I*a + I*b*x]*Log[-((b*(Sqrt[d] - Sqrt[-c]*x))/(I*Sqrt[-c] - a
*Sqrt[-c] - b*Sqrt[d]))])/(2*(-c)^(3/2)) + (Sqrt[d]*Log[1 + I*a + I*b*x]*L
og[(b*(Sqrt[d] + Sqrt[-c]*x))/(I*Sqrt[-c] - a*Sqrt[-c] + b*Sqrt[d])])/(2*(
-c)^(3/2)) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(I - a - b*x))/(I*Sqrt[-c] - a*
Sqrt[-c] - b*Sqrt[d])])/(2*(-c)^(3/2)) + (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1
+ I*a + I*b*x))/((1 + I*a)*Sqrt[-c] - I*b*Sqrt[d])])/(2*(-c)^(3/2))) + (I/
2)*(-(x/c) + (I*(1 - I*a - I*b*x)*Log[(-I)*(I + a + b*x)])/(b*c) - (Sqrt[d
]*Log[1 - I*a - I*b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/(I*Sqrt[-c] + a*Sqrt
[-c] + b*Sqrt[d])])/(2*(-c)^(3/2)) + (Sqrt[d]*Log[1 - I*a - I*b*x]*Log[-((
b*(Sqrt[d] + Sqrt[-c]*x))/((I + a)*Sqrt[-c] - b*Sqrt[d]))])/(2*(-c)^(3/2))
 + (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(I + a + b*x))/(I*Sqrt[-c] + a*Sqrt[-c] -
 b*Sqrt[d])])/(2*(-c)^(3/2)) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(I + a + b*x)
)/(I*Sqrt[-c] + a*Sqrt[-c] + b*Sqrt[d])])/(2*(-c)^(3/2)))
```

3.56.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 5574
```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[
I/2   Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Simp[I/2   Int[Log[1 +
 I*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

3.56.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 647, normalized size of antiderivative = 0.97

method	result
risch	$\frac{i\ln(-bxi-ia+1)x}{2c}+\frac{i\ln(-bxi-ia+1)a}{2bc}-\frac{i\ln(bxi+ia+1)x}{2c}-\frac{i\ln(bxi+ia+1)a}{2bc}-\frac{\ln(-bxi-ia+1)}{2bc}+\frac{1}{bc}-\ln(-$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(arctan(b*x+a)/(c+d/x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/c*ln(1-I*a-I*b*x)*x+1/2*I/b/c*ln(1-I*a-I*b*x)*a-1/2*I/c*ln(1+I*a+I*b
*x)*x-1/2*I/b/c*ln(1+I*a+I*b*x)*a-1/2/b/c*ln(1-I*a-I*b*x)+1/b/c-1/4/c^2*ln
(1-I*a-I*b*x)*(c*d)^(1/2)*ln((I*a*c-b*(c*d)^(1/2)+(1-I*a-I*b*x)*c-c)/(I*a*
c-b*(c*d)^(1/2)-c))+1/4/c^2*ln(1-I*a-I*b*x)*(c*d)^(1/2)*ln((I*a*c+b*(c*d)^
(1/2)+(1-I*a-I*b*x)*c-c)/(I*a*c+b*(c*d)^(1/2)-c))-1/4/c^2*dilog((I*a*c-b*(
c*d)^(1/2)+(1-I*a-I*b*x)*c-c)/(I*a*c-b*(c*d)^(1/2)-c))*(c*d)^(1/2)+1/4/c^2
*dilog((I*a*c+b*(c*d)^(1/2)+(1-I*a-I*b*x)*c-c)/(I*a*c+b*(c*d)^(1/2)-c))*(c
*d)^(1/2)-1/2/b/c*ln(1+I*a+I*b*x)-1/4/c^2*ln(1+I*a+I*b*x)*(c*d)^(1/2)*ln((
I*a*c+b*(c*d)^(1/2)-(1+I*a+I*b*x)*c+c)/(I*a*c+b*(c*d)^(1/2)+c))+1/4/c^2*ln
(1+I*a+I*b*x)*(c*d)^(1/2)*ln((I*a*c-b*(c*d)^(1/2)-(1+I*a+I*b*x)*c+c)/(I*a*
c-b*(c*d)^(1/2)+c))-1/4/c^2*(c*d)^(1/2)*dilog((I*a*c+b*(c*d)^(1/2)-(1+I*a+
I*b*x)*c+c)/(I*a*c+b*(c*d)^(1/2)+c))+1/4/c^2*(c*d)^(1/2)*dilog((I*a*c-b*(c
*d)^(1/2)-(1+I*a+I*b*x)*c+c)/(I*a*c-b*(c*d)^(1/2)+c))
```

3.56.5 Fricas [F]

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^2}}\,dx = \int \frac{\arctan(bx+a)}{c+\frac{d}{x^2}}\,dx$$

input
```
integrate(arctan(b*x+a)/(c+d/x^2),x, algorithm="fricas")
```

output
```
integral(x^2*arctan(b*x + a)/(c*x^2 + d), x)
```

3.56.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^2}}\,dx = \text{Timed out}$$

input
```
integrate(atan(b*x+a)/(c+d/x**2),x)
```

output
```
Timed out
```

3.56.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8518 vs. 2(466) = 932.

Time = 0.91 (sec) , antiderivative size = 8518, normalized size of antiderivative = 12.75

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^2}}\,dx = \text{Too large to display}$$

input
```
integrate(arctan(b*x+a)/(c+d/x^2),x, algorithm="maxima")
```

output
```
-(d*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c) - x/c)*arctan(b*x + a) + 1/8*(8*a*
c*arctan(b*x + a) + (4*b*arctan(sqrt(c)*x/sqrt(d))*arctan2((2*a*b^2*c*d +
(a*b^3*d + (a^3 + a)*b*c + (b^4*d + (a^2 + 3)*b^2*c)*x)*sqrt(c)*sqrt(d) +
(3*b^3*c*d + (a^2 + 1)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2
*a^2 + 1)*c^2 + 4*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2
*c*d + (a^4 + 2*a^2 + 1)*c^2 + (2*a*b^2*c*x + b^3*d + 3*(a^2 + 1)*b*c)*sqr
t(c)*sqrt(d) + (a*b^3*c*d + (a^3 + a)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2
*c*d + (a^4 + 2*a^2 + 1)*c^2 + 4*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d)))
 + 4*b*arctan(sqrt(c)*x/sqrt(d))*arctan2((2*a*b^2*c*d - (a*b^3*d + (a^3 +
a)*b*c + (b^4*d + (a^2 + 3)*b^2*c)*x)*sqrt(c)*sqrt(d) + (3*b^3*c*d + (a^2
+ 1)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*c^2 - 4*
(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2*c*d + (a^4 + 2*a^
2 + 1)*c^2 - (2*a*b^2*c*x + b^3*d + 3*(a^2 + 1)*b*c)*sqrt(c)*sqrt(d) + (a*
b^3*c*d + (a^3 + a)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^
2 + 1)*c^2 - 4*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d))) + b*log(c*x^2 + d
)*log(((a^2 + 1)*b^22*c*d^11 + 11*(a^4 + 22*a^2 + 21)*b^20*c^2*d^10 + 55*(
a^6 + 39*a^4 + 171*a^2 + 133)*b^18*c^3*d^9 + 33*(5*a^8 + 260*a^6 + 1870*a^
4 + 3876*a^2 + 2261)*b^16*c^4*d^8 + 330*(a^10 + 61*a^8 + 570*a^6 + 1802*a^
4 + 2261*a^2 + 969)*b^14*c^5*d^7 + 22*(21*a^12 + 1386*a^10 + 15015*a^8 + 6
0060*a^6 + 109395*a^4 + 92378*a^2 + 29393)*b^12*c^6*d^6 + 22*(21*a^14 +...
```

3.56.8 Giac [F]

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^2}}\,dx = \int \frac{\arctan(bx+a)}{c+\frac{d}{x^2}}\,dx$$

input
```
integrate(arctan(b*x+a)/(c+d/x^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^2}}\,dx=\int \frac{\mathrm{atan}(a+b\,x)}{c+\frac{d}{x^2}}\,dx$$

input
```
int(atan(a + b*x)/(c + d/x^2),x)
```

output
```
int(atan(a + b*x)/(c + d/x^2), x)
```

3.57 $\int \frac{\arctan(a+bx)}{c+\frac{d}{x^3}}\,dx$

3.57.1 Optimal result

Integrand size = 16, antiderivative size = 933

$$
\begin{aligned}
\int \frac{\arctan(a+bx)}{c+\frac{d}{x^3}}\,dx = &-\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} \\
&-\frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} \\
&-\frac{i\sqrt[3]{d}\log(1-ia-ibx)\log\left(-\frac{b\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{(i+a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&+\frac{i\sqrt[3]{d}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{(i-a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&-\frac{\sqrt[6]{-1}\sqrt[3]{d}\log(1+ia+ibx)\log\left(-\frac{b\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{c}x\right)}{\sqrt[3]{-1}(i-a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&+\frac{\sqrt[6]{-1}\sqrt[3]{d}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{c}x\right)}{\sqrt[3]{-1}(i+a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&-\frac{(-1)^{5/6}\sqrt[3]{d}\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{c}x\right)}{(-1)^{2/3}(i-a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&+\frac{(-1)^{5/6}\sqrt[3]{d}\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{c}x\right)}{\sqrt[6]{-1}(1-ia)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&-\frac{\sqrt[6]{-1}\sqrt[3]{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{c}(i-a-bx)}{\sqrt[3]{-1}(i-a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&-\frac{(-1)^{5/6}\sqrt[3]{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[6]{-1}\sqrt[3]{c}(i-a-bx)}{\sqrt[6]{-1}(i-a)\sqrt[3]{c}-ib\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&+\frac{i\sqrt[3]{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{c}(i-a-bx)}{(i-a)\sqrt[3]{c}+b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&-\frac{i\sqrt[3]{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{c}(i+a+bx)}{(i+a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&+\frac{(-1)^{5/6}\sqrt[3]{d}\,\mathrm{PolyLog}\left(2,\frac{(-1)^{2/3}\sqrt[3]{c}(i+a+bx)}{(-1)^{2/3}(i+a)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{6c^{4/3}} \\
&\sqrt[6]{-1}\sqrt[3]{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{-1}\sqrt[3]{c}(i+a+bx)}{\cdots}\right.
\end{aligned}
$$

output
```
-1/2*(1+I*a+I*b*x)*ln(1+I*a+I*b*x)/b/c-1/2*(1-I*a-I*b*x)*ln(-I*(I+a+b*x))/
b/c-1/6*I*d^(1/3)*ln(1-I*a-I*b*x)*ln(-b*(d^(1/3)+c^(1/3)*x)/((I+a)*c^(1/3)
-b*d^(1/3)))/c^(4/3)+1/6*I*d^(1/3)*ln(1+I*a+I*b*x)*ln(b*(d^(1/3)+c^(1/3)*x
)/((I-a)*c^(1/3)+b*d^(1/3)))/c^(4/3)-1/6*(-1)^(1/6)*d^(1/3)*ln(1+I*a+I*b*x
)*ln(-b*(d^(1/3)-(-1)^(1/3)*c^(1/3)*x)/((-1)^(1/3)*(I-a)*c^(1/3)-b*d^(1/3)
))/c^(4/3)+1/6*(-1)^(1/6)*d^(1/3)*ln(1-I*a-I*b*x)*ln(b*(d^(1/3)-(-1)^(1/3)
*c^(1/3)*x)/((-1)^(1/3)*(I+a)*c^(1/3)+b*d^(1/3)))/c^(4/3)-1/6*(-1)^(5/6)*d
^(1/3)*ln(1+I*a+I*b*x)*ln(b*(d^(1/3)+(-1)^(2/3)*c^(1/3)*x)/((-1)^(2/3)*(I-
a)*c^(1/3)+b*d^(1/3)))/c^(4/3)+1/6*(-1)^(5/6)*d^(1/3)*ln(1-I*a-I*b*x)*ln(b
*(d^(1/3)+(-1)^(2/3)*c^(1/3)*x)/((-1)^(1/6)*(1-I*a)*c^(1/3)+b*d^(1/3)))/c^
(4/3)-1/6*(-1)^(1/6)*d^(1/3)*polylog(2,(-1)^(1/3)*c^(1/3)*(I-a-b*x)/((-1)^
(1/3)*(I-a)*c^(1/3)-b*d^(1/3)))/c^(4/3)-1/6*(-1)^(5/6)*d^(1/3)*polylog(2,(
-1)^(1/6)*c^(1/3)*(I-a-b*x)/((-1)^(1/6)*(I-a)*c^(1/3)-I*b*d^(1/3)))/c^(4/3
)+1/6*I*d^(1/3)*polylog(2,c^(1/3)*(I-a-b*x)/((I-a)*c^(1/3)+b*d^(1/3)))/c^(
4/3)-1/6*I*d^(1/3)*polylog(2,c^(1/3)*(I+a+b*x)/((I+a)*c^(1/3)-b*d^(1/3)))/
c^(4/3)+1/6*(-1)^(5/6)*d^(1/3)*polylog(2,(-1)^(2/3)*c^(1/3)*(I+a+b*x)/((-1
)^(2/3)*(I+a)*c^(1/3)-b*d^(1/3)))/c^(4/3)+1/6*(-1)^(1/6)*d^(1/3)*polylog(2
,(-1)^(1/3)*c^(1/3)*(I+a+b*x)/((-1)^(1/3)*(I+a)*c^(1/3)+b*d^(1/3)))/c^(4/3
)
```

3.57.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 896, normalized size of antiderivative = 0.96

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^3}}\,dx$$
$$= \frac{i\left(3i\sqrt[3]{c}\log(1+ia+ibx)-3a\sqrt[3]{c}\log(1+ia+ibx)-3b\sqrt[3]{c}x\log(1+ia+ibx)+3i\sqrt[3]{c}\log(-i(i+a+bx)\right.}{}$$

input
```
Integrate[ArcTan[a + b*x]/(c + d/x^3),x]
```

output
```
((I/6)*((3*I)*c^(1/3)*Log[1 + I*a + I*b*x] - 3*a*c^(1/3)*Log[1 + I*a + I*b
*x] - 3*b*c^(1/3)*x*Log[1 + I*a + I*b*x] + (3*I)*c^(1/3)*Log[(-I)*(I + a +
 b*x)] + 3*a*c^(1/3)*Log[(-I)*(I + a + b*x)] + 3*b*c^(1/3)*x*Log[(-I)*(I +
 a + b*x)] + b*d^(1/3)*Log[1 + I*a + I*b*x]*Log[(b*(d^(1/3) + c^(1/3)*x))/
(-((-I + a)*c^(1/3)) + b*d^(1/3))] - b*d^(1/3)*Log[(-I)*(I + a + b*x)]*Log
[(b*(d^(1/3) + c^(1/3)*x))/(-((I + a)*c^(1/3)) + b*d^(1/3))] + (-1)^(2/3)*
b*d^(1/3)*Log[1 + I*a + I*b*x]*Log[(b*(d^(1/3) - (-1)^(1/3)*c^(1/3)*x))/((
-1)^(1/3)*(-I + a)*c^(1/3) + b*d^(1/3))] - (-1)^(2/3)*b*d^(1/3)*Log[(-I)*(
I + a + b*x)]*Log[(b*(d^(1/3) - (-1)^(1/3)*c^(1/3)*x))/((-1)^(1/3)*(I + a)
*c^(1/3) + b*d^(1/3))] + (-1)^(1/3)*b*d^(1/3)*Log[(-I)*(I + a + b*x)]*Log[
(b*(d^(1/3) + (-1)^(2/3)*c^(1/3)*x))/((-1)^(1/6)*(1 - I*a)*c^(1/3) + b*d^(
1/3))] - (-1)^(1/3)*b*d^(1/3)*Log[1 + I*a + I*b*x]*Log[(b*(d^(1/3) + (-1)^
(2/3)*c^(1/3)*x))/(-((-1)^(2/3)*(-I + a)*c^(1/3)) + b*d^(1/3))] + b*d^(1/3
)*PolyLog[2, (c^(1/3)*(-I + a + b*x))/((-I + a)*c^(1/3) - b*d^(1/3))] - (-
1)^(1/3)*b*d^(1/3)*PolyLog[2, ((-1)^(1/6)*c^(1/3)*(-I + a + b*x))/((-1)^(1
/6)*(-I + a)*c^(1/3) + I*b*d^(1/3))] + (-1)^(2/3)*b*d^(1/3)*PolyLog[2, ((-
1)^(1/3)*c^(1/3)*(-I + a + b*x))/((-1)^(1/3)*(-I + a)*c^(1/3) + b*d^(1/3))
] - b*d^(1/3)*PolyLog[2, (c^(1/3)*(I + a + b*x))/((I + a)*c^(1/3) - b*d^(1
/3))] + (-1)^(1/3)*b*d^(1/3)*PolyLog[2, ((-1)^(2/3)*c^(1/3)*(I + a + b*x))
/((-1)^(2/3)*(I + a)*c^(1/3) - b*d^(1/3))] - (-1)^(2/3)*b*d^(1/3)*PolyL...
```

3.57.3 Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 951, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5574, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^3}}dx$$

$$\Big\downarrow \text{5574}$$

$$\frac{1}{2}i\int \frac{\log(-ia-ibx+1)}{c+\frac{d}{x^3}}dx-\frac{1}{2}i\int \frac{\log(ia+ibx+1)}{c+\frac{d}{x^3}}dx$$

$$\Big\downarrow \text{2856}$$

$$\frac{1}{2}i\int \left(\frac{\log(-ia-ibx+1)}{c}-\frac{d\log(-ia-ibx+1)}{c\,(cx^3+d)}\right)dx-\frac{1}{2}i\int \left(\frac{\log(ia+ibx+1)}{c}-\frac{d\log(ia+ibx+1)}{c\,(cx^3+d)}\right)dx$$

$\downarrow$ 2009

$$\frac{1}{2}i\left(-\frac{x}{c}+\frac{i(-ia-ibx+1)\log(-i(a+bx+i))}{bc}-\frac{\sqrt[3]{d}\log(-ia-ibx+1)\log\left(-\frac{b\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{(a+i)\sqrt[3]{c}-b\sqrt[3]{d}}\right)}{3c^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{d}\log\right.$$

$$\frac{1}{2}i\left(-\frac{x}{c}-\frac{i(ia+ibx+1)\log(ia+ibx+1)}{bc}-\frac{\sqrt[3]{d}\log(ia+ibx+1)\log\left(\frac{b\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{\sqrt[3]{c}(i-a)+b\sqrt[3]{d}}\right)}{3c^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{d}\log(ia+\right.$$

input
```
Int[ArcTan[a + b*x]/(c + d/x^3),x]
```

output
```
(-1/2*I)*(-(x/c) - (I*(1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(b*c) - (d^(
1/3)*Log[1 + I*a + I*b*x]*Log[(b*(d^(1/3) + c^(1/3)*x))/((I - a)*c^(1/3) +
 b*d^(1/3))])/(3*c^(4/3)) - ((-1)^(2/3)*d^(1/3)*Log[1 + I*a + I*b*x]*Log[-
((b*(d^(1/3) - (-1)^(1/3)*c^(1/3)*x))/((-1)^(1/3)*(I - a)*c^(1/3) - b*d^(1
/3)))])/(3*c^(4/3)) + ((-1)^(1/3)*d^(1/3)*Log[1 + I*a + I*b*x]*Log[(b*(d^(
1/3) + (-1)^(2/3)*c^(1/3)*x))/((-1)^(2/3)*(I - a)*c^(1/3) + b*d^(1/3))])/(
3*c^(4/3)) - ((-1)^(2/3)*d^(1/3)*PolyLog[2, ((-1)^(1/3)*c^(1/3)*(I - a - b
*x))/((-1)^(1/3)*(I - a)*c^(1/3) - b*d^(1/3))])/(3*c^(4/3)) + ((-1)^(1/3)*
d^(1/3)*PolyLog[2, ((-1)^(1/6)*c^(1/3)*(I - a - b*x))/((-1)^(1/6)*(I - a)*
c^(1/3) - I*b*d^(1/3))])/(3*c^(4/3)) - (d^(1/3)*PolyLog[2, (c^(1/3)*(I - a
 - b*x))/((I - a)*c^(1/3) + b*d^(1/3))])/(3*c^(4/3))) + (I/2)*(-(x/c) + (I
*(1 - I*a - I*b*x)*Log[(-I)*(I + a + b*x)])/(b*c) - (d^(1/3)*Log[1 - I*a -
 I*b*x]*Log[-((b*(d^(1/3) + c^(1/3)*x))/((I + a)*c^(1/3) - b*d^(1/3)))])/(
3*c^(4/3)) - ((-1)^(2/3)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[(b*(d^(1/3) - (-
1)^(1/3)*c^(1/3)*x))/((-1)^(1/3)*(I + a)*c^(1/3) + b*d^(1/3))])/(3*c^(4/3)
) + ((-1)^(1/3)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[(b*(d^(1/3) + (-1)^(2/3)*
c^(1/3)*x))/((-1)^(1/6)*(1 - I*a)*c^(1/3) + b*d^(1/3))])/(3*c^(4/3)) - (d^
(1/3)*PolyLog[2, (c^(1/3)*(I + a + b*x))/((I + a)*c^(1/3) - b*d^(1/3))])/(
3*c^(4/3)) + ((-1)^(1/3)*d^(1/3)*PolyLog[2, ((-1)^(2/3)*c^(1/3)*(I + a + b
*x))/((-1)^(2/3)*(I + a)*c^(1/3) - b*d^(1/3))])/(3*c^(4/3)) - ((-1)^(2/...
```

3.57. $\int \frac{\arctan(a+bx)}{c+\frac{d}{x^3}}\,dx$

3.57.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 5574
```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[
I/2   Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Simp[I/2   Int[Log[1 +
 I*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

3.57.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.09 (sec) , antiderivative size = 511, normalized size of antiderivative = 0.55

method	result
risch	$\frac{i\ln(-bxi-ia+1)x}{2c}+\frac{i\ln(-bxi-ia+1)a}{2bc}-\frac{\ln(-bxi-ia+1)}{2bc}+\frac{1}{bc}+\frac{ib^2d\left(\_R1=\text{RootOf}\left(c\_Z^3+\left(3\,\text{RootOf}\left(\_Z^2+1,\text{i}\right.\right.\right.\right.}{}$
derivativedivides	$\frac{\arctan(bx+a)(bx+a)}{c}+\frac{\arctan(bx+a)\left(\sum_{\_R=\text{RootOf}\left(c\_Z^3-3ac\_Z^2+3a^2c\_Z-a^3c+b^3d\right)}\frac{\ln\left(bx-\_R+a\right)}{-\_R^2+2\_Ra-a^2}\right)db^3}{3c^2}-\frac{db^3\left(\text{arc}\right.}{}$
default	$\frac{\arctan(bx+a)(bx+a)}{c}+\frac{\arctan(bx+a)\left(\sum_{\_R=\text{RootOf}\left(c\_Z^3-3ac\_Z^2+3a^2c\_Z-a^3c+b^3d\right)}\frac{\ln\left(bx-\_R+a\right)}{-\_R^2+2\_Ra-a^2}\right)db^3}{3c^2}-\frac{db^3\left(\text{arc}\right.}{}$

input
```
int(arctan(b*x+a)/(c+d/x^3),x,method=_RETURNVERBOSE)
```

3.57. $\int \frac{\arctan(a+bx)}{c+\frac{d}{x^3}}\,dx$

output
```
1/2*I/c*ln(1-I*a-I*b*x)*x+1/2*I/b/c*ln(1-I*a-I*b*x)*a-1/2/b/c*ln(1-I*a-I*b
*x)+1/b/c+1/6*I*b^2*d/c^2*sum(1/(1+2*I*a*_R1-2*I*a+_R1^2-a^2-2*_R1)*(ln(1-
I*a-I*b*x)*ln((_R1+I*b*x+I*a-1)/_R1)+dilog((_R1+I*b*x+I*a-1)/_R1)),_R1=Roo
tOf(c*_Z^3+(3*RootOf(_Z^2+1,index=1)*a*c-3*c)*_Z^2+(-6*RootOf(_Z^2+1,index
=1)*a*c-3*a^2*c+3*c)*_Z-RootOf(_Z^2+1,index=1)*a^3*c+RootOf(_Z^2+1,index=1
)*b^3*d+3*RootOf(_Z^2+1,index=1)*a*c+3*a^2*c-c))-1/2*I/c*ln(1+I*a+I*b*x)*x
-1/2*I/b/c*ln(1+I*a+I*b*x)*a-1/2/b/c*ln(1+I*a+I*b*x)-1/6*I*b^2*d/c^2*sum(1
/(1-2*I*a*_R1+2*I*a+_R1^2-a^2-2*_R1)*(ln(1+I*a+I*b*x)*ln((_R1-I*b*x-I*a-1)
/_R1)+dilog((_R1-I*b*x-I*a-1)/_R1)),_R1=RootOf(c*_Z^3+(-3*RootOf(_Z^2+1,in
dex=1)*a*c-3*c)*_Z^2+(6*RootOf(_Z^2+1,index=1)*a*c-3*a^2*c+3*c)*_Z+RootOf(
_Z^2+1,index=1)*a^3*c-RootOf(_Z^2+1,index=1)*b^3*d-3*RootOf(_Z^2+1,index=1
)*a*c+3*a^2*c-c))
```

3.57.5 Fricas [F]

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^3}}\,dx = \int \frac{\arctan\left(bx+a\right)}{c+\frac{d}{x^3}}\,dx$$

input
```
integrate(arctan(b*x+a)/(c+d/x^3),x, algorithm="fricas")
```

output
```
integral(x^3*arctan(b*x + a)/(c*x^3 + d), x)
```

3.57.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^3}}\,dx = \text{Timed out}$$

input
```
integrate(atan(b*x+a)/(c+d/x**3),x)
```

output
```
Timed out
```

3.57.7 Maxima [F]

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^3}}\,dx = \int \frac{\arctan(bx+a)}{c+\frac{d}{x^3}}\,dx$$

input
```
integrate(arctan(b*x+a)/(c+d/x^3),x, algorithm="maxima")
```

output
```
integrate(arctan(b*x + a)/(c + d/x^3), x)
```

3.57.8 Giac [F]

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^3}}\,dx = \int \frac{\arctan(bx+a)}{c+\frac{d}{x^3}}\,dx$$

input
```
integrate(arctan(b*x+a)/(c+d/x^3),x, algorithm="giac")
```

output
```
sage0*x
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{x^3}}\,dx = \int \frac{\operatorname{atan}(a+b\,x)}{c+\frac{d}{x^3}}\,dx$$

input
```
int(atan(a + b*x)/(c + d/x^3),x)
```

output
```
int(atan(a + b*x)/(c + d/x^3), x)
```

3.58 $\int \frac{\arctan(a+bx)}{c+d\sqrt{x}}\,dx$

3.58.1 Optimal result

Integrand size = 18, antiderivative size = 673

$$\begin{aligned}\int \frac{\arctan(a+bx)}{c+d\sqrt{x}}\,dx &= \frac{2i\sqrt{i+a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}d}-\frac{2i\sqrt{i-a}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}d}\\
&+\frac{ic\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&-\frac{ic\log\left(\frac{d\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&+\frac{ic\log\left(-\frac{d\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&-\frac{ic\log\left(-\frac{d\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c-\sqrt{i-a}d}\right)\log\left(c+d\sqrt{x}\right)}{d^2}\\
&+\frac{i\sqrt{x}\log(1-ia-ibx)}{d}-\frac{ic\log\left(c+d\sqrt{x}\right)\log(1-ia-ibx)}{d^2}\\
&-\frac{i\sqrt{x}\log(1+ia+ibx)}{d}+\frac{ic\log\left(c+d\sqrt{x}\right)\log(1+ia+ibx)}{d^2}\\
&+\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-i-a}d}\right)}{d^2}+\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{-i-a}d}\right)}{d^2}\\
&-\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{i-a}d}\right)}{d^2}-\frac{ic\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{i-a}d}\right)}{d^2}\end{aligned}$$

output
```
-I*c*ln(1-I*a-I*b*x)*ln(c+d*x^(1/2))/d^2+I*c*ln(1+I*a+I*b*x)*ln(c+d*x^(1/2
))/d^2+I*c*ln(c+d*x^(1/2))*ln(d*((-I-a)^(1/2)-b^(1/2)*x^(1/2))/(d*(-I-a)^(
1/2)+c*b^(1/2)))/d^2-I*c*ln(c+d*x^(1/2))*ln(d*((I-a)^(1/2)-b^(1/2)*x^(1/2)
)/(d*(I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*ln(c+d*x^(1/2))*ln(-d*((-I-a)^(1/2)+b
^(1/2)*x^(1/2))/(-d*(-I-a)^(1/2)+c*b^(1/2)))/d^2-I*c*ln(c+d*x^(1/2))*ln(-d
*((I-a)^(1/2)+b^(1/2)*x^(1/2))/(-d*(I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*polylog
(2,b^(1/2)*(c+d*x^(1/2))/(-d*(-I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*polylog(2,b^
(1/2)*(c+d*x^(1/2))/(d*(-I-a)^(1/2)+c*b^(1/2)))/d^2-I*c*polylog(2,b^(1/2)*
(c+d*x^(1/2))/(-d*(I-a)^(1/2)+c*b^(1/2)))/d^2-I*c*polylog(2,b^(1/2)*(c+d*x
^(1/2))/(d*(I-a)^(1/2)+c*b^(1/2)))/d^2-2*I*arctanh(b^(1/2)*x^(1/2)/(I-a)^(
1/2))*(I-a)^(1/2)/d/b^(1/2)+2*I*arctan(b^(1/2)*x^(1/2)/(I+a)^(1/2))*(I+a)^
(1/2)/d/b^(1/2)+I*ln(1-I*a-I*b*x)*x^(1/2)/d-I*ln(1+I*a+I*b*x)*x^(1/2)/d
```

3.58.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 604, normalized size of antiderivative = 0.90

$$\int \frac{\arctan(a+bx)}{c+d\sqrt{x}}\,dx = \frac{i\left(\frac{2\sqrt{i+a}d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}} - \frac{2\sqrt{i-a}d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}} + c\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{-i-a}d}\right)\log\left(c+d\sqrt{x}\right) - c\log\left(\frac{d\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}c+\sqrt{i-a}d}\right.\right.}{}$$

input
```
Integrate[ArcTan[a + b*x]/(c + d*Sqrt[x]),x]
```

output
```
(I*((2*Sqrt[I + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/Sqrt[b] - (2*S
qrt[I - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/Sqrt[b] + c*Log[(d*(S
qrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)]*Log[c + d*Sq
rt[x]] - c*Log[(d*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[I - a
]*d)]*Log[c + d*Sqrt[x]] + c*Log[(d*(Sqrt[-I - a] + Sqrt[b]*Sqrt[x]))/(-(S
qrt[b]*c) + Sqrt[-I - a]*d)]*Log[c + d*Sqrt[x]] - c*Log[(d*(Sqrt[I - a] +
Sqrt[b]*Sqrt[x]))/(-(Sqrt[b]*c) + Sqrt[I - a]*d)]*Log[c + d*Sqrt[x]] - d*S
qrt[x]*Log[1 + I*a + I*b*x] + c*Log[c + d*Sqrt[x]]*Log[1 + I*a + I*b*x] +
d*Sqrt[x]*Log[(-I)*(I + a + b*x)] - c*Log[c + d*Sqrt[x]]*Log[(-I)*(I + a +
 b*x)] + c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-I - a]*
d)] + c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)]
 - c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[I - a]*d)] - c
*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[I - a]*d)]))/d^2
```

3.58.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 665, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5574, 2855, 2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(a+bx)}{c+d\sqrt{x}}dx$$

$$\Big\downarrow \text{5574}$$

$$\frac{1}{2}i\int \frac{\log(-ia-ibx+1)}{c+d\sqrt{x}}dx-\frac{1}{2}i\int \frac{\log(ia+ibx+1)}{c+d\sqrt{x}}dx$$

$$\Big\downarrow \text{2855}$$

$$i\int \frac{\sqrt{x}\log(-ia-ibx+1)}{c+d\sqrt{x}}d\sqrt{x}-i\int \frac{\sqrt{x}\log(ia+ibx+1)}{c+d\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{2916}$$

$$i\int \left(\frac{\log(-ia-ibx+1)}{d}-\frac{c\log(-ia-ibx+1)}{d\left(c+d\sqrt{x}\right)}\right)d\sqrt{x}- i\int \left(\frac{\log(ia+ibx+1)}{d}-\frac{c\log(ia+ibx+1)}{d\left(c+d\sqrt{x}\right)}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{2\sqrt{a+i}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+i}}\right)}{\sqrt{b}d}+\frac{c\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-a-i}d}\right)}{d^2}+\frac{c\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{-a-i}d}\right)}{d^2}+\frac{c\log\left(c+d\sqrt{x}\right)\log\left(\right.}{d^2}\right.$$

$$i\left(\frac{2\sqrt{-a+i}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a+i}}\right)}{\sqrt{b}d}+\frac{c\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{i-a}d}\right)}{d^2}+\frac{c\,\text{PolyLog}\left(2,\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{i-a}d}\right)}{d^2}+\frac{c\log\left(c+d\sqrt{x}\right)\log\left(\right.}{d}\right.$$

input
```
Int[ArcTan[a + b*x]/(c + d*Sqrt[x]),x]
```

output
```
I*((-2*Sqrt[x])/d + (2*Sqrt[I + a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/
(Sqrt[b]*d) + (c*Log[(d*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqr
t[-I - a]*d)]*Log[c + d*Sqrt[x]])/d^2 + (c*Log[-((d*(Sqrt[-I - a] + Sqrt[b
]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-I - a]*d))]*Log[c + d*Sqrt[x]])/d^2 + (Sqrt
[x]*Log[1 - I*a - I*b*x])/d - (c*Log[c + d*Sqrt[x]]*Log[1 - I*a - I*b*x])/
d^2 + (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-I - a]*d)
])/d^2 + (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]
*d)])/d^2) - I*((-2*Sqrt[x])/d + (2*Sqrt[I - a]*ArcTanh[(Sqrt[b]*Sqrt[x])/
Sqrt[I - a]])/(Sqrt[b]*d) + (c*Log[(d*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sq
rt[b]*c + Sqrt[I - a]*d)]*Log[c + d*Sqrt[x]])/d^2 + (c*Log[-((d*(Sqrt[I -
a] + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[I - a]*d))]*Log[c + d*Sqrt[x]])/d
^2 + (Sqrt[x]*Log[1 + I*a + I*b*x])/d - (c*Log[c + d*Sqrt[x]]*Log[1 + I*a
+ I*b*x])/d^2 + (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[
I - a]*d)])/d^2 + (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqr
t[I - a]*d)])/d^2)
```

3.58.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2855
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_)^(r_))^(q_.), x_Symbol] :> With[{k = Denominator[r]}, Simp[k   Subst
[Int[x^(k - 1)*(f + g*x^(k*r))^q*(a + b*Log[c*(d + e*x^k)^n])^p, x], x, x^(
1/k)], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && FractionQ[r] && I
GtQ[p, 0]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

rule 5574
```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[
I/2   Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Simp[I/2   Int[Log[1 +
 I*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

3.58.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.54

method	result
derivativedivides	$\frac{2\arctan(bx+a)\sqrt{x}}{d}-\frac{2\arctan(bx+a)c\ln(c+d\sqrt{x})}{d^2}-\frac{4b\left(\frac{d^2\left(\sum_{\_R=\mathrm{RootOf}\left(b^2\_Z^4-4b^2c\_Z^3+\left(2ab\,d^2+6b^2c^2\right)\_Z^2+\left(-\ldots\right.\right.}\ldots\right)}{\ldots}\ldots\right)}{\ldots}$
default	$\frac{2\arctan(bx+a)\sqrt{x}}{d}-\frac{2\arctan(bx+a)c\ln(c+d\sqrt{x})}{d^2}-\frac{4b\left(\frac{d^2\left(\sum_{\_R=\mathrm{RootOf}\left(b^2\_Z^4-4b^2c\_Z^3+\left(2ab\,d^2+6b^2c^2\right)\_Z^2+\left(-\ldots\right.\right.}\ldots\right)}{\ldots}\ldots\right)}{\ldots}$

input
```
int(arctan(b*x+a)/(c+d*x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2*arctan(b*x+a)/d*x^(1/2)-2*arctan(b*x+a)*c/d^2*ln(c+d*x^(1/2))-4*b/d^2*(1
/4*d^2/b*sum((_R^2-2*_R*c+c^2)/(_R^3*b-3*_R^2*b*c+_R*a*d^2+3*_R*b*c^2-a*c*
d^2-b*c^3)*ln(d*x^(1/2)-_R+c),_R=RootOf(b^2*_Z^4-4*b^2*c*_Z^3+(2*a*b*d^2+6
*b^2*c^2)*_Z^2+(-4*a*b*c*d^2-4*b^2*c^3)*_Z+a^2*d^4+2*a*b*c^2*d^2+b^2*c^4+d
^4))-1/4*c*d^2/b*sum(1/(_R1^2*b-2*_R1*b*c+a*d^2+b*c^2)*(ln(c+d*x^(1/2))*ln
((-d*x^(1/2)+_R1-c)/_R1)+dilog((-d*x^(1/2)+_R1-c)/_R1)),_R1=RootOf(b^2*_Z^
4-4*b^2*c*_Z^3+(2*a*b*d^2+6*b^2*c^2)*_Z^2+(-4*a*b*c*d^2-4*b^2*c^3)*_Z+a^2*
d^4+2*a*b*c^2*d^2+b^2*c^4+d^4)))
```

3.58.5 Fricas [F]

$$\int \frac{\arctan(a+bx)}{c+d\sqrt{x}}\,dx=\int \frac{\arctan\left(bx+a\right)}{d\sqrt{x}+c}\,dx$$

input
```
integrate(arctan(b*x+a)/(c+d*x^(1/2)),x, algorithm="fricas")
```

output
```
integral((d*sqrt(x)*arctan(b*x + a) - c*arctan(b*x + a))/(d^2*x - c^2), x)
```

3.58.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+d\sqrt{x}}\,dx = \text{Timed out}$$

input
```
integrate(atan(b*x+a)/(c+d*x**(1/2)),x)
```

output
```
Timed out
```

3.58.7 Maxima [F]

$$\int \frac{\arctan(a+bx)}{c+d\sqrt{x}}\,dx = \int \frac{\arctan(bx+a)}{d\sqrt{x}+c}\,dx$$

input
```
integrate(arctan(b*x+a)/(c+d*x^(1/2)),x, algorithm="maxima")
```

output
```
integrate(arctan(b*x + a)/(d*sqrt(x) + c), x)
```

3.58.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\arctan(a+bx)}{c+d\sqrt{x}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(b*x+a)/(c+d*x^(1/2)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0]W
arning, replacing 0 by -24, a substitution variable should perhaps be purg
ed.Warnin
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+d\sqrt{x}}\,dx = \int \frac{\operatorname{atan}(a+b\,x)}{c+d\,\sqrt{x}}\,dx$$

input
```
int(atan(a + b*x)/(c + d*x^(1/2)),x)
```

output
```
int(atan(a + b*x)/(c + d*x^(1/2)), x)
```

3.59 $\int \frac{\arctan(a+bx)}{c+\frac{d}{\sqrt{x}}} \, dx$

3.59.1 Optimal result

Integrand size = 18, antiderivative size = 770

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = -\frac{2i\sqrt{i+a}\,d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}c^2} + \frac{2i\sqrt{i-a}\,d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}c^2}$$
$$-\frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$
$$+\frac{id^2\log\left(\frac{c\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c+\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$
$$-\frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$
$$+\frac{id^2\log\left(\frac{c\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}c-\sqrt{b}d}\right)\log\left(d+c\sqrt{x}\right)}{c^3}$$
$$-\frac{id\sqrt{x}\log(1-ia-ibx)}{c^2} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log(1-ia-ibx)}{c^3}$$
$$+\frac{id\sqrt{x}\log(1+ia+ibx)}{c^2} - \frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc}$$
$$-\frac{id^2\log\left(d+c\sqrt{x}\right)\log(1+ia+ibx)}{c^3}$$
$$-\frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc}$$
$$-\frac{id^2\,\text{PolyLog}\left(2,-\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{-i-a}c-\sqrt{b}d}\right)}{c^3} + \frac{id^2\,\text{PolyLog}\left(2,-\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{i-a}c-\sqrt{b}d}\right)}{c^3}$$
$$-\frac{id^2\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{-i-a}c+\sqrt{b}d}\right)}{c^3} + \frac{id^2\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{i-a}c+\sqrt{b}d}\right)}{c^3}$$

output
```
-1/2*(1+I*a+I*b*x)*ln(1+I*a+I*b*x)/b/c-1/2*(1-I*a-I*b*x)*ln(-I*(I+a+b*x))/
b/c+I*d^2*ln(d+c*x^(1/2))*ln(c*((I-a)^(1/2)+b^(1/2)*x^(1/2))/(c*(I-a)^(1/2
)-d*b^(1/2)))/c^3-I*d^2*polylog(2,b^(1/2)*(d+c*x^(1/2))/(c*(-I-a)^(1/2)+d*
b^(1/2)))/c^3-I*d^2*ln(d+c*x^(1/2))*ln(c*((-I-a)^(1/2)+b^(1/2)*x^(1/2))/(c
*(-I-a)^(1/2)-d*b^(1/2)))/c^3+I*d^2*polylog(2,b^(1/2)*(d+c*x^(1/2))/(c*(I-
a)^(1/2)+d*b^(1/2)))/c^3-I*d^2*ln(d+c*x^(1/2))*ln(c*((-I-a)^(1/2)-b^(1/2)*
x^(1/2))/(c*(-I-a)^(1/2)+d*b^(1/2)))/c^3+I*d^2*ln(1-I*a-I*b*x)*ln(d+c*x^(1
/2))/c^3+I*d*ln(1+I*a+I*b*x)*x^(1/2)/c^2+I*d^2*ln(d+c*x^(1/2))*ln(c*((I-a)
^(1/2)-b^(1/2)*x^(1/2))/(c*(I-a)^(1/2)+d*b^(1/2)))/c^3-2*I*d*arctan(b^(1/2
)*x^(1/2)/(I+a)^(1/2))*(I+a)^(1/2)/c^2/b^(1/2)-I*d^2*polylog(2,-b^(1/2)*(d
+c*x^(1/2))/(c*(-I-a)^(1/2)-d*b^(1/2)))/c^3-I*d*ln(1-I*a-I*b*x)*x^(1/2)/c^
2-I*d^2*ln(1+I*a+I*b*x)*ln(d+c*x^(1/2))/c^3+I*d^2*polylog(2,-b^(1/2)*(d+c*
x^(1/2))/(c*(I-a)^(1/2)-d*b^(1/2)))/c^3+2*I*d*arctanh(b^(1/2)*x^(1/2)/(I-a
)^(1/2))*(I-a)^(1/2)/c^2/b^(1/2)
```

3.59.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 770, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx =$$

$$-\frac{i\left(4\sqrt{i+a}\sqrt{b}cd\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)-4\sqrt{i-a}\sqrt{b}cd\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)+2bd^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}c+\sqrt{b}d}\right)\log\left(d+c\right.\right.}{}$$

input
```
Integrate[ArcTan[a + b*x]/(c + d/Sqrt[x]),x]
```

output
```
((-1/2*I)*(4*Sqrt[I + a]*Sqrt[b]*c*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]]
 - 4*Sqrt[I - a]*Sqrt[b]*c*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]] + 2*b*
d^2*Log[(c*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)]
*Log[d + c*Sqrt[x]] - 2*b*d^2*Log[(c*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqr
t[I - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]] + 2*b*d^2*Log[(c*(Sqrt[-I - a]
 + Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[x]] - 2*
b*d^2*Log[(c*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c - Sqrt[b]*d)]
*Log[d + c*Sqrt[x]] - I*c^2*Log[1 + I*a + I*b*x] + a*c^2*Log[1 + I*a + I*b
*x] - 2*b*c*d*Sqrt[x]*Log[1 + I*a + I*b*x] + b*c^2*x*Log[1 + I*a + I*b*x]
+ 2*b*d^2*Log[d + c*Sqrt[x]]*Log[1 + I*a + I*b*x] - I*c^2*Log[(-I)*(I + a
+ b*x)] - a*c^2*Log[(-I)*(I + a + b*x)] + 2*b*c*d*Sqrt[x]*Log[(-I)*(I + a
+ b*x)] - b*c^2*x*Log[(-I)*(I + a + b*x)] - 2*b*d^2*Log[d + c*Sqrt[x]]*Log
[(-I)*(I + a + b*x)] + 2*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sqr
t[-I - a]*c) + Sqrt[b]*d)] + 2*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/
(Sqrt[-I - a]*c + Sqrt[b]*d)] - 2*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]
))/(-(Sqrt[I - a]*c) + Sqrt[b]*d)] - 2*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sq
rt[x]))/(Sqrt[I - a]*c + Sqrt[b]*d)]))/(b*c^3)
```

3.59.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 792, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5574, 2855, 2005, 2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{\sqrt{x}}}dx$$

$$\Big\downarrow \text{ 5574}$$

$$\frac{1}{2}i\int \frac{\log(-ia-ibx+1)}{c+\frac{d}{\sqrt{x}}}dx-\frac{1}{2}i\int \frac{\log(ia+ibx+1)}{c+\frac{d}{\sqrt{x}}}dx$$

$$\Big\downarrow \text{ 2855}$$

$$i\int \frac{\sqrt{x}\log(-ia-ibx+1)}{c+\frac{d}{\sqrt{x}}}d\sqrt{x}-i\int \frac{\sqrt{x}\log(ia+ibx+1)}{c+\frac{d}{\sqrt{x}}}d\sqrt{x}$$

$$\Big\downarrow \text{ 2005}$$

$$i\int \frac{x\log(-ia-ibx+1)}{\sqrt{x}c+d}d\sqrt{x}-i\int \frac{x\log(ia+ibx+1)}{\sqrt{x}c+d}d\sqrt{x}$$

$$\Big\downarrow \text{2916}$$

$$i\int \left(\frac{\log(-ia-ibx+1)d^2}{c^2\left(\sqrt{x}c+d\right)}-\frac{\log(-ia-ibx+1)d}{c^2}+\frac{\sqrt{x}\log(-ia-ibx+1)}{c}\right)d\sqrt{x}-i\int \left(\frac{\log(ia+ibx+1)d^2}{c^2\left(\sqrt{x}c+d\right)}-\frac{\log(ia+ibx+1)d}{c^2}+\frac{\sqrt{x}\log(ia+ibx+1)}{c}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{2\sqrt{a+i}d\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+i}}\right)}{\sqrt{b}c^2}-\frac{d^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{b}(\sqrt{x}c+d)}{\sqrt{-a-i}c-\sqrt{b}d}\right)}{c^3}-\frac{d^2\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(\sqrt{x}c+d)}{\sqrt{-a-i}c+\sqrt{b}d}\right)}{c^3}-\frac{d^2\log\left(c\sqrt{x}+\right.}{}\right.$$

$$i\left(-\frac{2\sqrt{-a+i}d\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{-a+i}}\right)}{\sqrt{b}c^2}-\frac{d^2\operatorname{PolyLog}\left(2,-\frac{\sqrt{b}(\sqrt{x}c+d)}{\sqrt{i-a}c-\sqrt{b}d}\right)}{c^3}-\frac{d^2\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(\sqrt{x}c+d)}{\sqrt{i-a}c+\sqrt{b}d}\right)}{c^3}-\frac{d^2\log\left(c\sqrt{x}+\right.}{}\right.$$

input
```
Int[ArcTan[a + b*x]/(c + d/Sqrt[x]),x]
```

output
```
I*((2*d*Sqrt[x])/c^2 - x/(2*c) - (2*Sqrt[I + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])
/Sqrt[I + a]])/(Sqrt[b]*c^2) - (d^2*Log[(c*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]
))/(Sqrt[-I - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (d^2*Log[(c*(Sq
rt[-I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d)]*Log[d + c*Sqr
t[x]])/c^3 - (d*Sqrt[x]*Log[1 - I*a - I*b*x])/c^2 + (d^2*Log[d + c*Sqrt[x]
]*Log[1 - I*a - I*b*x])/c^3 + ((I/2)*(1 - I*a - I*b*x)*Log[(-I)*(I + a + b
*x)])/(b*c) - (d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*c
- Sqrt[b]*d))])/c^3 - (d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-I -
 a]*c + Sqrt[b]*d)])/c^3) - I*((2*d*Sqrt[x])/c^2 - x/(2*c) - (2*Sqrt[I - a
]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/(Sqrt[b]*c^2) - (d^2*Log[(c*(S
qrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt
[x]])/c^3 - (d^2*Log[(c*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c -
Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (d*Sqrt[x]*Log[1 + I*a + I*b*x])/c^2
 - ((I/2)*(1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(b*c) + (d^2*Log[d + c*S
qrt[x]]*Log[1 + I*a + I*b*x])/c^3 - (d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt
[x]))/(Sqrt[I - a]*c - Sqrt[b]*d))])/c^3 - (d^2*PolyLog[2, (Sqrt[b]*(d + c
*Sqrt[x]))/(Sqrt[I - a]*c + Sqrt[b]*d)])/c^3)
```

3.59. $\int \frac{\arctan(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx$

3.59.3.1 Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2855
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_)^(r_))^(q_.), x_Symbol] :> With[{k = Denominator[r]}, Simp[k   Subst
[Int[x^(k - 1)*(f + g*x^(k*r))^q*(a + b*Log[c*(d + e*x^k)^n])^p, x], x, x^(
1/k)], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && FractionQ[r] && I
GtQ[p, 0]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

rule 5574
```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[
I/2   Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Simp[I/2   Int[Log[1 +
 I*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

3.59.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.50

method	result
derivativedivides	$\frac{\arctan(bx+a)x}{c} - \frac{2\arctan(bx+a)d\sqrt{x}}{c^2} + \frac{2\arctan(bx+a)d^2\ln(d+c\sqrt{x})}{c^3} - \frac{4b\left(-\frac{c\left(\sum_{\_R=\mathrm{RootOf}\left(b^2\_Z^4-4b^2d\_Z^3+\right.}\right.}{}\right.}{}$
default	$\frac{\arctan(bx+a)x}{c} - \frac{2\arctan(bx+a)d\sqrt{x}}{c^2} + \frac{2\arctan(bx+a)d^2\ln(d+c\sqrt{x})}{c^3} - \frac{4b\left(-\frac{c\left(\sum_{\_R=\mathrm{RootOf}\left(b^2\_Z^4-4b^2d\_Z^3+\right.}\right.}{}\right.}{}$

input
```
int(arctan(b*x+a)/(c+d/x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
arctan(b*x+a)*x/c-2*arctan(b*x+a)/c^2*d*x^(1/2)+2*arctan(b*x+a)*d^2/c^3*ln
(d+c*x^(1/2))-4*b/c^2*(-1/8*c/b*sum((-_R^3+5*_R^2*d-7*_R*d^2+3*d^3)/(_R^3*
b-3*_R^2*b*d+_R*a*c^2+3*_R*b*d^2-a*c^2*d-b*d^3)*ln(c*x^(1/2)-_R+d),_R=Root
Of(b^2*_Z^4-4*b^2*d*_Z^3+(2*a*b*c^2+6*b^2*d^2)*_Z^2+(-4*a*b*c^2*d-4*b^2*d^
3)*_Z+a^2*c^4+2*a*b*c^2*d^2+b^2*d^4+c^4))+1/4*c*d^2/b*sum(1/(_R1^2*b-2*_R1
*b*d+a*c^2+b*d^2)*(ln(d+c*x^(1/2))*ln((-c*x^(1/2)+_R1-d)/_R1)+dilog((-c*x^
(1/2)+_R1-d)/_R1)),_R1=RootOf(b^2*_Z^4-4*b^2*d*_Z^3+(2*a*b*c^2+6*b^2*d^2)*
_Z^2+(-4*a*b*c^2*d-4*b^2*d^3)*_Z+a^2*c^4+2*a*b*c^2*d^2+b^2*d^4+c^4)))
```

3.59.5 Fricas [F]

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \int \frac{\arctan(bx+a)}{c+\frac{d}{\sqrt{x}}}\,dx$$

input
```
integrate(arctan(b*x+a)/(c+d/x^(1/2)),x, algorithm="fricas")
```

output
```
integral((c*x*arctan(b*x + a) - d*sqrt(x)*arctan(b*x + a))/(c^2*x - d^2),
x)
```

3.59.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \text{Timed out}$$

input
```
integrate(atan(b*x+a)/(c+d/x**(1/2)),x)
```

output
```
Timed out
```

3.59.7 Maxima [F]

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \int \frac{\arctan(bx+a)}{c+\frac{d}{\sqrt{x}}}\,dx$$

input
```
integrate(arctan(b*x+a)/(c+d/x^(1/2)),x, algorithm="maxima")
```

output
```
integrate(arctan(b*x + a)/(c + d/sqrt(x)), x)
```

3.59.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(b*x+a)/(c+d/x^(1/2)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0]W
arning, replacing 0 by -24, a substitution variable should perhaps be purg
ed.Warnin
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \int \frac{\mathrm{atan}(a+b\,x)}{c+\frac{d}{\sqrt{x}}}\,dx$$

input
```
int(atan(a + b*x)/(c + d/x^(1/2)),x)
```

output
```
int(atan(a + b*x)/(c + d/x^(1/2)), x)
```

3.60 $\int \frac{\arctan(a+bx)}{1+x^2} dx$

3.60.1 Optimal result

Integrand size = 14, antiderivative size = 274

$$\begin{aligned}\int \frac{\arctan(a+bx)}{1+x^2} dx = &\frac{1}{4}\log\left(\frac{b(i-x)}{a+i(1+b)}\right)\log(1-ia-ibx) \\ &-\frac{1}{4}\log\left(-\frac{b(i+x)}{a+i(1-b)}\right)\log(1-ia-ibx) \\ &-\frac{1}{4}\log\left(\frac{b(i-x)}{a-i(1-b)}\right)\log(1+ia+ibx) \\ &+\frac{1}{4}\log\left(-\frac{b(i+x)}{a-i(1+b)}\right)\log(1+ia+ibx) \\ &-\frac{1}{4}\operatorname{PolyLog}\left(2,-\frac{i-a-bx}{a-i(1-b)}\right)+\frac{1}{4}\operatorname{PolyLog}\left(2,-\frac{i-a-bx}{a-i(1+b)}\right) \\ &-\frac{1}{4}\operatorname{PolyLog}\left(2,\frac{i+a+bx}{i+a-ib}\right)+\frac{1}{4}\operatorname{PolyLog}\left(2,\frac{i+a+bx}{a+i(1+b)}\right)\end{aligned}$$

output
```
1/4*ln(b*(I-x)/(a+I*(1+b)))*ln(1-I*a-I*b*x)-1/4*ln(-b*(I+x)/(a+I*(1-b)))*l
n(1-I*a-I*b*x)-1/4*ln(b*(I-x)/(a-I*(1-b)))*ln(1+I*a+I*b*x)+1/4*ln(-b*(I+x)
/(a-I*(1+b)))*ln(1+I*a+I*b*x)-1/4*polylog(2,(-I+a+b*x)/(a-I*(1-b)))+1/4*po
lylog(2,(-I+a+b*x)/(a-I*(1+b)))-1/4*polylog(2,(I+a+b*x)/(I+a-I*b))+1/4*pol
ylog(2,(I+a+b*x)/(a+I*(1+b)))
```

3.60.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.03

$$\int \frac{\arctan(a+bx)}{1+x^2}\,dx = \frac{1}{4}\log\left(\frac{b(i-x)}{a+i(1+b)}\right)\log(1-ia-ibx) - \frac{1}{4}\log\left(-\frac{b(i+x)}{a+i(1-b)}\right)\log(1-ia-ibx) - \frac{1}{4}\log\left(\frac{b(i-x)}{a-i(1-b)}\right)\log(1+ia+ibx) + \frac{1}{4}\log\left(-\frac{b(i+x)}{a-i(1+b)}\right)\log(1+ia+ibx) - \frac{1}{4}\text{PolyLog}\left(2,\frac{1-ia-ibx}{1-ia-b}\right) + \frac{1}{4}\text{PolyLog}\left(2,\frac{1-ia-ibx}{1-ia+b}\right) - \frac{1}{4}\text{PolyLog}\left(2,\frac{1+ia+ibx}{1+ia-b}\right) + \frac{1}{4}\text{PolyLog}\left(2,\frac{1+ia+ibx}{1+ia+b}\right)$$

input
```
Integrate[ArcTan[a + b*x]/(1 + x^2),x]
```

output
```
(Log[(b*(I - x))/(a + I*(1 + b))]*Log[1 - I*a - I*b*x])/4 - (Log[-((b*(I +
 x))/(a + I*(1 - b)))]*Log[1 - I*a - I*b*x])/4 - (Log[(b*(I - x))/(a - I*(
1 - b))]*Log[1 + I*a + I*b*x])/4 + (Log[-((b*(I + x))/(a - I*(1 + b)))]*Lo
g[1 + I*a + I*b*x])/4 - PolyLog[2, (1 - I*a - I*b*x)/(1 - I*a - b)]/4 + Po
lyLog[2, (1 - I*a - I*b*x)/(1 - I*a + b)]/4 - PolyLog[2, (1 + I*a + I*b*x)
/(1 + I*a - b)]/4 + PolyLog[2, (1 + I*a + I*b*x)/(1 + I*a + b)]/4
```

3.60.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5574, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(a+bx)}{x^2+1}\,dx$$

$$\Big\downarrow \text{5574}$$

$$\frac{1}{2}i\int \frac{\log(-ia-ibx+1)}{x^2+1}dx - \frac{1}{2}i\int \frac{\log(ia+ibx+1)}{x^2+1}dx$$

$$\Big\downarrow \text{2856}$$

$$\frac{1}{2}i\int\left(\frac{i\log(-ia-ibx+1)}{2(i-x)}+\frac{i\log(-ia-ibx+1)}{2(x+i)}\right)dx-\frac{1}{2}i\int\left(\frac{i\log(ia+ibx+1)}{2(i-x)}+\frac{i\log(ia+ibx+1)}{2(x+i)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}i\left(\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{a+bx+i}{a-ib+i}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{a+bx+i}{a+i(b+1)}\right)-\frac{1}{2}i\log\left(\frac{b(-x+i)}{a+i(b+1)}\right)\log(-ia-ibx+1)+\frac{1}{2}\right.$$
$$\frac{1}{2}i\left(-\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{-a-bx+i}{a-i(1-b)}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{-a-bx+i}{a-i(b+1)}\right)-\frac{1}{2}i\log\left(\frac{b(-x+i)}{a-i(1-b)}\right)\log(ia+ibx+1\right.$$

input
```
Int[ArcTan[a + b*x]/(1 + x^2),x]
```

output
```
(-1/2*I)*((-1/2*I)*Log[(b*(I - x))/(a - I*(1 - b))]*Log[1 + I*a + I*b*x] +
 (I/2)*Log[-((b*(I + x))/(a - I*(1 + b)))]*Log[1 + I*a + I*b*x] - (I/2)*Po
lyLog[2, -((I - a - b*x)/(a - I*(1 - b)))] + (I/2)*PolyLog[2, -((I - a - b
*x)/(a - I*(1 + b)))]) + (I/2)*((-1/2*I)*Log[(b*(I - x))/(a + I*(1 + b))]*
Log[1 - I*a - I*b*x] + (I/2)*Log[-((b*(I + x))/(a + I*(1 - b)))]*Log[1 - I
*a - I*b*x] + (I/2)*PolyLog[2, (I + a + b*x)/(I + a - I*b)] - (I/2)*PolyLo
g[2, (I + a + b*x)/(a + I*(1 + b))])
```

3.60.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 5574
```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[
I/2   Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Simp[I/2   Int[Log[1 +
 I*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

3.60.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{\ln(-bxi-ia+1)\ln\left(\frac{-bxi+b}{ia+b-1}\right)}{4}-\frac{\operatorname{dilog}\left(\frac{-bxi+b}{ia+b-1}\right)}{4}+\frac{\ln(-bxi-ia+1)\ln\left(\frac{-bxi-b}{ia-b-1}\right)}{4}+\frac{\operatorname{dilog}\left(\frac{-bxi-b}{ia-b-1}\right)}{4}+\frac{\ln(bxi+ia\cdots}{\cdots}$
default	$\arctan(x)\arctan(bx+a)-b\left(-\frac{i\arctan(x)\ln\left(1-\frac{(-ib+a-i)(ix+1)^2}{(x^2+1)(-ib-a+i)}\right)}{2b}-\frac{\arctan(x)^2}{2b}-\frac{\operatorname{polylog}\left(2,\frac{(-\cdots}{(x^2\cdots}\right)}{\cdots}\cdots\right.$
parts	$\arctan(x)\arctan(bx+a)-b\left(-\frac{i\arctan(x)\ln\left(1-\frac{(-ib+a-i)(ix+1)^2}{(x^2+1)(-ib-a+i)}\right)}{2b}-\frac{\arctan(x)^2}{2b}-\frac{\operatorname{polylog}\left(2,\frac{(-\cdots}{(x^2\cdots}\right)}{\cdots}\cdots\right.$
derivativedivides	$\frac{b\arctan(x)\arctan(bx+a)-b^2\left(-\frac{\arctan\left(b\left(\frac{bx+a}{b}-\frac{a}{b}\right)+a\right)\arctan\left(-\frac{bx+a}{b}+\frac{a}{b}\right)}{b}-\frac{-\arctan\left(-\frac{bx+a}{b}+\frac{a}{b}\right)\arctan\left(b\left(\frac{bx+a}{b}-\frac{a}{b}\right)+\cdots\right)\cdots}{\cdots}\cdots\right.}{\cdots}$

input
```
int(arctan(b*x+a)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(1-I*a-I*b*x)*ln((-I*b*x+b)/(I*a+b-1))-1/4*dilog((-I*b*x+b)/(I*a+b-
1))+1/4*ln(1-I*a-I*b*x)*ln((-I*b*x-b)/(I*a-b-1))+1/4*dilog((-I*b*x-b)/(I*a
-b-1))+1/4*ln(1+I*a+I*b*x)*ln((I*b*x-b)/(-I*a-b-1))+1/4*dilog((I*b*x-b)/(-
I*a-b-1))-1/4*ln(1+I*a+I*b*x)*ln((I*b*x+b)/(-I*a+b-1))-1/4*dilog((I*b*x+b)
/(-I*a+b-1))
```

3.60.5 Fricas [F]

$$\int \frac{\arctan(a+bx)}{1+x^2}\,dx = \int \frac{\arctan(bx+a)}{x^2+1}\,dx$$

input
```
integrate(arctan(b*x+a)/(x^2+1),x, algorithm="fricas")
```

output
```
integral(arctan(b*x + a)/(x^2 + 1), x)
```

3.60.6 Sympy [F]

$$\int \frac{\arctan(a+bx)}{1+x^2}\,dx = \int \frac{\operatorname{atan}(a+bx)}{x^2+1}\,dx$$

input
```
integrate(atan(b*x+a)/(x**2+1),x)
```

output
```
Integral(atan(a + b*x)/(x**2 + 1), x)
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.20

$$\int \frac{\arctan(a+bx)}{1+x^2}\,dx$$
$$= \frac{1}{8}b\left(\frac{8\arctan(x)\arctan\left(\frac{b^2x+ab}{b}\right)}{b} - \frac{4\arctan(x)\arctan\left(\frac{ab+(b^2+b)x}{a^2+b^2+2b+1}, \frac{abx+a^2+b+1}{a^2+b^2+2b+1}\right) - 4\arctan(x)\arctan\left(\right.}{}\right.$$
$$+\arctan(bx+a)\arctan(x) - \arctan(x)\arctan\left(\frac{b^2x+ab}{b}\right)$$

input
```
integrate(arctan(b*x+a)/(x^2+1),x, algorithm="maxima")
```

output
```
1/8*b*(8*arctan(x)*arctan((b^2*x + a*b)/b)/b - (4*arctan(x)*arctan2((a*b +
 (b^2 + b)*x)/(a^2 + b^2 + 2*b + 1), (a*b*x + a^2 + b + 1)/(a^2 + b^2 + 2*
b + 1)) - 4*arctan(x)*arctan2((a*b + (b^2 - b)*x)/(a^2 + b^2 - 2*b + 1), (
a*b*x + a^2 - b + 1)/(a^2 + b^2 - 2*b + 1)) + log(x^2 + 1)*log((b^2*x^2 +
2*a*b*x + a^2 + 1)/(a^2 + b^2 + 2*b + 1)) - log(x^2 + 1)*log((b^2*x^2 + 2*
a*b*x + a^2 + 1)/(a^2 + b^2 - 2*b + 1)) + 2*dilog(-(I*b*x - b)/(I*a + b +
1)) - 2*dilog(-(I*b*x - b)/(I*a + b - 1)) + 2*dilog((I*b*x + b)/(-I*a + b
+ 1)) - 2*dilog((I*b*x + b)/(-I*a + b - 1)))/b) + arctan(b*x + a)*arctan(x
) - arctan(x)*arctan((b^2*x + a*b)/b)
```

3.60.8 Giac [F]

$$\int \frac{\arctan(a+bx)}{1+x^2}\,dx = \int \frac{\arctan(bx+a)}{x^2+1}\,dx$$

input
```
integrate(arctan(b*x+a)/(x^2+1),x, algorithm="giac")
```

output
```
sage0*x
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{1+x^2}\,dx = \int \frac{\mathrm{atan}(a+b\,x)}{x^2+1}\,dx$$

input
```
int(atan(a + b*x)/(x^2 + 1),x)
```

output
```
int(atan(a + b*x)/(x^2 + 1), x)
```

3.61 $\int \frac{\arctan(d+ex)}{a+bx^2} dx$

3.61.1 Optimal result

Integrand size = 16, antiderivative size = 543

$$\begin{aligned}\int \frac{\arctan(d+ex)}{a+bx^2} dx =& \frac{i\log\left(\frac{e(\sqrt{-a}-\sqrt{b}x)}{\sqrt{b}(i+d)+\sqrt{-a}e}\right)\log(1-id-iex)}{4\sqrt{-a}\sqrt{b}} \\ &- \frac{i\log\left(-\frac{e(\sqrt{-a}+\sqrt{b}x)}{\sqrt{b}(i+d)-\sqrt{-a}e}\right)\log(1-id-iex)}{4\sqrt{-a}\sqrt{b}} \\ &- \frac{i\log\left(-\frac{e(\sqrt{-a}-\sqrt{b}x)}{\sqrt{b}(i-d)-\sqrt{-a}e}\right)\log(1+id+iex)}{4\sqrt{-a}\sqrt{b}} \\ &+ \frac{i\log\left(\frac{e(\sqrt{-a}+\sqrt{b}x)}{\sqrt{b}(i-d)+\sqrt{-a}e}\right)\log(1+id+iex)}{4\sqrt{-a}\sqrt{b}} \\ &- \frac{i\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(i-d-ex)}{\sqrt{b}(i-d)-\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{b}} + \frac{i\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(i-d-ex)}{\sqrt{b}(i-d)+\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{b}} \\ &- \frac{i\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(i+d+ex)}{\sqrt{b}(i+d)-\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{b}} + \frac{i\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(i+d+ex)}{\sqrt{b}(i+d)+\sqrt{-a}e}\right)}{4\sqrt{-a}\sqrt{b}}\end{aligned}$$

output
```
-1/4*I*ln(1+I*d+I*e*x)*ln(-e*((-a)^(1/2)-x*b^(1/2))/(-e*(-a)^(1/2)+(I-d)*b
^(1/2)))/(-a)^(1/2)/b^(1/2)+1/4*I*ln(1-I*d-I*e*x)*ln(e*((-a)^(1/2)-x*b^(1/
2))/(e*(-a)^(1/2)+(I+d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)+1/4*I*ln(1+I*d+I*e*x)
*ln(e*((-a)^(1/2)+x*b^(1/2))/(e*(-a)^(1/2)+(I-d)*b^(1/2)))/(-a)^(1/2)/b^(1
/2)-1/4*I*ln(1-I*d-I*e*x)*ln(-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(1/2)+(I+d
)*b^(1/2)))/(-a)^(1/2)/b^(1/2)-1/4*I*polylog(2,(I-d-e*x)*b^(1/2)/(-e*(-a)^
(1/2)+(I-d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)+1/4*I*polylog(2,(I-d-e*x)*b^(1/2)
/(e*(-a)^(1/2)+(I-d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)-1/4*I*polylog(2,(I+d+e*x
)*b^(1/2)/(-e*(-a)^(1/2)+(I+d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)+1/4*I*polylog(
2,(I+d+e*x)*b^(1/2)/(e*(-a)^(1/2)+(I+d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)
```

3.61.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.75

$$\int \frac{\arctan(d+ex)}{a+bx^2}\,dx = \frac{i\left(-\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(-i+d)+\sqrt{-a}e}\right)\log(1+id+iex)+\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{-\sqrt{b}(-i+d)+\sqrt{-a}e}\right)\log(1+id+iex)+\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}(i+d)+\sqrt{-a}e}\right.\right.}{}$$

input
```
Integrate[ArcTan[d + e*x]/(a + b*x^2),x]
```

output
```
((I/4)*(-(Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*(-I + d) + Sqrt[-a]*e)]*
Log[1 + I*d + I*e*x]) + Log[(e*(Sqrt[-a] + Sqrt[b]*x))/(-(Sqrt[b]*(-I + d)
) + Sqrt[-a]*e)]*Log[1 + I*d + I*e*x] + Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sq
rt[b]*(I + d) + Sqrt[-a]*e)]*Log[(-I)*(I + d + e*x)] - Log[(e*(Sqrt[-a] +
Sqrt[b]*x))/(-(Sqrt[b]*(I + d)) + Sqrt[-a]*e)]*Log[(-I)*(I + d + e*x)] + P
olyLog[2, (Sqrt[b]*(-I + d + e*x))/(Sqrt[b]*(-I + d) - Sqrt[-a]*e)] - Poly
Log[2, (Sqrt[b]*(-I + d + e*x))/(Sqrt[b]*(-I + d) + Sqrt[-a]*e)] - PolyLog
[2, (Sqrt[b]*(I + d + e*x))/(Sqrt[b]*(I + d) - Sqrt[-a]*e)] + PolyLog[2, (
Sqrt[b]*(I + d + e*x))/(Sqrt[b]*(I + d) + Sqrt[-a]*e)]))/(Sqrt[-a]*Sqrt[b]
)
```

3.61.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5574, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(d+ex)}{a+bx^2}dx$$

$$\Big\downarrow \text{5574}$$

$$\frac{1}{2}i\int \frac{\log(-id-iex+1)}{bx^2+a}dx-\frac{1}{2}i\int \frac{\log(id+iex+1)}{bx^2+a}dx$$

$$\Big\downarrow \text{2856}$$

$$\frac{1}{2}i\int \left(\frac{\sqrt{-a}\log(-id-iex+1)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\log(-id-iex+1)}{2a\left(\sqrt{b}x+\sqrt{-a}\right)}\right)dx-\frac{1}{2}i\int \left(\frac{\sqrt{-a}\log(id+iex+1)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\log(id+iex+1)}{2a\left(\sqrt{b}x+\sqrt{-a}\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}i\left(-\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex+i)}{\sqrt{b}(d+i)-\sqrt{-a}e}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex+i)}{\sqrt{b}(d+i)+\sqrt{-a}e}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\log(-id-iex+1)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}(d+i)}\right)}{2\sqrt{-a}\sqrt{b}}-\ldots\right)$$

$$\frac{1}{2}i\left(\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(-d-ex+i)}{\sqrt{b}(i-d)-\sqrt{-a}e}\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(-d-ex+i)}{\sqrt{b}(i-d)+\sqrt{-a}e}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\log(id+iex+1)\log\left(-\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{-\sqrt{-a}e+\sqrt{b}(-d+i)}\right)}{2\sqrt{-a}\sqrt{b}}-\ldots\right)$$

input
```
Int[ArcTan[d + e*x]/(a + b*x^2),x]
```

output
```
(-1/2*I)*((Log[-((e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*(I - d) - Sqrt[-a]*e)
)]*Log[1 + I*d + I*e*x])/(2*Sqrt[-a]*Sqrt[b]) - (Log[(e*(Sqrt[-a] + Sqrt[b
]*x))/(Sqrt[b]*(I - d) + Sqrt[-a]*e)]*Log[1 + I*d + I*e*x])/(2*Sqrt[-a]*Sq
rt[b]) + PolyLog[2, (Sqrt[b]*(I - d - e*x))/(Sqrt[b]*(I - d) - Sqrt[-a]*e)
]/(2*Sqrt[-a]*Sqrt[b]) - PolyLog[2, (Sqrt[b]*(I - d - e*x))/(Sqrt[b]*(I -
d) + Sqrt[-a]*e)]/(2*Sqrt[-a]*Sqrt[b])) + (I/2)*((Log[(e*(Sqrt[-a] - Sqrt[
b]*x))/(Sqrt[b]*(I + d) + Sqrt[-a]*e)]*Log[1 - I*d - I*e*x])/(2*Sqrt[-a]*S
qrt[b]) - (Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*(I + d) - Sqrt[-a]*e)
)]*Log[1 - I*d - I*e*x])/(2*Sqrt[-a]*Sqrt[b]) - PolyLog[2, (Sqrt[b]*(I + d
 + e*x))/(Sqrt[b]*(I + d) - Sqrt[-a]*e)]/(2*Sqrt[-a]*Sqrt[b]) + PolyLog[2,
 (Sqrt[b]*(I + d + e*x))/(Sqrt[b]*(I + d) + Sqrt[-a]*e)]/(2*Sqrt[-a]*Sqrt[
b]))
```

3.61.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 5574
```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[
I/2   Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Simp[I/2   Int[Log[1 +
 I*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

3.61.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.00

method	result
risch	$\frac{\ln(-iex-id+1)\ln\left(\frac{ibd-e\sqrt{ab}+b(-iex-id+1)-b}{ibd-e\sqrt{ab}-b}\right)\sqrt{ab}}{4ab}-\frac{\ln(-iex-id+1)\ln\left(\frac{ibd+e\sqrt{ab}+b(-iex-id+1)-b}{ibd+e\sqrt{ab}-b}\right)\sqrt{ab}}{4ab}+\frac{\mathrm{dilog}(}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(arctan(e*x+d)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(1-I*d-I*e*x)/a/b*ln((I*b*d-e*(a*b)^(1/2)+b*(1-I*d-I*e*x)-b)/(I*b*d-
e*(a*b)^(1/2)-b))*(a*b)^(1/2)-1/4*ln(1-I*d-I*e*x)/a/b*ln((I*b*d+e*(a*b)^(1
/2)+b*(1-I*d-I*e*x)-b)/(I*b*d+e*(a*b)^(1/2)-b))*(a*b)^(1/2)+1/4/a/b*dilog(
(I*b*d-e*(a*b)^(1/2)+b*(1-I*d-I*e*x)-b)/(I*b*d-e*(a*b)^(1/2)-b))*(a*b)^(1/
2)-1/4/a/b*dilog((I*b*d+e*(a*b)^(1/2)+b*(1-I*d-I*e*x)-b)/(I*b*d+e*(a*b)^(1
/2)-b))*(a*b)^(1/2)+1/4*ln(1+I*d+I*e*x)/a/b*ln((I*b*d+e*(a*b)^(1/2)-b*(1+I
*d+I*e*x)+b)/(I*b*d+e*(a*b)^(1/2)+b))*(a*b)^(1/2)-1/4*ln(1+I*d+I*e*x)/a/b*
ln((I*b*d-e*(a*b)^(1/2)-b*(1+I*d+I*e*x)+b)/(I*b*d-e*(a*b)^(1/2)+b))*(a*b)^
(1/2)+1/4/a/b*dilog((I*b*d+e*(a*b)^(1/2)-b*(1+I*d+I*e*x)+b)/(I*b*d+e*(a*b)
^(1/2)+b))*(a*b)^(1/2)-1/4/a/b*dilog((I*b*d-e*(a*b)^(1/2)-b*(1+I*d+I*e*x)+
b)/(I*b*d-e*(a*b)^(1/2)+b))*(a*b)^(1/2)
```

3.61.5 Fricas [F]

$$\int \frac{\arctan(d+ex)}{a+bx^2}\,dx = \int \frac{\arctan\left(ex+d\right)}{bx^2+a}\,dx$$

input
```
integrate(arctan(e*x+d)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral(arctan(e*x + d)/(b*x^2 + a), x)
```

3.61.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(d+ex)}{a+bx^2}\,dx = \text{Timed out}$$

input
```
integrate(atan(e*x+d)/(b*x**2+a),x)
```

output
```
Timed out
```

3.61.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 14300 vs. 2(369) = 738.

Time = 2.00 (sec) , antiderivative size = 14300, normalized size of antiderivative = 26.34

$$\int \frac{\arctan(d+ex)}{a+bx^2}\,dx = \text{Too large to display}$$

input
```
integrate(arctan(e*x+d)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/8*e*(8*arctan(b*x/sqrt(a*b))*arctan((e^2*x + d*e)/e)/e - (4*arctan(sqrt(
b)*x/sqrt(a))*arctan2((2*a*b*d*e^2 + (a*d*e^3 + (b*d^3 + b*d)*e + (a*e^4 +
 (b*d^2 + 3*b)*e^2)*x)*sqrt(a)*sqrt(b) + (3*a*b*e^3 + (b^2*d^2 + b^2)*e)*x
)/(b^2*d^4 + a^2*e^4 + 2*b^2*d^2 + 2*(a*b*d^2 + 3*a*b)*e^2 + 4*(a*e^3 + (b
*d^2 + b)*e)*sqrt(a)*sqrt(b) + b^2), (b^2*d^4 + 2*b^2*d^2 + (a*b*d^2 + 3*a
*b)*e^2 + (2*b*d*e^2*x + a*e^3 + 3*(b*d^2 + b)*e)*sqrt(a)*sqrt(b) + b^2 +
(a*b*d*e^3 + (b^2*d^3 + b^2*d)*e)*x)/(b^2*d^4 + a^2*e^4 + 2*b^2*d^2 + 2*(a
*b*d^2 + 3*a*b)*e^2 + 4*(a*e^3 + (b*d^2 + b)*e)*sqrt(a)*sqrt(b) + b^2)) +
4*arctan(sqrt(b)*x/sqrt(a))*arctan2((2*a*b*d*e^2 - (a*d*e^3 + (b*d^3 + b*d
)*e + (a*e^4 + (b*d^2 + 3*b)*e^2)*x)*sqrt(a)*sqrt(b) + (3*a*b*e^3 + (b^2*d
^2 + b^2)*e)*x)/(b^2*d^4 + a^2*e^4 + 2*b^2*d^2 + 2*(a*b*d^2 + 3*a*b)*e^2 -
 4*(a*e^3 + (b*d^2 + b)*e)*sqrt(a)*sqrt(b) + b^2), (b^2*d^4 + 2*b^2*d^2 +
(a*b*d^2 + 3*a*b)*e^2 - (2*b*d*e^2*x + a*e^3 + 3*(b*d^2 + b)*e)*sqrt(a)*sq
rt(b) + b^2 + (a*b*d*e^3 + (b^2*d^3 + b^2*d)*e)*x)/(b^2*d^4 + a^2*e^4 + 2*
b^2*d^2 + 2*(a*b*d^2 + 3*a*b)*e^2 - 4*(a*e^3 + (b*d^2 + b)*e)*sqrt(a)*sqrt
(b) + b^2)) + log(b*x^2 + a)*log((b^12*d^24 + 12*b^12*d^22 + 66*b^12*d^20
+ 220*b^12*d^18 + 495*b^12*d^16 + 792*b^12*d^14 + 924*b^12*d^12 + (a^11*b*
d^2 + a^11*b)*e^22 + 792*b^12*d^10 + 11*(a^10*b^2*d^4 + 22*a^10*b^2*d^2 +
21*a^10*b^2)*e^20 + 495*b^12*d^8 + 55*(a^9*b^3*d^6 + 39*a^9*b^3*d^4 + 171*
a^9*b^3*d^2 + 133*a^9*b^3)*e^18 + 220*b^12*d^6 + 33*(5*a^8*b^4*d^8 + 26...
```

3.61.8 Giac [F]

$$\int \frac{\arctan(d+ex)}{a+bx^2}\,dx = \int \frac{\arctan(ex+d)}{bx^2+a}\,dx$$

input
```
integrate(arctan(e*x+d)/(b*x^2+a),x, algorithm="giac")
```

output
```
sage0*x
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(d+ex)}{a+bx^2}\,dx = \int \frac{\mathrm{atan}(d+e\,x)}{b\,x^2+a}\,dx$$

input
```
int(atan(d + e*x)/(a + b*x^2),x)
```

output
```
int(atan(d + e*x)/(a + b*x^2), x)
```

3.62 $\int \frac{\arctan(d+ex)}{a+bx+cx^2}\,dx$

3.62.1 Optimal result

Integrand size = 19, antiderivative size = 367

$$\int \frac{\arctan(d+ex)}{a+bx+cx^2}\,dx = \frac{\arctan(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2c(i-d)+\left(b-\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}} - \frac{\arctan(d+ex)\log\left(\frac{2e\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}} - \frac{i\,\mathrm{PolyLog}\left(2,1+\frac{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2ic-2cd+be-\sqrt{b^2-4ac}e\right)(1-i(d+ex))}\right)}{2\sqrt{b^2-4ac}} + \frac{i\,\mathrm{PolyLog}\left(2,1+\frac{2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}\right)}{2\sqrt{b^2-4ac}}$$

output
```
arctan(e*x+d)*ln(2*e*(b+2*c*x-(-4*a*c+b^2)^(1/2))/(1-I*(e*x+d))/(2*c*(I-d)
+e*(b-(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)-arctan(e*x+d)*ln(2*e*(b+2*c
*x+(-4*a*c+b^2)^(1/2))/(1-I*(e*x+d))/(2*c*(I-d)+e*(b+(-4*a*c+b^2)^(1/2))))
/(-4*a*c+b^2)^(1/2)-1/2*I*polylog(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b-(-4*a*c+b^
2)^(1/2)))/(1-I*(e*x+d))/(2*I*c-2*c*d+b*e-e*(-4*a*c+b^2)^(1/2)))/(-4*a*c+b
^2)^(1/2)+1/2*I*polylog(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b+(-4*a*c+b^2)^(1/2)))
/(1-I*(e*x+d))/(2*c*(I-d)+e*(b+(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)
```

3.62.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.21

$$\int \frac{\arctan(d+ex)}{a+bx+cx^2}\,dx$$
$$= \frac{i\left(\log\left(\frac{e\left(-b+\sqrt{b^2-4ac}-2cx\right)}{2c(i+d)+\left(-b+\sqrt{b^2-4ac}\right)e}\right)\log(1-i(d+ex))-\log\left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2c(i+d)+\left(b+\sqrt{b^2-4ac}\right)e}\right)\log(1-i(d+ex))-\log\left(\right.\right.}{}$$

input
```
Integrate[ArcTan[d + e*x]/(a + b*x + c*x^2),x]
```

output
```
((I/2)*(Log[(e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*(I + d) + (-b + Sqrt
[b^2 - 4*a*c])*e)]*Log[1 - I*(d + e*x)] - Log[(e*(b + Sqrt[b^2 - 4*a*c] +
2*c*x))/(-2*c*(I + d) + (b + Sqrt[b^2 - 4*a*c])*e)]*Log[1 - I*(d + e*x)] -
 Log[(e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*(-I + d) + (-b + Sqrt[b^2 -
 4*a*c])*e)]*Log[1 + I*(d + e*x)] + Log[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
)/(-2*c*(-I + d) + (b + Sqrt[b^2 - 4*a*c])*e)]*Log[1 + I*(d + e*x)] - Poly
Log[2, (2*c*(-I + d + e*x))/(2*c*(-I + d) + (-b + Sqrt[b^2 - 4*a*c])*e)] +
 PolyLog[2, (2*c*(-I + d + e*x))/(2*c*(-I + d) - (b + Sqrt[b^2 - 4*a*c])*e
)] + PolyLog[2, (2*c*(I + d + e*x))/(2*c*(I + d) + (-b + Sqrt[b^2 - 4*a*c]
)*e)] - PolyLog[2, (2*c*(I + d + e*x))/(2*c*(I + d) - (b + Sqrt[b^2 - 4*a*
c])*e)]))/Sqrt[b^2 - 4*a*c]
```

3.62.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(d+ex)}{a+bx+cx^2}\,dx$$
$$\Big\downarrow \ 7279$$
$$\int \left(\frac{2c\arctan(d+ex)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}+b+2cx\right)}-\frac{2c\arctan(d+ex)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b+2cx\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan(d+ex)\log\left(-\frac{2\left(-e\left(b-\sqrt{b^2-4ac}\right)-2c(d+ex)+2cd\right)}{(1-i(d+ex))\left(-e\sqrt{b^2-4ac}+be-2cd+2ic\right)}\right)}{\sqrt{b^2-4ac}}-\frac{\arctan(d+ex)\log\left(-\frac{2\left(-e\left(\sqrt{b^2-4ac}+b\right)-2c(d+ex)+2cd\right)}{(1-i(d+ex))\left(e\left(\sqrt{b^2-4ac}+b\right)+2c(-d+i)\right)}\right)}{\sqrt{b^2-4ac}}-\frac{i\,\text{PolyLog}\left(2,\frac{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(-2dc+2ic+be-\sqrt{b^2-4ac}e\right)(1-i(d+ex))}+1\right)}{2\sqrt{b^2-4ac}}+\frac{i\,\text{PolyLog}\left(2,\frac{2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}+1\right)}{2\sqrt{b^2-4ac}}$$

input
```
Int[ArcTan[d + e*x]/(a + b*x + c*x^2),x]
```

output
```
(ArcTan[d + e*x]*Log[(-2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x
)))/(((2*I)*c - 2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e)*(1 - I*(d + e*x)))])/Sq
rt[b^2 - 4*a*c] - (ArcTan[d + e*x]*Log[(-2*(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e - 2*c*(d + e*x)))/((2*c*(I - d) + (b + Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d
 + e*x)))])/Sqrt[b^2 - 4*a*c] - ((I/2)*PolyLog[2, 1 + (2*(2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/(((2*I)*c - 2*c*d + b*e - Sqrt[b^2 - 4
*a*c]*e)*(1 - I*(d + e*x)))])/Sqrt[b^2 - 4*a*c] + ((I/2)*PolyLog[2, 1 + (2
*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/((2*c*(I - d) + (b +
 Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))])/Sqrt[b^2 - 4*a*c]
```

3.62.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.62.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(329) = 658$.

Time = 1.36 (sec) , antiderivative size = 890, normalized size of antiderivative = 2.43

method	result
risch	$\frac{e\ln(-iex-id+1)\ln\left(\frac{ibe-2icd-2(-iex-id+1)c+\sqrt{4ac\,e^2-b^2e^2}+2c}{ibe-2icd+2c+\sqrt{4ac\,e^2-b^2e^2}}\right)}{2\sqrt{4ac\,e^2-b^2e^2}} - \frac{e\ln(-iex-id+1)\ln\left(\frac{ibe-2icd-2(-iex-id+1)c-\sqrt{}}{ibe-2icd+2c-\sqrt{4ac\,e}}\right.}{2\sqrt{4ac\,e^2-b^2e^2}}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(arctan(e*x+d)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*e*ln(1-I*d-I*e*x)/(4*a*c*e^2-b^2*e^2)^(1/2)*ln((I*b*e-2*I*c*d-2*(1-I*d
-I*e*x)*c+(4*a*c*e^2-b^2*e^2)^(1/2)+2*c)/(I*b*e-2*I*c*d+2*c+(4*a*c*e^2-b^2
*e^2)^(1/2)))-1/2*e*ln(1-I*d-I*e*x)/(4*a*c*e^2-b^2*e^2)^(1/2)*ln((I*b*e-2*
I*c*d-2*(1-I*d-I*e*x)*c-(4*a*c*e^2-b^2*e^2)^(1/2)+2*c)/(I*b*e-2*I*c*d+2*c-
(4*a*c*e^2-b^2*e^2)^(1/2)))+1/2*e/(4*a*c*e^2-b^2*e^2)^(1/2)*dilog((I*b*e-2
*I*c*d-2*(1-I*d-I*e*x)*c+(4*a*c*e^2-b^2*e^2)^(1/2)+2*c)/(I*b*e-2*I*c*d+2*c
+(4*a*c*e^2-b^2*e^2)^(1/2)))-1/2*e/(4*a*c*e^2-b^2*e^2)^(1/2)*dilog((I*b*e-
2*I*c*d-2*(1-I*d-I*e*x)*c-(4*a*c*e^2-b^2*e^2)^(1/2)+2*c)/(I*b*e-2*I*c*d+2*
c-(4*a*c*e^2-b^2*e^2)^(1/2)))+1/2*e*ln(1+I*d+I*e*x)/(4*a*c*e^2-b^2*e^2)^(1
/2)*ln((I*b*e-2*I*c*d+2*(1+I*d+I*e*x)*c-(4*a*c*e^2-b^2*e^2)^(1/2)-2*c)/(I*
b*e-2*I*c*d-(4*a*c*e^2-b^2*e^2)^(1/2)-2*c))-1/2*e*ln(1+I*d+I*e*x)/(4*a*c*e
^2-b^2*e^2)^(1/2)*ln((I*b*e-2*I*c*d+2*(1+I*d+I*e*x)*c+(4*a*c*e^2-b^2*e^2)^
(1/2)-2*c)/(I*b*e-2*I*c*d+(4*a*c*e^2-b^2*e^2)^(1/2)-2*c))+1/2*e/(4*a*c*e^2
-b^2*e^2)^(1/2)*dilog((I*b*e-2*I*c*d+2*(1+I*d+I*e*x)*c-(4*a*c*e^2-b^2*e^2)
^(1/2)-2*c)/(I*b*e-2*I*c*d-(4*a*c*e^2-b^2*e^2)^(1/2)-2*c))-1/2*e/(4*a*c*e^
2-b^2*e^2)^(1/2)*dilog((I*b*e-2*I*c*d+2*(1+I*d+I*e*x)*c+(4*a*c*e^2-b^2*e^2
)^(1/2)-2*c)/(I*b*e-2*I*c*d+(4*a*c*e^2-b^2*e^2)^(1/2)-2*c))
```

3.62.5 Fricas [F]

$$\int \frac{\arctan(d+ex)}{a+bx+cx^2}\,dx = \int \frac{\arctan(ex+d)}{cx^2+bx+a}\,dx$$

input
```
integrate(arctan(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral(arctan(e*x + d)/(c*x^2 + b*x + a), x)
```

3.62.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(d+ex)}{a+bx+cx^2}\,dx = \text{Timed out}$$

input
```
integrate(atan(e*x+d)/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

3.62.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(d+ex)}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(arctan(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.62.8 Giac [F]

$$\int \frac{\arctan(d+ex)}{a+bx+cx^2}\,dx = \int \frac{\arctan(ex+d)}{cx^2+bx+a}\,dx$$

input
```
integrate(arctan(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
sage0*x
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(d+ex)}{a+bx+cx^2}\,dx = \int \frac{\mathrm{atan}(d+e\,x)}{c\,x^2+b\,x+a}\,dx$$

input
```
int(atan(d + e*x)/(a + b*x + c*x^2),x)
```

output
```
int(atan(d + e*x)/(a + b*x + c*x^2), x)
```

3.63 $\int \frac{\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

3.63.1 Optimal result

Integrand size = 28, antiderivative size = 132

$$\int \frac{\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = -\frac{2i\arctan(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} + \frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} - \frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

output
```
-2*I*arctan(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b+I*pol
ylog(2,-I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b-I*polylog(2,I*(1+I*(b
*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b
```

3.63.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.51

$$\int \frac{\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = -\frac{i\left(2\arctan\left(e^{i\arctan(a+bx)}\right)\arctan(a+bx) - \mathrm{PolyLog}\left(2,-ie^{i\arctan(a+bx)}\right) + \mathrm{PolyLog}\left(2,ie^{i\arctan(a+bx)}\right)\right)}{b}$$

input
```
Integrate[ArcTan[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((-I)*(2*ArcTan[E^(I*ArcTan[a + b*x])]*ArcTan[a + b*x] - PolyLog[2, (-I)*E
^(I*ArcTan[a + b*x])] + PolyLog[2, I*E^(I*ArcTan[a + b*x])]))/b
```

3.63.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5578, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(a+bx)}{\sqrt{a^2+2abx+b^2x^2+1}}dx$$

$$\Big\downarrow \text{5578}$$

$$\frac{\int \frac{\arctan(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)}{b}$$

$$\Big\downarrow \text{5421}$$

$$\frac{-2i\arctan(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)+i\,\text{PolyLog}\left(2,-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)-i\,\text{PolyLog}\left(2,\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

input
```
Int[ArcTan[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
((-2*I)*ArcTan[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]
] + I*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]] - I*P
olyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/b
```

3.63.3.1 Defintions of rubi rules used

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5578
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_) + (
C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2)
^q*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
p, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.63.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02

method	result
default	$\frac{-\arctan(bx+a)\ln\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)+\arctan(bx+a)\ln\left(1-\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)+i\,\mathrm{dilog}\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)-i\,\mathrm{dilog}\left(1-\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)}{b}$

input
```
int(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-arctan(b*x+a)*ln(1+I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))+arctan(b*x+a
)*ln(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))+I*dilog(1+I*(1+I*(b*x+a))/(1+(
b*x+a)^2)^(1/2))-I*dilog(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)))
```

3.63.5 Fricas [F]

$$\int \frac{\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx=\int \frac{\arctan(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

input
```
integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fricas
")
```

output
```
integral(arctan(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

3.63.6 Sympy [F]

$$\int \frac{\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{atan}(a+bx)}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx$$

input
```
integrate(atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

output
```
Integral(atan(a + b*x)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x)
```

3.63.7 Maxima [F]

$$\int \frac{\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\arctan(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

input
```
integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(arctan(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

3.63.8 Giac [F]

$$\int \frac{\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\arctan(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

input
```
integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx=\int \frac{\mathrm{atan}(a+b\,x)}{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\,dx$$

input
```
int(atan(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

output
```
int(atan(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

3.64 $\int \frac{\arctan(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}}\,dx$

3.64.1 Optimal result

Integrand size = 33, antiderivative size = 216

$$\int \frac{\arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = -\frac{2i\sqrt{1+(a+bx)^2}\arctan(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} + \frac{i\sqrt{1+(a+bx)^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} - \frac{i\sqrt{1+(a+bx)^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}}$$

output
```
-2*I*arctan(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x
+a)^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)+I*polylog(2,-I*(1+I*(b*x+a))^(1/2)/(1
-I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)-I*polylog(2
,I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b*
x+a)^2)^(1/2)
```

3.64.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.44

$$\int \frac{\arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx =$$
$$-\frac{i\sqrt{1+(a+bx)^2}\left(2\arctan\left(e^{i\arctan(a+bx)}\right)\arctan(a+bx)-\operatorname{PolyLog}\left(2,-ie^{i\arctan(a+bx)}\right)+\operatorname{PolyLog}\left(2,i\right.\right.}{b\sqrt{c\,(1+(a+bx)^2)}}$$

input
```
Integrate[ArcTan[a + b*x]/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2],x]
```

output
```
((-I)*Sqrt[1 + (a + b*x)^2]*(2*ArcTan[E^(I*ArcTan[a + b*x])]*ArcTan[a + b*
x] - PolyLog[2, (-I)*E^(I*ArcTan[a + b*x])] + PolyLog[2, I*E^(I*ArcTan[a +
 b*x])]))/(b*Sqrt[c*(1 + (a + b*x)^2)])
```

3.64.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5578, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(a+bx)}{\sqrt{(a^2+1)\,c+2abcx+b^2cx^2}}\,dx$$
$$\Big\downarrow \ 5578$$
$$\frac{\int \frac{\arctan(a+bx)}{\sqrt{c(a+bx)^2+c}}d(a+bx)}{b}$$
$$\Big\downarrow \ 5425$$
$$\frac{\sqrt{(a+bx)^2+1}\int \frac{\arctan(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)}{b\sqrt{c(a+bx)^2+c}}$$
$$\Big\downarrow \ 5421$$
$$\frac{\sqrt{(a+bx)^2+1}\left(-2i\arctan(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)+i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)-i\operatorname{PolyLog}\left(2,\frac{i\sqrt{i(a+bx}}{\sqrt{1-i(a+}}\right.\right.}{b\sqrt{c(a+bx)^2+c}}$$

input
```
Int[ArcTan[a + b*x]/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2],x]
```

output
```
(Sqrt[1 + (a + b*x)^2]*((-2*I)*ArcTan[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)
]/Sqrt[1 - I*(a + b*x)]] + I*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[
1 - I*(a + b*x)]] - I*PolyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a +
 b*x)]]))/(b*Sqrt[c + c*(a + b*x)^2])
```

3.64.3.1 Defintions of rubi rules used

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5578
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_) + (
C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2)
^q*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
p, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.64.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.81

method	result
default	$-\frac{\left(\arctan(bx+a)\ln\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)-\arctan(bx+a)\ln\left(1-\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)-i\,\mathrm{dilog}\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)+i\,\mathrm{dilog}\left(1-\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)\right.}{\sqrt{b^2x^2+2abx+a^2+1}\,bc}$

input
```
int(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
-(arctan(b*x+a)*ln(1+I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-arctan(b*x+a)*ln
(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-I*dilog(1+I*(1+I*(b*x+a))/(1+(b*x+
a)^2)^(1/2))+I*dilog(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)))*(c*(-I+a+b*x)
*(I+a+b*x))^(1/2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b/c
```

3.64.5 Fricas [F]

$$\int \frac{\arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\arctan\left(bx+a\right)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

input
```
integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x, algorithm
="fricas")
```

output
```
integral(arctan(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c), x)
```

3.64.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\text{Timed out}$$

input
```
integrate(atan(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/2),x)
```

output
```
Timed out
```

3.64.7 Maxima [F]

$$\int \frac{\arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\arctan\left(bx+a\right)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

input
```
integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(arctan(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c), x)
```

3.64.8 Giac [F]

$$\int \frac{\arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\arctan(bx+a)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

input
```
integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x, algorithm
="giac")
```

output
```
sage0*x
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\operatorname{atan}(a+b\,x)}{\sqrt{c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)}}\,dx$$

input
```
int(atan(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2),x)
```

output
```
int(atan(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2), x)
```

3.65 $\int \frac{\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$

3.65.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \text{Int}\left(\frac{\arctan(a+bx)}{\sqrt[3]{1+(a+bx)^2}},x\right)$$

output
```
Unintegrable(arctan(b*x+a)/(1+(b*x+a)^2)^(1/3),x)
```

3.65.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 163 vs. 2(23) = 46.

Time = 0.31 (sec) , antiderivative size = 163, normalized size of antiderivative = 5.82

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$$
$$= \frac{6\,\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)\left(15+10(a+bx)\arctan(a+bx)+\frac{4(a+bx)\arctan(a+bx)\,\text{Hypergeometric2F1}\left(1,\frac{4}{3},\frac{11}{6},\frac{1}{1+(a}\right.}{1+(a+bx)^2}\right.}{20b\sqrt[3]{1+a^2+2abx+b^2x^2}\,\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)}$$

input
```
Integrate[ArcTan[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

output
```
(6*Gamma[11/6]*Gamma[7/3]*(15 + 10*(a + b*x)*ArcTan[a + b*x] + (4*(a + b*x
)*ArcTan[a + b*x]*Hypergeometric2F1[1, 4/3, 11/6, (1 + (a + b*x)^2)^(-1)])
/(1 + (a + b*x)^2)) + (5*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1,
 4/3, 4/3}, {11/6, 7/3}, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2))/(20*b
*(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3)*Gamma[11/6]*Gamma[7/3])
```

3.65.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5578, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{a^2+2abx+b^2x^2+1}}dx$$
$$\Big\downarrow \text{5578}$$
$$\frac{\int \frac{\arctan(a+bx)}{\sqrt[3]{(a+bx)^2+1}}d(a+bx)}{b}$$
$$\Big\downarrow \text{5560}$$
$$\frac{\int \frac{\arctan(a+bx)}{\sqrt[3]{(a+bx)^2+1}}d(a+bx)}{b}$$

input
```
Int[ArcTan[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

output
```
$Aborted
```

3.65.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

rule 5578
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_) + (
C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2)
^q*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
p, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.65.4 Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\arctan(bx+a)}{(b^2x^2+2abx+a^2+1)^{\frac{1}{3}}}dx$$

input
```
int(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

output
```
int(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

3.65.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\arctan(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="fricas
")
```

output
```
integral(arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

3.65.6 Sympy [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{atan}(a+bx)}{\sqrt[3]{a^2+2abx+b^2x^2+1}}\,dx$$

input
```
integrate(atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/3),x)
```

output
```
Integral(atan(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3), x)
```

3.65.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\arctan(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="maxima
")
```

output
```
integrate(arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

3.65.8 Giac [N/A]

Not integrable

Time = 56.78 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.11

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\arctan(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="giac")
```

output
```
sage0*x
```

3.65.9 Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\mathrm{atan}(a+b\,x)}{{(a^2+2\,a\,b\,x+b^2\,x^2+1)}^{1/3}}\,dx$$

input
```
int(atan(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3),x)
```

output
```
int(atan(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3), x)
```

3.66 $\int \frac{\arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$

3.66.1 Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\text{Int}\left(\frac{\arctan(a+bx)}{\sqrt[3]{c+c(a+bx)^2}},x\right)$$

output
```
Unintegrable(arctan(b*x+a)/(c+c*(b*x+a)^2)^(1/3),x)
```

3.66.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 165 vs. 2(25) = 50.

Time = 0.15 (sec) , antiderivative size = 165, normalized size of antiderivative = 5.00

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$$
$$=\frac{6\,\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)\left(15+10(a+bx)\arctan(a+bx)+\frac{4(a+bx)\arctan(a+bx)\,\text{Hypergeometric2F1}\left(1,\frac{4}{3},\frac{11}{6},\frac{\ }{1+(a}\right.}{1+(a+bx)^2}\right.}{20b\sqrt[3]{c\,(1+a^2+2abx+b^2x^2)}\,\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)}$$

input
```
Integrate[ArcTan[a + b*x]/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/3),x]
```

output
```
(6*Gamma[11/6]*Gamma[7/3]*(15 + 10*(a + b*x)*ArcTan[a + b*x] + (4*(a + b*x
)*ArcTan[a + b*x]*Hypergeometric2F1[1, 4/3, 11/6, (1 + (a + b*x)^2)^(-1)])
/(1 + (a + b*x)^2)) + (5*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1,
 4/3, 4/3}, {11/6, 7/3}, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2))/(20*b
*(c*(1 + a^2 + 2*a*b*x + b^2*x^2))^(1/3)*Gamma[11/6]*Gamma[7/3])
```

3.66.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5578, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{(a^2+1)\,c+2abcx+b^2cx^2}}\,dx$$

$$\Big\downarrow \ 5578$$

$$\frac{\int \frac{\arctan(a+bx)}{\sqrt[3]{c(a+bx)^2+c}}d(a+bx)}{b}$$

$$\Big\downarrow \ 5560$$

$$\frac{\int \frac{\arctan(a+bx)}{\sqrt[3]{c(a+bx)^2+c}}d(a+bx)}{b}$$

input
```
Int[ArcTan[a + b*x]/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/3),x]
```

output
```
$Aborted
```

3.66.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

rule 5578
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_) + (
C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subst[Int[(C/d^2 + (C/d^2)*x^2)
^q*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
p, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.66.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{\arctan(bx+a)}{\left((a^2+1)\,c+2abcx+b^2c\,x^2\right)^{\frac{1}{3}}}dx$$

input
```
int(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

output
```
int(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

3.66.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\arctan(bx+a)}{\left(b^2cx^2+2\,abcx+(a^2+1)c\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x, algorithm
="fricas")
```

output
```
integral(arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

3.66.6 Sympy [N/A]

Not integrable

Time = 6.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\operatorname{atan}{(a+bx)}}{\sqrt[3]{c\,(a^2+2abx+b^2x^2+1)}}\,dx$$

input
```
integrate(atan(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/3),x)
```

output
```
Integral(atan(a + b*x)/(c*(a**2 + 2*a*b*x + b**2*x**2 + 1))**(1/3), x)
```

3.66.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\arctan{(bx+a)}}{(b^2cx^2+2\,abcx+(a^2+1)c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x, algorithm
="maxima")
```

output
```
integrate(arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

3.66.8 Giac [N/A]

Not integrable

Time = 59.74 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.09

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\arctan{(bx+a)}}{(b^2cx^2+2\,abcx+(a^2+1)c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x, algorithm
="giac")
```

output
```
sage0*x
```

3.66.9 Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\int \frac{\mathrm{atan}(a+b\,x)}{{\left(c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)\right)}^{1/3}}\,dx$$

input
```
int(atan(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3),x)
```

output
```
int(atan(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3), x)
```

3.67 $\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

3.67.1 Optimal result

Integrand size = 35, antiderivative size = 187

$$\begin{aligned}\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = &-\frac{\sqrt{1+(a+bx)^2}}{2b} \\ &+\frac{(a+bx)\sqrt{1+(a+bx)^2}\arctan(a+bx)}{2b} \\ &+\frac{i\arctan(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} \\ &-\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b} \\ &+\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b}\end{aligned}$$

output
```
I*arctan(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b-1/2*I*po
lylog(2,-I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b+1/2*I*polylog(2,I*(1
+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b-1/2*(1+(b*x+a)^2)^(1/2)/b+1/2*(b*
x+a)*arctan(b*x+a)*(1+(b*x+a)^2)^(1/2)/b
```

3.67.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$$
$$= \frac{-\sqrt{1+(a+bx)^2}+(a+bx)\sqrt{1+(a+bx)^2}\arctan(a+bx)-\arctan(a+bx)\log\left(1-ie^{i\arctan(a+bx)}\right)+\text{a}}{2b}$$

input
```
Integrate[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],
x]
```

output
```
(-Sqrt[1 + (a + b*x)^2] + (a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcTan[a + b*x]
- ArcTan[a + b*x]*Log[1 - I*E^(I*ArcTan[a + b*x])] + ArcTan[a + b*x]*Log[1
 + I*E^(I*ArcTan[a + b*x])] - I*PolyLog[2, (-I)*E^(I*ArcTan[a + b*x])] + I
*PolyLog[2, I*E^(I*ArcTan[a + b*x])])/(2*b)
```

3.67.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {5580, 5487, 241, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx$$
$$\Big\downarrow \text{5580}$$
$$\frac{\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)}{b}$$
$$\Big\downarrow \text{5487}$$
$$\frac{-\frac{1}{2}\int \frac{\arctan(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)-\frac{1}{2}\int \frac{a+bx}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}(a+bx)\sqrt{(a+bx)^2+1}\arctan(a+bx)}{b}$$
$$\Big\downarrow \text{241}$$
$$\frac{-\frac{1}{2}\int \frac{\arctan(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}(a+bx)\sqrt{(a+bx)^2+1}\arctan(a+bx)-\frac{1}{2}\sqrt{(a+bx)^2+1}}{b}$$

$$\Big\downarrow \text{5421}$$

$$\frac{\frac{1}{2}\left(2i\arctan(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)-i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)+i\operatorname{PolyLog}\left(2,\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)\right)+\frac{1}{2}(a+b}{b}$$

input
```
Int[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

output
```
(-1/2*Sqrt[1 + (a + b*x)^2] + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcTan[a +
b*x])/2 + ((2*I)*ArcTan[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(
a + b*x)]] - I*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x
)]] + I*PolyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/2)/b
```

3.67.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5580
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTan[x])
^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &
& EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.67.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.96

method	result
default	$\frac{(\arctan(bx+a)bx+a\arctan(bx+a)-1)\sqrt{b^2x^2+2abx+a^2+1}}{2b}+\frac{\arctan(bx+a)\ln\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)-\arctan(bx+a)\ln\left(1-\frac{i(1+i(bx+a}{\sqrt{1+(bx+a}}\right.}{2b}$

input
```
int((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
1/2*(arctan(b*x+a)*b*x+a*arctan(b*x+a)-1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b+
1/2*(arctan(b*x+a)*ln(1+I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-arctan(b*x+a)
*ln(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-I*dilog(1+I*(1+I*(b*x+a))/(1+(b
*x+a)^2)^(1/2))+I*dilog(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)))/b
```

3.67.5 Fricas [F]

$$\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2\arctan(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

input
```
integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorit
hm="fricas")
```

output
```
integral((b^2*x^2 + 2*a*b*x + a^2)*arctan(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1), x)
```

3.67.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**2*atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

output
```
Timed out
```

3.67.7 Maxima [F]

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2 \arctan(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

input
```
integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate((b*x + a)^2*arctan(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x
)
```

3.67.8 Giac [F]

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2 \arctan(bx+a)}{\sqrt{b^2x^2+2\,abx+a^2+1}}\,dx$$

input
```
integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorit
hm="giac")
```

output
```
sage0*x
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\operatorname{atan}(a+b\,x)\,(a+b\,x)^2}{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\,dx$$

input
```
int((atan(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

output
```
int((atan(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

3.68 $\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}}\,dx$

3.68.1 Optimal result

Integrand size = 40, antiderivative size = 281

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = -\frac{\sqrt{c+c(a+bx)^2}}{2bc} + \frac{(a+bx)\sqrt{c+c(a+bx)^2}\arctan(a+bx)}{2bc} + \frac{i\sqrt{1+(a+bx)^2}\arctan(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} - \frac{i\sqrt{1+(a+bx)^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c+c(a+bx)^2}} + \frac{i\sqrt{1+(a+bx)^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c+c(a+bx)^2}}$$

output
```
I*arctan(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+a)
^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)-1/2*I*polylog(2,-I*(1+I*(b*x+a))^(1/2)/(
1-I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)+1/2*I*poly
log(2,I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+
c*(b*x+a)^2)^(1/2)-1/2*(c+c*(b*x+a)^2)^(1/2)/b/c+1/2*(b*x+a)*arctan(b*x+a)
*(c+c*(b*x+a)^2)^(1/2)/b/c
```

3.68.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$$
$$= \frac{\sqrt{1+a^2+2abx+b^2x^2}\left(-\sqrt{1+(a+bx)^2}+(a+bx)\sqrt{1+(a+bx)^2}\arctan(a+bx)-\arctan(a+bx)\log\right.}{2b\sqrt{}}$$

input
```
Integrate[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2
*c*x^2],x]
```

output
```
(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(-Sqrt[1 + (a + b*x)^2] + (a + b*x)*Sqr
t[1 + (a + b*x)^2]*ArcTan[a + b*x] - ArcTan[a + b*x]*Log[1 - I*E^(I*ArcTan
[a + b*x])] + ArcTan[a + b*x]*Log[1 + I*E^(I*ArcTan[a + b*x])] - I*PolyLog
[2, (-I)*E^(I*ArcTan[a + b*x])] + I*PolyLog[2, I*E^(I*ArcTan[a + b*x])]))/
(2*b*Sqrt[c*(1 + a^2 + 2*a*b*x + b^2*x^2)])
```

3.68.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5580, 5487, 241, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt{(a^2+1)\,c+2abcx+b^2cx^2}}\,dx$$

$$\Big\downarrow \text{5580}$$

$$\frac{\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt{c(a+bx)^2+c}}d(a+bx)}{b}$$

$$\Big\downarrow \text{5487}$$

$$\frac{-\frac{1}{2}\int \frac{\arctan(a+bx)}{\sqrt{c(a+bx)^2+c}}d(a+bx)-\frac{1}{2}\int \frac{a+bx}{\sqrt{c(a+bx)^2+c}}d(a+bx)+\frac{(a+bx)\arctan(a+bx)\sqrt{c(a+bx)^2+c}}{2c}}{b}$$

$$\Big\downarrow \text{241}$$

$$\frac{-\frac{1}{2}\int \frac{\arctan(a+bx)}{\sqrt{c(a+bx)^2+c}}d(a+bx)+\frac{(a+bx)\arctan(a+bx)\sqrt{c(a+bx)^2+c}}{2c}-\frac{\sqrt{c(a+bx)^2+c}}{2c}}{b}$$

$$\Big\downarrow \text{5425}$$

$$\frac{-\frac{\sqrt{(a+bx)^2+1}\int \frac{\arctan(a+bx)}{\sqrt{(a+bx)^2+1}}d(a+bx)}{2\sqrt{c(a+bx)^2+c}}+\frac{(a+bx)\arctan(a+bx)\sqrt{c(a+bx)^2+c}}{2c}-\frac{\sqrt{c(a+bx)^2+c}}{2c}}{b}$$

$$\Big\downarrow \text{5421}$$

$$\frac{-\frac{\sqrt{(a+bx)^2+1}\left(-2i\arctan(a+bx)\arctan\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)+i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)-i\operatorname{PolyLog}\left(2,\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)\right)}{2\sqrt{c(a+bx)^2+c}}+\frac{(a+bx)\arctan(a+ba}{2c}}{b}$$

input
```
Int[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2
],x]
```

output
```
(-1/2*Sqrt[c + c*(a + b*x)^2]/c + ((a + b*x)*Sqrt[c + c*(a + b*x)^2]*ArcTa
n[a + b*x])/(2*c) - (Sqrt[1 + (a + b*x)^2]*((-2*I)*ArcTan[a + b*x]*ArcTan[
Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]] + I*PolyLog[2, ((-I)*Sqrt[1 +
 I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]] - I*PolyLog[2, (I*Sqrt[1 + I*(a + b*
x)])/Sqrt[1 - I*(a + b*x)]]))/(2*Sqrt[c + c*(a + b*x)^2]))/b
```

3.68.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5580
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTan[x])
^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &
& EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.68.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.79

method	result
default	$\frac{(\arctan(bx+a)bx+a\arctan(bx+a)-1)\sqrt{c(bx+a-i)(bx+a+i)}}{2bc}+\frac{\left(\arctan(bx+a)\ln\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)-\arctan(bx+a)\ln\left(1-\frac{i(1+i}{\sqrt{1+(}}\right.\right.}{2}$

input
```
int((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
1/2*(arctan(b*x+a)*b*x+a*arctan(b*x+a)-1)*(c*(-I+a+b*x)*(I+a+b*x))^(1/2)/b
/c+1/2*(arctan(b*x+a)*ln(1+I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-arctan(b*x
+a)*ln(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-I*dilog(1+I*(1+I*(b*x+a))/(1
+(b*x+a)^2)^(1/2))+I*dilog(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)))*(c*(-I+
a+b*x)*(I+a+b*x))^(1/2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b/c
```

3.68.5 Fricas [F]

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(bx+a)^2 \arctan(bx+a)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

input
```
integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,
 algorithm="fricas")
```

output
```
integral((b^2*x^2 + 2*a*b*x + a^2)*arctan(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*
c*x + (a^2 + 1)*c), x)
```

3.68.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**2*atan(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/2)
,x)
```

output
```
Timed out
```

3.68.7 Maxima [F]

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(bx+a)^2 \arctan(bx+a)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

input
```
integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,
 algorithm="maxima")
```

output
```
integrate((b*x + a)^2*arctan(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 +
1)*c), x)
```

3.68.8 Giac [F]

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(bx+a)^2 \arctan(bx+a)}{\sqrt{b^2cx^2+2\,abcx+(a^2+1)c}}\,dx$$

input
```
integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,
 algorithm="giac")
```

output
```
sage0*x
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\operatorname{atan}(a+b\,x)\,(a+b\,x)^2}{\sqrt{c\,b^2\,x^2+2\,a\,c\,b\,x+c\,(a^2+1)}}\,dx$$

input
```
int((atan(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2
),x)
```

output
```
int((atan(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2
), x)
```

3.69 $\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} \, dx$

3.69.1 Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} \, dx = \text{Int}\left(\frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{1+(a+bx)^2}}, x\right)$$

output
```
Unintegrable((b*x+a)^2*arctan(b*x+a)/(1+(b*x+a)^2)^(1/3),x)
```

3.69.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(30) = 60.

Time = 4.75 (sec) , antiderivative size = 181, normalized size of antiderivative = 5.17

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} \, dx =$$
$$-\frac{3(1+(a+bx)^2)^{2/3} \left(\frac{5\sqrt[3]{2}\sqrt{\pi}\,\text{Gamma}\left(\frac{5}{3}\right) {}_3F_2\left(1,\frac{4}{3},\frac{4}{3};\frac{11}{6},\frac{7}{3};\frac{1}{1+(a+bx)^2}\right)}{(1+(a+bx)^2)^2} + \text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)\left(15+\frac{90}{1+(a+bx)\ldots}\right.\right.}{140b\,\text{Gam}\ldots}$$

input
```
Integrate[((a + b*x)^2*ArcTan[a + b*x])/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3
),x]
```

output
```
(-3*(1 + (a + b*x)^2)^(2/3)*((5*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*Hypergeometric
PFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2)
^2 + Gamma[11/6]*Gamma[7/3]*(15 + 90/(1 + (a + b*x)^2) + (24*(a + b*x)*Arc
Tan[a + b*x]*Hypergeometric2F1[1, 4/3, 11/6, (1 + (a + b*x)^2)^(-1)])/(1 +
 (a + b*x)^2)^2 + 5*ArcTan[a + b*x]*(-4*(a + b*x) + 6*Sin[2*ArcTan[a + b*x
]]))))/(140*b*Gamma[11/6]*Gamma[7/3])
```

3.69.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5580, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{a^2+2abx+b^2x^2+1}} \, dx$$

$$\Big\downarrow \; 5580$$

$$\frac{\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{(a+bx)^2+1}} d(a+bx)}{b}$$

$$\Big\downarrow \; 5560$$

$$\frac{\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{(a+bx)^2+1}} d(a+bx)}{b}$$

input
```
Int[((a + b*x)^2*ArcTan[a + b*x])/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

output
```
$Aborted
```

3.69.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

rule 5580
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTan[x])
^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &
& EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.69.4 Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{(bx+a)^2 \arctan(bx+a)}{(b^2x^2+2abx+a^2+1)^{\frac{1}{3}}}dx$$

input
```
int((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

output
```
int((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

3.69.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2 \arctan(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorit
hm="fricas")
```

output
```
integral((b^2*x^2 + 2*a*b*x + a^2)*arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^
2 + 1)^(1/3), x)
```

3.69.6 Sympy [N/A]

Not integrable

Time = 3.72 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(a+bx)^2\operatorname{atan}(a+bx)}{\sqrt[3]{a^2+2abx+b^2x^2+1}}\,dx$$

input
```
integrate((b*x+a)**2*atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/3),x)
```

output
```
Integral((a + b*x)**2*atan(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3
), x)
```

3.69.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2\arctan(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorit
hm="maxima")
```

output
```
integrate((b*x + a)^2*arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3),
 x)
```

3.69.8 Giac [N/A]

Not integrable

Time = 172.49 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.09

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{(bx+a)^2 \arctan(bx+a)}{(b^2x^2+2\,abx+a^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorit
hm="giac")
```

output
```
sage0*x
```

3.69.9 Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \int \frac{\mathrm{atan}(a+b\,x)\,(a+b\,x)^2}{(a^2+2\,a\,b\,x+b^2\,x^2+1)^{1/3}}\,dx$$

input
```
int((atan(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3),x)
```

output
```
int((atan(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3), x)
```

3.70 $\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$

3.70.1 Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \text{Int}\left(\frac{(a+bx)^2\arctan(a+bx)}{\sqrt[3]{c+c(a+bx)^2}},x\right)$$

output
```
Unintegrable((b*x+a)^2*arctan(b*x+a)/(c+c*(b*x+a)^2)^(1/3),x)
```

3.70.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 225 vs. 2(32) = 64.

Time = 0.73 (sec) , antiderivative size = 225, normalized size of antiderivative = 5.62

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = -\frac{3\sqrt[3]{1+a^2+2abx+b^2x^2}\,(1+(a+bx)^2)^{2/3}\left(\frac{5\sqrt[3]{2}\sqrt{\pi}\,\text{Gamma}\left(\frac{5}{3}\right)\,{}_3F_2\left(1,\frac{4}{3},\frac{4}{3};\frac{11}{6},\frac{7}{3};\frac{1}{1+(a+bx)^2}\right)}{(1+(a+bx)^2)^2}+\text{Gamma}\left(\frac{11}{6}\right)\,\text{Gar}\cdots\right.}{140b\sqrt[3]{c\,(1\cdots}}$$

input
```
Integrate[((a + b*x)^2*ArcTan[a + b*x])/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x
^2)^(1/3),x]
```

output
```
(-3*(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3)*(1 + (a + b*x)^2)^(2/3)*((5*2^(1/3
)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + (
a + b*x)^2)^(-1)])/(1 + (a + b*x)^2)^2 + Gamma[11/6]*Gamma[7/3]*(15 + 90/(
1 + (a + b*x)^2) + (24*(a + b*x)*ArcTan[a + b*x]*Hypergeometric2F1[1, 4/3,
 11/6, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2)^2 + 5*ArcTan[a + b*x]*(-
4*(a + b*x) + 6*Sin[2*ArcTan[a + b*x]]))))/(140*b*(c*(1 + a^2 + 2*a*b*x +
b^2*x^2))^(1/3)*Gamma[11/6]*Gamma[7/3])
```

3.70.3 Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5580, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt[3]{(a^2+1)\,c+2abcx+b^2cx^2}}dx$$

$$\Big\downarrow \; 5580$$

$$\frac{\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt[3]{c(a+bx)^2+c}}d(a+bx)}{b}$$

$$\Big\downarrow \; 5560$$

$$\frac{\int \frac{(a+bx)^2\arctan(a+bx)}{\sqrt[3]{c(a+bx)^2+c}}d(a+bx)}{b}$$

input
```
Int[((a + b*x)^2*ArcTan[a + b*x])/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1
/3),x]
```

output
```
$Aborted
```

3.70.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

rule 5580
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/d   Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTan[x])
^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &
& EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

3.70.4 Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{(bx+a)^2 \arctan(bx+a)}{\left((a^2+1)\,c+2abcx+b^2c\,x^2\right)^{\frac{1}{3}}}dx$$

input
```
int((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

output
```
int((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

3.70.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(bx+a)^2 \arctan(bx+a)}{\left(b^2cx^2+2\,abcx+(a^2+1)c\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x,
 algorithm="fricas")
```

output
```
integral((b^2*x^2 + 2*a*b*x + a^2)*arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x
+ (a^2 + 1)*c)^(1/3), x)
```

3.70.6 Sympy [N/A]

Not integrable

Time = 26.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\left(a+bx\right)^2 \operatorname{atan}\left(a+bx\right)}{\sqrt[3]{c\left(a^2+2abx+b^2x^2+1\right)}}\,dx$$

input
```
integrate((b*x+a)**2*atan(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/3)
,x)
```

output
```
Integral((a + b*x)**2*atan(a + b*x)/(c*(a**2 + 2*a*b*x + b**2*x**2 + 1))**
(1/3), x)
```

3.70.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\left(bx+a\right)^2 \arctan\left(bx+a\right)}{\left(b^2cx^2+2\,abcx+(a^2+1)c\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x,
 algorithm="maxima")
```

output
```
integrate((b*x + a)^2*arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c
)^(1/3), x)
```

3.70.8 Giac [N/A]

Not integrable

Time = 173.56 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.08

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{(bx+a)^2 \arctan\left(bx+a\right)}{\left(b^2cx^2+2\,abcx+(a^2+1)c\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x,
 algorithm="giac")
```

output
```
sage0*x
```

3.70.9 Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \arctan(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a+b\,x)\,\left(a+b\,x\right)^2}{{\left(c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)\right)}^{1/3}}\,dx$$

input
```
int((atan(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3
),x)
```

output
```
int((atan(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3
), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```